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An Application of a Random Level Shifts Model to the Volatility of Peruvian Stock and Exchange Rate Returns

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Abstract

The literature has shown that the volatility of Stock and Forex rate market returns shows the characteristic of long memory. Another fact that is shown in the literature is that this feature may be spurious and volatility actually consists of a short memory process contaminated with random level shifts. In this paper, we follow the approach of Lu and Perron (2010) and Li and Perron (2013) estimating a model of random level shifts (RLS) to the logarithm of the absolute value of Stock and Forex returns. The model consists of the sum of a short term memory component and a component of level shifts. The second component is specified as the cumulative sum of a process that is zero with probability $1 - \alpha$ and is a random variable with probability α. The results show that there are level shifts that are rare but once they are taken into account, the characteristic or property of long memory disappears. Also, the presence of GARCH effects is eliminated when included or deducted level shifts. An exercise of out-of-sample forecasting shows that the RLS model has better performance than traditional models for modeling long memory such as the models ARFIMA (p,d q).

JEL Classification: C22

Keywords: Returns, Volatility, Long Memory, Random Level Shifts, Kalman Filter, Forecasting

Resumen

La literatura ha mostrado que la volatilidad de los retornos bursátiles y cambiarios muestra la característica de larga memoria. Otro hecho mostrado en la literatura es que dicha característica puede ser espúria y que en realidad la volatilidad está compuesta de un proceso de corta memoria contaminado con cambios de nivel aleatorios. En este documento, seguimos el enfoque de Lu y Perron (2010) y Li y Perron (2013) estimando un modelo de cambios de nivel aleatorios (RLS) al logaritmo del valor absoluto de los retornos bursátiles y cambiarios del Perú. El modelo consta de la suma de un componente de corta memoria y un componente de cambios de nivel. El segundo componente es especificado como la suma acumulada de un proceso que es cero con probabilidad $1 - \alpha$ y es una variable aleatoria con probabilidad α. Los resultados muestran que existen cambios de nivel que son infrecuentes pero una vez que son tomados en cuenta, la característica o propiedad de larga memoria desaparece. Asimismo, la presencia de efectos GARCH es eliminada cuando se incluyen o descuentan los cambios de nivel. Un ejercicio de predicción fuera de la muestra indica que el modelo RLS tiene mejor performance que modelos tradicionalmente utilizados para modelar larga memoria como los modelos ARFIMA(p,d,q).

Classificación JEL: C22

Palabras Claves: Retornos, Volatilidad, Larga Memoria, Cambios de Nivel Aleatorios, Filtro de Kalman, Forecasting

An Application of a Random Level Shifts Model to the Volatility of Peruvian Stock and Exchange Rate Returns[1]

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1 Introduction

A stylized fact frequently tackled by economists on the volatility of stock and foreign exchange (Forex) rate returns is the long memory behavior that they display[3]. However, in recent years it has been proposed that processes with occasional shifts may cast a behavior that can be mistakenly taken for long memory.

The concept of long memory or fractional integration applied to economics was first put forward by Granger and Joyeux (1980) and Hosking (1981). The former suggest that when we apply a $(1-L)^d$ filter, being L the lag operator, to a white noise, we get a series with long memory characteristics. In turn, Hosking (1981) also suggests a model that allows the fractional operator d to assume values between 0 and 1, to apply it to the well-known ARIMA(p,d,q) model, which leads to the model ARFIMA(p,d,q). In both cases, the value that d assumes in a range from 0 to 1 will be crucial for the modeling of long term persistence; if d assumes a value of $0.5 \leq d < 1$ we can make use of the fractional operator $(1-L)^d$, and generate series that would prove long memory for $0 < d < 0.5$. Geweke and Porter-Hudak (1983), based on a linear regression of the log-periodogram with a deterministic regressor, show that the asymptotic distribution of the long memory parameter d has a Normal distribution; see also Robinson (1995).

On the basis of these developments, new methods for modeling long-range dependence in volatility were introduced; this is the case of Baillie et al. (1996) and Bollerslev and Mikkelsen (1996), who presented the FIGARCH and FIEGARCH models respectively, where the conditional variance of the analyzed process decays at a hyperbolic or long dependence rate.

Continuing the research of long memory characteristics in the stock market, Ding et al. (1993) study the characteristics of the correlations of absolute returns. In particular, they observe that the transformation $|r_t|^\delta$ shows a high autocorrelation in distant lags, which implies long memory. To a greater extent, this occurs when δ is close to 1. Also, the authors use the S&P 500 end-of-day data for 1928 to 1991 in order to show a jump in the volatility for 1929 and 1930, providing evidence for a stylized fact of structural change brought on by the Great Depression.

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[3] Shocks to a series that has long memory will be more durable than a series with short memory. This difference is important to determine the impact of economic policy on the series analyzed.

From the second half of the 90s onward, emphasis was placed on the analysis of estimations previously carried out and on the possibility that these representations did not really show the behavior of returns and their volatility. Perron (1989, 1990) had shown that when the real data-generating process includes a structural break, the null hypothesis of a unit root cannot be rejected; i.e. processes of level shifts or structural changes can be mistaken with processes of random walk or with coefficients close to the unit. However, Teverovsky and Taqqu (1997) allow us to state that a method had been implemented to distinguish the effects of level shift from those of long memory. Lobato and Savin (1998) also evaluate the presence of long memory in the S&P500 stock returns, using a semi-parametric test with short memory as null hypothesis and long memory as alternative hypothesis. The results of the stock returns do not reject the null hypothesis by showing short memory. However, the squared returns do reflect long memory, showing a stronger effect for the returns in absolute value. Furthermore, Lobato and Savin (1998) divide the sample, taking 1973 as break point, and conclude that the series displays long memory as well as a structural change.

On the other hand, Gourieroux and Jasiak (2001) focus on studying the autocorrelogram estimation rather than the fractional integration parameter. They find that long memory characteristics can be found in non-linear series with infrequent breaks, which would suggest that the hyperbolic decay reflected in the autocorrelograms would not be due to the fractional dynamics of the series, but to the non-linear dynamics with infrequent regime-switching. This has implications for the variables to be analyzed, since if we use a fractional or long memory model, the prediction will depend on past information (distant lags); however, if we use a model with infrequent breaks, the prediction will depend on a moving-average process based on data from the same regime (short memory).

Following the same framework, Diebold and Inoue (2001) seek to shorten the relation between long memory studies and regime switching. They show that for simple mixture models, for the permanent stochastic breaks model by Engle and Smith (1999) and for Hamilton's Markov-Switching model (1989), the long memory estimation may be confused with stochastic regime-switching, which also occurs asymptotically and with few changes. Based on Monte Carlo simulations, the authors verify that under certain circumstances and with few structural changes in a particular period, i.e. with a low probability of change occurring, a long memory process and a structural change process may be confused. Diebold and Inoue (2001) use the estimator proposed by Geweke and Porter-Hudak (1983) and they show that as the probability of finding a break increases, the parameter value of $d \Rightarrow 1$.

Granger and Hyung (2004) hold that because occasional breaks generate slow decays in the autocorrelations, as well as other distinctive characteristics of long-memory processes, it is difficult to discern whether these characteristics are produced by long memory processes or by structural break processes. They use a daily sample for S&P500 returns for the years 1928 to 2002. Using the fractional parameter estimation method proposed by Geweke and Porter-Hudak (1983), they show that an occasional break model has a better adjustment than an I(d) model, when d is fractional. Moreover, it can be proven that at least part of the long memory effect is produced by changes in the series.

On the other hand, Mikosh and Stărică (2004a) seek to explain that long range dependence can be represented by non-stationary models. This is because the estimators used to deduce these aspects are not very helpful in identifying whether they are stationary, long-memory or non-stationary; therefore, certain shifts in the conditional variance may lead to confusions with the IGARCH models. In other work, Mikosh and Stărică (2004b) propose a goodness of fit test, which can verify that the logarithm of the returns can be modeled by a GARCH process. Furthermore, they asymptotically analyze the behavior of the test, concluding that it displays a distribution which depends on the variance of the variable to be explained. This would make it difficult to generate the goodness of fit procedure, since it requires total independence of the null hypothesis to be analyzed. By means of Monte Carlo simulations it can be concluded that the null hypothesis (GARCH modeling) is rejected and is related to the existence of shifts in the unconditional variance. It shows that a GARCH model is not a good representation for the afore-mentioned period of time. However, if the observations for four years from 1973 onward -related to the oil crisis- were to be left out, the ACF would show a short memory behavior. This leads to the conclusion that the long dependence behavior would be due to structural changes in the logarithm of the returns.

Meanwhile, Stărică and Granger (2005) analyze the behavior of non-stationary financial data by means of stationary models. They perform this for the S&P500 and find that the series is full of shifts in the unconditional variance. These shifts would explain the long memory characteristics, as when they are taken into account in the estimation, every trace of long dependence is eliminated.

One of the most recent works on the analysis of long memory and level shift or structural break characteristics is that of Perron and Qu (2010). They present a method that allows long memory to be distinguished from level shifts by means of studying the ACF, the periodogram and the fractional integration parameter d. Perron and Qu (2010) propose a simple mixture model that combines a short memory process and a component that reflects level shifts, the latter being determined by a variable of occurrence related to a Bernoulli process. Drawing on the logarithm of squared returns as a proxy for volatility and using four stock market indexes (S&P 500, NASDAQ, AMEX and Dow Jones), they conclude that the model that best describes volatility of returns is the one that considers a short memory process with level shifts.

Proceeding with the methodology of level shifts, Lu and Perron (2010) and Li and Perron (2013) model the volatility of stock market returns and of exchange rate returns, respectively. The former develop a model composed of a short memory process and a level shift component, which is defined by the cumulative sum of a process that is 1 with probability α or is 0 with probability $(1-\alpha)$; this is the so-called random level shifts (RLS) model. The method is applied in order to model the logarithm of the daily absolute returns for the S&P500, AMEX, Dow Jones and NASDAQ indexes. Once the probability of level shift occurrence for each series is estimated, the dates of the shifts for each series are found using the method developed by Bai and Perron (1998, 2003). By including the shifts in the series, every trace of long memory and conditional heteroskedasticity is eliminated. Meanwhile, Li and Perron (2013) use the same procedure but apply it to the volatilities of the dollar-mark and dollar-yen exchange rate returns. As in the case of stock market series, they find that the long memory behavior observed is because the series develops as a short memory process with level shifts.

Empirical studies applied to financial series in Peru are very scarce. Humala and Rodríguez (2013) discussed the stylized facts of stock and exchange rate markets using daily, weekly and monthly data. From this paper, a work agenda related to different aspects of the returns and volatilities of the two markets is emphasized. This study forms part of that empirical agenda. Recent work related to long memory for Peru was undertaken by Herrera Aramburú and Rodríguez (2014). From a testing perspective, the authors attempt to discern whether the volatilities of these markets are characterized as a long memory process or a short memory process with level shifts. The authors follow Perron and Qu (2010) and despite the graphic evidence, the test results are inconclusive. Slightly more conclusive results are obtained by Pardo Figueroa and Rodríguez

(2014) using stock market volatilities of several Latin American countries and a larger number of tests, and their conclusion is more in favor of a short memory process with level shifts. In this paper, we follow Lu and Perron (2010) and Li and Perron (2013) and apply a RLS model to the volatilities in both markets. Although we have fewer observations compared to developed countries, our results are quite conclusive, as in Lu and Perron (2010). These results can be summarized as follows: (i) the probability of level shifts is small but is responsible for the presence of long memory in the volatilities of the analyzed series. Having estimated the probability of level shifts, the exact number of such level shifts can be calculated. Thus, the component obtained as a subtraction between volatility and the level shift component has an ACF indicating no long memory; (ii) estimates of autoregressive conditional heteroskedasticity models discounting level shifts show that these components are artificially introduced by the level shifts; (iii) estimates of fractional models for the discounted level shift series show that the fractional parameter is near to zero; (iv) the performance of the RLS model in terms of forecast is better in comparison with standard ARFIMA(p,d,q) models.

This paper is structured as follows. Section 2 presents the model and some description of the estimation method. Section 3 describes and analyzes the empirical results, which are divided into the effects of the level shifts on the long memory behavior and the effects of the level shifts on the GARCH behavior. Section 4 discusses the performance of the RLS model in terms of forecasting. Finally in the Section 5, the conclusions are presented.

2 The Model

To perform this study we employed a simple mixture model, which is the combination of a short memory process and a level shift component that depends on a binomial distribution. We describe the RLS model using the notation of Lu and Perron (2014). It is specified as follows:

$$\begin{aligned} y_t &= a + \tau_t + c_t, \qquad (1) \\ \tau_t &= \tau_{t-1} + \delta_t, \\ \delta_t &= \pi_t \eta_t, \end{aligned}$$

where a is a constant, τ_t is the level shift component and c_t is the short-memory process, π_t is a binomial variable, which assumes the value 1 with a probability α and the value 0 with probability $(1-\alpha)$. Therefore, for the third equation in (1), when π_t assumes the value 1, a random level shift η_t occurs with a distribution $\eta_t \sim i.i.d.N(0, \sigma_\eta^2)$. The short-memory process (in its most general form) is defined by the process $c_t = C(L)e_t$, being $e_t \sim i.i.d.\ N(0, \sigma_e^2)$ and $E|e_t|^r < \infty$ for values $r > 2$ and where $C(L) = \sum_{i=0}^{\infty} c_i L^i$, $\sum_{i=0}^{\infty} i|c_i| < \infty$ y $C(1) \neq 0$. Moreover, it is assumed that π_t, η_t and c_t are mutually independent. Based on the results by Lu and Perron (2010) and Li and Perron (2013), even if it were considered best to propose the e_t component as a random variable, in this study we model it as originally proposed in the afore-mentioned work, i.e. as a AR(1) process: $c_t = \phi c_{t-1} + e_t$.

In comparison with Hamilton's Markov regime switching model (1989), this model does not limit the magnitude of level shifts, so that any number of regimes is possible. Also, the probability 0 or 1 does not depend on past facts, whereas the Markov switching models do. Note that the δ_t process may be described as $\delta_t = \pi_t \eta_{1t} + (1-\pi_t)\eta_{2t}$, with $\eta_{it} \sim i.i.d.N(0, \sigma_{\eta_i}^2)$ for $i = 1, 2$ and $\sigma_{\eta_1}^2 = \sigma_\eta^2$, $\sigma_{\eta_2}^2 = 0$. The model in first differences, in order to eliminate the autoregressive process of the level shift component, only depends on the binomial process: $\Delta y_t = \tau_t - \tau_{t-1} + c_t - c_{t-1} = c_t - c_{t-1} + \delta_t$. In addition, by focusing on the space-state form, we obtain the measure equation and the transition equation respectively: $\Delta y_t = c_t - c_{t-1} + \delta_t$, $c_t = \phi c_{t-1} + e_t$. In the matrix form, we have $\Delta y_t = HX_t + \delta_t$and $X_t = FX_{t-1} + U_t$, where, $X_t = [c_t, c_{t-1}]$, $F = \begin{bmatrix} \phi & 0 \\ 1 & 0 \end{bmatrix}$, $H = [1, -1]'$. In this case, the first row of the matrix F presents the coefficient ϕ of the autoregressive component of the short memory process. Furthermore, U is a 2 dimensional Normally distributed vector with a mean of 0 and a variance: $Q = \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & 0 \end{bmatrix}$. In comparison with a standard state-space model, the important difference in the current model is that the distribution of δ_t is a mixture of two Normal distributions with variances σ_η^2 and 0, occurring with probabilities α and $1 - \alpha$, respectively[4].

The model described above is a special case of models considered in Wada and Perron (2006) and Perron and Wada (2009). Here, we only have shocks affecting the level of the series, and we need to impose the restriction that the variance of one component of the mixture of normal distributions is zero. The basic input for estimation is the augmentation of the states by the realizations of the mixture at time t so that the Kalman filter may be used to create the likelihood function, conditional on the realizations of the states. The latent states are eliminated from the final likelihood expression by summing over all possible state realizations. Therefore, despite their fundamental differences, the model takes a structure that is similar to the Markov-Switching model of Hamilton (1994). Let $Y_t = (\Delta y_1, ..., \Delta y_t)$ be the vector of available data up to time t and denote the vector of parameters by $\theta = [\sigma_\eta^2, \alpha, \sigma_e^2, \phi]$. Adopting the notation used in Hamilton (1994), $\mathbf{1}$ represents a (4×1) vector of ones, the symbol $\odot$ denotes element-by-element multiplication, $\widehat{\xi}_{t|t-1}^{ij} = vec(\widetilde{\xi}_{t|t-1})$ with the (i, j)th element of $\widetilde{\xi}_{t|t-1}$ being $\Pr(s_{t-1} = i, s_t = j|Y_{t-1}; \theta)$ and $\omega_t = vec(\widetilde{\omega}_t)$ with the (i, j)th element of $\widetilde{\omega}_t$ being $f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}; \theta)$ for $i, j \in \{1, 2\}$. Here $s_t = 1$ when $\pi_t = 1$; that is, a level shift occurs. Therefore, using the same notation as in Lu and Perron (2010), the log-likelihood function is $\ln(L) = \sum_{t=1}^T \ln f(\Delta y_t|Y_{t-1}; \theta)$,where $f(\Delta y_t|Y_{t-1}, \theta) = \sum_{i=1}^2 \sum_{j=1}^2 f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}, \theta) \Pr(s_{t-1} = i, s_t = j|Y_{t-1}, \theta) \equiv \mathbf{1}'(\widehat{\xi}_{t|t-1} \odot \omega_t)$. Applying rules for conditional probabilities, the rule of Bayes and the independence of s_t with past realizations, we have $\widetilde{\xi}_{t|t-1}^{ki} = \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1}; \theta)$. The evolution of $\widehat{\xi}_{t|t-1}$ can be expressed as:

$$\begin{bmatrix} \widetilde{\xi}_{t+1|t}^{11} \\ \widetilde{\xi}_{t+1|t}^{21} \\ \widetilde{\xi}_{t+1|t}^{12} \\ \widetilde{\xi}_{t+1|t}^{22} \end{bmatrix} = \begin{bmatrix} \alpha & \alpha & 0 & 0 \\ 0 & 0 & \alpha & \alpha \\ 1-\alpha & 1-\alpha & 0 & 0 \\ 0 & 0 & 1-\alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} \widetilde{\xi}_{t|t}^{11} \\ \widetilde{\xi}_{t|t}^{21} \\ \widetilde{\xi}_{t|t}^{12} \\ \widetilde{\xi}_{t|t}^{22} \end{bmatrix},$$

which is equal to $\widehat{\xi}_{t+1|t} = \Pi \widehat{\xi}_{t|t}$ with $\widehat{\xi}_{t|t} = \frac{(\widehat{\xi}_{t|t-1} \odot \omega_t)}{\mathbf{1}'(\widehat{\xi}_{t|t-1} \odot \omega_t)}$. Consequently, the conditional likelihood

[4] Note that the model may be extended to have the short-memory component follow an ARMA process.

function for Δy_t follows the Normal density:

$$\widetilde{\omega}_t^{ij} = f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}} |f_t^{ij}|^{-1/2} \exp(-\frac{v_t^{ij\prime}(f_t^{ij})^{-1/2} v_t^{ij}}{2}), \tag{2}$$

where v_t^{ij} is the forecast error and f_t^{ij} is its variance and they are defined as: $v_t^{ij} = \Delta y_t - \Delta y_{t|t-1}^i = \Delta y_t - E[\Delta y_t | s_t = i, Y_{t-1}; \theta]$, and $f_t^{ij} = E(v_t^{ij} v_t^{ij\prime})$. The best predictions for the state variable and the respective conditional variance at $s_{t-1} = i$ are $X_{t|t-1}^i = F X_{t-1|t-1}^i$, and $P_{t|t-1}^i = F P_{t-1|t-1}^i F' + Q$, respectively.

The measurement equation is $\Delta y_t = H X_t + \delta_t$, where the error δ_t has a mean of 0 and a variance which can assume the values $R_1 = \sigma_\eta^2$ with probability α or $R_2 = 0$ with probability $(1-\alpha)$. Then, the prediction error is $v_t^{ij} = \Delta y_t - H X_{t|t-1}^i$ and its variance is $f_t^{ij} = H P_{t|t-1}^i H' + R_j$. Then, given $s_t = j$ and $s_{t-1} = i$ and using the updating formulae, we have $X_{t|t}^{ij} = X_{t|t-1}^i + P_{t|t-1}^i H'(H P_{t|t-1}^i H' + R_j)^{-1}(\Delta y_t - H X_{t|t-1}^i)$ and $P_{t|t-1}^{ij} = P_{t|t-1}^i - P_{t|t-1}^i H'(H P_{t|t-1}^i H' + R_j)^{-1} H P_{t|t-1}^i$. In order to reduce the dimension of the estimation problem, Lu and Perron (2010) use the re-collapsing procedure suggested by Harrison and Stevens (1976). In so doing, ω_t^{ij} is unaffected by the history of states before time $t-1$. We have four possible states corresponding to $S_t = 1$ when $(s_t = 1, s_{t-1} = 1)$, $S_t = 2$ when $(s_t = 1, s_{t-1} = 2)$, $S_t = 3$ when $(s_t = 2, s_{t-1} = 2)$ and $S_t = 4$ when $(s_t = 2, s_{t-1} = 2)$ and the matrix Π as defined before. Observing the definition of ω_t, $\widehat{\xi}_{t|t}$, $\widehat{\xi}_{t+1|t}$, the collection of conditional probabilities and its one-period ahead forecasts, we have the same structure as a version of a Markov regime switching model used in Hamilton (1994). However, the EM algorithm does not apply. It is because the mean and variance in the conditional density function are non-linear functions of the fundamental parameters θ and past realizations $\{\Delta y_{t-j}; j \geq 1\}$. Furthermore, the conditional probability of being in a given regime $\widehat{\xi}_{t|t}$ is not separable from the conditional densities ω_t. For further details, see Lu and Perron (2010), Li and Perron (2013) and Wada and Perron (2006).

Once we have the point estimate of α, a possible approach would to use a smoothed estimate of the level shift component τ_t. In the current context of abrupt changes, the conventional smoother may performs poorly. Instead, we use the approach of Bai and Perron (1998, 2003) to obtain the dates on which the level shifts occurs as well as the means within each segment. Therefore we use the estimate of α to obtain a point estimate of the number of shifts, and we use Bai and Perron (1998, 2003) to obtain the estimates of the break dates that globally minimize the following sum of squared residuals $\sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [y_t - \mu_i]^2$, where m is the number of breaks, T_i $(i = 1, 2, ...; m)$ are the break dates with $T_0 = 0$ and $T_{m+1} = T$ and μ_i $(i = 1, 2, ..., m+1)$ are the means within each regime, which may be estimated once the break dates are estimated. This method is efficient and can handle a large number of observations; see Bai and Perron (2003) for further details[5].

3 Empirical Results

We use two series of daily data: the returns volatility series of the Lima Stock Exchange General Index and the returns volatility of the exchange rate. The stock series covers 03/01/1990 to

[5] Note that since the model allows for consecutive level shifts, we set the minimal length of a segment to just one observation.

13/06/2013 (5,832 observations) and the exchange rate series covers 03/01/1997 to 24/06/2013 (4,111 observations). The returns are generated as $r_t = \ln(P_t) - \ln(P_{t-1})$, where P_t are closed stock index or the exchange rate. Then, the volatility variable is $y_t = \ln(|r_t| + 0.001)$. Figure 1 shows the returns for stock returns (upper panel) and exchange rate returns (bottom panel) while Table 1 shows the descriptive statistics for the stock and the exchange rate returns and volatilities. In Figure 1 we observe the same high variation grouping of the returns at times of international and local crisis, and the effect of the last Greek debt crisis is also added. The statistics values show a mean value very close to 0 for both series. On the other hand, the standard deviation differs, being almost nine times larger in the case of the stocks. This is also observed in the previously mentioned Figure: the variation around the mean is much smaller in the case of the exchange rate than in the stocks, except for the time of the subprime crisis[6]. Both series display positive skewness, albeit very small ones. The kurtosis values show results that are consistent with those of the standard variation: the value of these statistics is larger than that of the stock exchange rate, which would show that the data in this series have a deviation that is smaller than the mean. The Jarque-Bera (JB) statistic signals that the distribution of these returns is very distant from a Normal distribution.

The descriptive statistics for the volatility (Table 1) show a mean of -4.858 for the stock index and of -6.192 for the exchange rate. Furthermore, the standard deviation is still too close to 0, being smaller - just as the in the returns - in the case of the exchange rate. The kurtosis is close to 3, which is consistent with the smaller value displayed by the JB test. However, it still cannot be said that distribution for both series is Normal. Figure 2 shows the ACF for both series of volatilities for 2,000 lags. In both cases, the evidence of long memory is clear.

3.1 Effects of Level Shifts on the Long Memory and ARFIMA Models

We first discuss the results for the stock volatility. The estimation results are presented in Table 2[7]. Every estimated coefficient is significant, especially the probability of level shifts and the coefficient of the autoregressive component, which would suggest that the stock series is well modeled by mixing random level shifts and a short memory process. The estimates of the standard deviations (level shift and short-memory components) are higher for the stock volatilities compared to those of the exchange rate volatility. Taking into consideration the estimated probability and the number of observations that were used, we find 26 shifts in stock volatility. This value indicates that level shifts are rare and occur with a duration of 216 days on average[8]. On the other hand, as can be seen in Table 2, the value of the autoregressive coefficient of the short memory component (ϕ) is small; this is also the case in the work of Lu and Perron (2010) and Li and Perron (2013). However, as has been previously mentioned, it is significant in the case of Peru, which means that it cannot be removed from the modeling of the volatility series.

[6] This may be due to the interventions of the Central Reserve Bank of Peru (BCRP) in the exchange rate market with the aim of reducing volatility.

[7] Given that all components of the state vector are stationary, we initialize the state vector and its covariance matrix by their unconditional expected values: $X_{0|0} = (0,0)'$ and $P_{0|0} = \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & 0 \end{bmatrix}$. In order to avoid the issue of local maximum, we re-estimate the model using a large set of random initial values and we select the estimates related with the largest likelihood value upon convergence.

[8] Observing the distribution of the level shifts, we find that the minimum occurrence of the level shifts is 3 days and their maximum is 1715 days.

Figure 3 presents the series of the smoothed level shift component and the level shift series with dates and regimes estimated using the method of Bai and Perron (1998, 2003). As can be seen, the smoothed estimates are quite erratic, though they generally follow the overall changing mean of the series as depicted using the method of Bai and Perron (1998, 2003). In Figure 3 we notice certain dates of level shifts that are very clear. Moreover, it displays groups of these shift dates where the volatility has undergone strong variations in short periods of time. For instance, the shifts observed in the years 1990 and 1991 reflect the political and economic changes that the country went through, when the period of hyperinflation in the 1980s was over and Alberto Fujimori was elected president. The first economic action that had an impact on the Peruvian economy was the so-called *"Fujishock"* in August 1990, which also generated instability in the Peruvian stock market. This action is reflected as level shift in the solid line that represents that same month in 1990. Furthermore, we observe a level shift in 1991, which reflects the impact of the change of currency that same year and displays lower volatility for this period.

In 1995 the Peruvian stock market was strongly affected by the Mexican crisis of the end of 1994. This impact was due to the lack of confidence of foreign investors in Latin-American markets, which decreased capital flows and funding for companies. Moreover, the conflict with Ecuador (the *Cenepa* War) and the uncertainty of the presidential election were national events that had an effect. The level shift observed in 1998 is due to the instability triggered by the Asian crisis of the second half of 1997, as well as the subsequent uncertainty related to the Russian crisis, which affected the countries that depended on the commodity prices -as was the case of Peru.

We can also note a group of level shifts posted from 2006 to the end of 2009. In 2006, as is the case of the majority of electoral times, the uncertainty in the stock market increased, which led to a level shift that disappeared as soon as the electoral period was over. In 2007 we observe level shifts, prompted by the mortgage crisis in the USA and the decrease in the price of metals, which led to periods of high volatility in Peru as in other countries. In 2008, while the effects of the international financial crisis continued, the Peruvian stock market faced periods of high volatility and negative profitability. In 2009 the Peruvian stock market entered a period of recovery, which is reflected in a level shift towards less volatility.

In the 2011 a level shift is also observed for the Stock returns volatility, which can be explained by external and internal factors. As external factors, we can cite the uncertainty generated by the international crisis and the risk of Greece leaving the Eurozone, which could also be said of Italy, Spain and Portugal. As internal factors, a period of high volatility occurred due to the presidential elections. However, this effect thinned out during the second half of the year.

It was previously stated that the level shifts might be the cause of the long memory detected in the volatility series. . The Figure 4 shows the ACF (top panel) of the original series minus the level shift process, leading to the observation that the long memory effect has disappeared. The same occurs in the cases where the level shift component smoothed by the Gaussian kernel and the series of volatility means estimated under the method of Bai and Perron(1998, 2003) are removed from the original volatility process. Therefore, it can be stated that by applying the RLS model to the stock return volatility series, the long memory component displayed in Figure 2 disappears completely, reflecting the short memory behavior that this series actually displays.

Observing the results for exchange rate volatility, Table 2 suggests the presence of 75 level shifts. This value indicates that level shifts are rare but more frequent than the stock market. Now, level shifts occur with a duration of 54 days on average[9].The interventions of the BCRP to

[9] Observing the distribution of the level shifts, we find that the minimum occurrence of the level shifts is 2 days

control exchange rate volatility could explain the larger number of shifts found in this series; i.e. when a shock in the volatility arises the BCRP will intervene in order to control it, which will generate a double level shift in the series[10].

Therefore, if these crashes are due to external events that are not controlled by the BCRP, the influence will be constant, producing more level shifts whenever the BCRP intervenes. Once the number of level shifts is found for each series, their dates are estimated using Bai and Perron's method (1998, 2003). By means of the identified dates of the level shifts, we calculate the specific measures of the regimes in order to find the short memory component.

The bottom panel of Figure 3 shows that exchange rate return volatility presents a conglomeration of very strong level shifts for 1997 and 1999. In the first half of 1998, the high volatility behavior observed was produced by the remainder of the uncertainty caused by the Asian crisis. Then, in the second half-year, the financial turbulence escalated due to the crisis in Russia. Short term credit lines were reduced, which led to a depreciation of the Peruvian Nuevo Sol. Nevertheless, in order to alleviate the trend of depreciation and in accordance with the system of exchange flotation with interventions, the BCRP took actions to reduce the marginal reserve ratio in foreign currency from 45% in July to 20% in December and sold \$82.6 million in foreign currency through the BCRP's *Mesa de Negociación*.

Similarly, in 1999 the conditions associated with adverse developments in the terms of trade, as well as the reduction of short term capital flows brought about by the preceding crisis, led to the level shifts observed in the Figure of that year. For 2001, 2005 and 2006, and in every election year, level shifts occurred, reflecting the uncertainty that accompanies an electoral process. Furthermore, in 2006 the greater volatility in the capital flow of emerging economies also had an influence. When facing this situation, the BCRP carried out interventions by buying and selling foreign currency in order to regulate the volatility of the exchange rate. The largest number of transactions were recorded in 2006 due to the appreciatory trend of the exchange rate, which led to purchases of \$ 4,237 million in the second half-year compared with \$ 62 million in the first half-year.

From the second half of 2007 to the first half of 2009, a set of level shifts is observed in Figure 3, caused by the decrease in the terms of trade -due mainly to the international financial crisis. Over these years the BCRP accumulated international reserves by means of purchase operations; this led to an increase in international reserves from \$27,689 million in 2007 to \$33,135 million in 2009. For 2011, in addition to the uncertainty caused by the electoral process, there were difficulties in solving the Greek debt problem and in formulating rescue plans for other European economies. In this period the BCRP accumulated reserves of \$ 4,711 million, reaching a total of \$48,816 million by the end of the year.

As was performed for the stock returns volatility, the ACF was estimated for the original volatility series set out in the first part of the research. The autocorrelation was also estimated after having eliminated the level shift component (bottom panels of Figures 2 and 4, respectively). Here we can see that the evidence of long memory disappears when these breaks have been excluded[11].

Complementarily to the analysis, the volatility series and the short memory component (measured as the volatility minus the component of level shifts as estimated by Bai and Perron (1998, 2003)) were modeled using ARFIMA(0,d,0) and ARFIMA(1,d,1)[12]. In the case of the ARFIMA(0,d,0),

and their maximum is 358 days.

[10] A similar conclusion, but using another approach, is suggested by Beine and Laurent (2003).

[11] Same results are obtained if we use the smoothed (Gaussian kernel) estimate of the level shift component.

[12] Results are available upon request.

the results for the volatility of stock returns show values that reflect the results presented previously, together with the autocorrelations. Concerning the fractional parameter (d), the volatility shows a long memory behavior as a stylized fact in the series. However, the short memory component presents a $\widehat{d}$ parameter with a positive but very low value to involve long memory.

The results are very similar in the case of the ARFIMA(1,d,1). The volatility shows a positive and significant fractional parameter, and the ϕ (autoregressive) and θ (moving average) parameters are small but significant. In the case of the volatility adjusted by the method of Bai and Perron (1998, 2003), the parameter $\widehat{d}$ also shows a negative value close to zero, confirming that once the level shifts have been introduced, the long memory behavior is eliminated. Moreover, the component c_t, the parameter ϕ and the parameter θ are all significant.

The results for the exchange rate returns lead to the same conclusions as the for the stock returns. The ARFIMA(0,d,0) presents a $\widehat{d}$ value larger than 0 for the volatility, which signals long memory. On the other hand, the fractional parameter is too small and not significant for the short memory component (volatility adjusted by Bai and Perron's estimation). In the case of the ARFIMA(1,d,1) modeling, the volatility parameters are significant, being value $\widehat{d}$, the most striking, as it is close to 0.5 and signals long memory as well as possible nonstationary behavior according to Hosking (1981). As far as the short memory component is concerned (volatility adjusted by the procedure of Bai and Perron (1998)), the fractional parameter is negative (anti persistence), very close to zero, and not significant, ruling out the presence of long memory. This is also signaled by the parameter ϕ which is significant and has a value far from 1 for both series.

3.2 Effects of Level Shifts on GARCH and CGARCH Models

Since GARCH models -as well as ARFIMA models- are the best means of portraying volatility, GARCH (1,1) and CGARCH are also estimated. The GARCH model is formulated as follows:

$$\widetilde{r}_t = \sigma_t \epsilon_t, \tag{3}$$
$$\sigma_t^2 = \mu + \beta_1 \widetilde{r}_{t-1}^2 + \beta_2 \sigma_{t-1}^2, \tag{4}$$

where $\widetilde{r}_t$ are the mean-corrected returns, ϵ_t is a *i.i.d.* t-Student distribution with mean 0 and variance 1. The CGARCH model (GARCH components) is specified as follows:

$$\widetilde{r}_t = \sigma_t \varepsilon_t, \tag{5}$$
$$(\sigma_t^2 - n_t) = \beta_1(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_2(\sigma_{t-1}^2 - n_{t-1}), \tag{6}$$
$$n_t = \mu + \rho(n_{t-1} - \mu) + \varphi(\widetilde{r}_{t-1}^2 - \sigma_{t-1}^2). \tag{7}$$

The important coefficients are β_1 and β_2, which reflect the conditional heteroskedasticity of the series. The parameter μ is a constant at which n_t converges, which represents the varying and long term component of the volatility. Therefore, equation (6) represents the transitory component of the volatility. Furthermore, the parameter ρ measures the persistence of shocks in the permanent component of the equation (7), as long as the persistence is measured by $(\beta_1 + \beta_2)$ in the equation (4) and the transitory component in the equation (6).

Otherwise, a CGARCH model is estimated, but enhanced by dummy variables:

$$\widetilde{r}_t = \sigma_t \varepsilon_t \tag{8}$$

$$(\sigma_t^2 - n_t) = \beta_1(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_2(\sigma_{t-1}^2 - n_{t-1}) \tag{9}$$

$$n_t = \mu + \rho(n_{t-1} - \mu) + \varphi(\widetilde{r}_{t-1}^2 - \sigma_{t-1}^2) + \sum_{i=2}^{m+1} D_{i,t}\gamma_i, \tag{10}$$

where $D_{i,t} = 1$ if t belongs to the regime i, and 0 if it does not belong to the regime, with $t \in \{T_i + 1, ..., T_{i+1}\}$ and T_i $(i = 1, ..., m)$ as the dates of the level shifts, which are estimated by the method of Bai and Perron (1998, 2003). The γ_i coefficients are estimated along with the other GARCH parameters and reflect the size of the level shifts.

The results are presented in Table 3 (stock returns) and Table 4 (exchange rate returns). The parameters β_1 and β_2 in the GARCH model are highly significant in both series. The β_2 parameter is high, since its value fluctuates between 0.73 and 0.79. The sum of β_1 and β_2 in the case of the stock returns series is very close to 1, which would indicate that the shocks decay at a very low rate and that the model is close to an IGARCH model (the half-life of the shocks is around 43 days). Meanwhile, for the exchange rate series the same sum is larger than 1, which reflects non-stationarity and that the shocks gain power with time. Strictly speaking, the half-life of shocks is infinite days.

In the CGARCH model, although the β_1 and β_2 values decrease, they are still significant. On the other hand, ρ adopts a value very close to one, as this estimator seeks to reflect the long memory effect of the series. Therefore, it can be stated that the long memory effect is dominant as the horizon of analysis increases. The half-lives of the shocks are 173 days and 693 days for the volatility in stock and exchange rate markets, respectively.

However, once the level shifts are introduced in the form of dummy variables, the parameters β_1 and β_2 are not significant again for either series. Furthermore, the parameter ρ drops to 0.730 for the stock return series and 0.460 for the exchange rate return series, which would signal that though still significant, the impact of the shocks decays faster than when level shifts are not considered. In fact, the half-lives of the shocks are now around 1 and 2 days for volatility in stocks and exchange rate, respectively.

We also assessed the sensitivity of the results using the smoothed estimate of the trend function. This is done by replacing the term $\sum_{i=2}^{m+1} D_{i,t}\gamma_i$ by the smoothed (Gaussian kernel) estimate of the level shift component. The results are very similar to those obtained above. The parameters β_1 and β_2 are no longer significant and the value of parameter ρ -even if still significant- drops radically, reducing the power of the permanent effect of the equation.

Some conclusions can be advanced so far: (i) the RLS model with an AR (1) stationary component seem to provide a safe description of the data; (ii) the level shift component is an important fact which explains both the long memory and conditional heteroskedasticity as they are generally perceived as stylized facts. As a final test, we will look at whether the RLS model provides reasonable predictions compared with some traditional models.

4 Forecasting

In this section we evaluate the RLS model in comparison to ARFIMA models, with regard to the accuracy of their predictions. The predictions are based on Varneksov and Perron (2014).

Therefore, the predictions τ-periods ahead are given by:

$$\widehat{y}_{t+\tau|t} = y_t + HF^{\tau}[\sum_{i=1}^{2}\sum_{j=1}^{2} \Pr(s_{t+1} = j) \Pr(s_t = i|Y_t) X_{t|t}^{ij}], \tag{11}$$

where $E_t(y_{t+\tau}) = \widehat{y}_{t+\tau|t}$ is the prediction of the volatility in time $t+\tau$, conditional to the information until time t, and the matrices F and H are as defined in Section 2 and the prediction horizons $\tau = 1, 5, 10, 20, 50$ and 100 are used. Furthermore, as a criterion for measuring precision, we use the mean of the squared prediction errors (MSFE), which was proposed by Hansen and Lunde (2006) and is defined as:

$$MSFE_{\tau,i} = \frac{1}{T_{out}} \sum_{t=1}^{T_{out}} (\overline{\sigma}_{t,\tau}^2 - \overline{y}_{t+\tau,i|t})^2, \tag{12}$$

where T_{out} is the number of predictions $\overline{\sigma}_{t,\tau}^2 = \sum_{s=1}^{\tau} y_{t+s}$, and $\overline{y}_{t+\tau,i|t} = \sum_{s=1}^{\tau} \widehat{y}_{t+s,i|t}$, with i representing each model. The evaluations and comparisons are performed considering 5% of the Model Confidence Set (MCS), as proposed by Hansen et al. (2011). The MCS enables better evaluations of the models than do pair comparisons based on the p–values. One of the advantages of this procedure is that evaluations are made by taking into account the limitations of the data. This means that if the data is clear, a unique model will be selected, whereas if the data is not sufficiently informative, a MCS with several models would be the result. Therefore, in these cases we can state that more than one model provide a good prediction, which is not the case with other types of comparisons.

In this experiment, two different sizes of samples were utilized for each series in order to make observations against which to contrast the predictions. As small samples, 10% of the total sample was used, which is also the number of predictions. As large samples, the data from 02/01/2006 until the end of the series was used -a time period that comprises the last subprime crisis, and which will allow us to observe whether or not the RLS model is a good predictor, in during times of normality (short sample), as well as of crisis (long sample). Therefore, for the stock market series the latest 550 observations were first made, followed by the latest 1866. In the case of the exchange rate series, the latest 400 observations were made, followed by the latest 1856. In this way, the predictions for the lowest T_{out} for the stock returns series start on 06/04/2011 and on 09/11/2011 for the exchange rate series. The models were estimated without the above-mentioned latest observations for each series, and the predictions were obtained based on the newly estimated parameters.

The results are presented in Table 5. They lead to the conclusion that the best prediction is obtained with the RLS model. For the stock market series, the level shifts model is the only one within the 5% of the MCS for all prediction horizons. The prediction errors in all horizons are lower than in the ARFIMA models.

For the exchange rate series, the RLS model is also a good predictor for short horizons (1 to 20 periods ahead). However, for distant periods, the best predictor is the ARFIMA(1,d,1) model. This result can be explained by considering the interventions carried out by the BCRP in order to control exchange rate return volatility; therefore, it is possible to obtain good predictions with the RLS model, if the period of time is relatively short. However, these predictions lose precision as the time period increases; this is because the afore-mentioned interventions must be taken into consideration since they lead to trajectories that differ from those the series would have naturally followed, as a result of the general effects of the economy.

The results set out previously are repeated for the predictions relating to the crisis, except that this time, in the case of the exchange rate series, the RLS model is better for all forecast horizons. For this period, the MSFE are larger than the values of the previous experiments. This is due to the financial crisis of 2007 onward, which added more volatility to the stock returns, and to the exchange rate returns.

5 Conclusions

Empirical studies applied to financial series in Peru are very scarce. Humala and Rodríguez (2013) discussed the stylized facts of exchange rate and stock markets returns and volatility using daily, weekly and monthly data. From this paper, a work agenda related to different aspects of the returns and volatilities of the two markets is emphasized. This study forms part of that empirical agenda. We follow Lu and Perron (2010) and Li and Perron (2013) and apply a RLS model to the volatilities in the Stock and Forex rate markets. Although we have fewer observations compared to developed countries, our results are quite as conclusive as Lu and Perron (2010). The results can be summarized as follows: (i) the probability of level shifts is small but is responsible for the presence of long memory in volatilities of the analyzed series. Having estimated the probability of level shifts, the exact number of such shifts can be calculated. Thus, the component obtained as a subtraction between volatility and the level shift component has an ACF indicating no long memory; (ii) estimates of autoregressive conditional heteroskedasticity models discounting level shifts show that these components are artificially introduced by the level shifts; (iii) estimates of fractional series for the discounted level shift series show that the fractional parameter is less than zero or near to zero implying the non existence of long memory; and (iv) the performance of the RLS model in terms of forecast is better in comparison with standard ARFIMA(p,d,q) models.

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Table 1. Descriptive Statistics

	Stock		Forex Rate	
	Returns	Volatility	Returns	Volatility
Mean	0.001	-4.858	0.000	-6.192
SD	0.017	0.951	0.002	0.554
Maximum	0.143	-1.931	0.022	-3.727
Minimum	-0.132	-6.907	-0.023	-6.907
Skewness	0.519	-0.027	0.188	0.930
Kurtosis	11.088	2.622	16.331	3.666
JB	16159.29	35.375	30466.23	669.765
p-value	(0.000)	(0.000)	(0.000)	(0.000)

Table 2. Estimates of the RLS Model

	σ_η	α	σ_e	ϕ	Likelihood
Stock	0.874^a	0.00446^a	0.841^a	0.115^a	7421.853
(sd=0.951)	(0.127)	(0.00164)	(0.008)	(0.015)	
Forex Rate	0.462^a	0.01824^a	0.441^a	0.0968^a	2711.282
(sd=0.554)	(0.077)	(0.00498)	(0.006)	(0.020)	

Standar errors are in parentheses; ^{a,b,c} denote significance at the 1.0%, 5.0% and 10.0%, respectively.

Tabla 3. Estimates of GARCH and CGARCH Models for Stock Series

	parameter	value	s.e.	p-values
GARCH	β_1	0.255	0.010	0.000
	β_2	0.729	0.008	0.000
CGARCH	β_1	0.232	0.023	0.000
	β_2	0.583	0.043	0.000
	ρ	0.996	0.004	0.000
	φ	0.104	0.022	0.000
CGARCH (using τ_t from Bai and Perron)	β_1	0.023	0.093	0.804
	β_2	0.426	1.169	0.716
	ρ	0.730	0.040	0.000
	φ	0.230	0.094	0.014
CGARCH (using smoothed estimate of τ_t)	β_1	-0.024	0.018	0.171
	β_2	0.024	0.448	0.957
	ρ	0.723	0.000	0.000
	φ	0.306	0.021	0.000

Tabla 4. Estimates of GARCH and CGARCH Models for Forex Rate Series

	parameter	value	s.e.	p-values
GARCH	β_1	0.261	0.022	0.000
	β_2	0.794	0.012	0.000
CGARCH	β_1	0.172	0.017	0.000
	β_2	0.328	0.072	0.000
	ρ	0.999	0.000	0.000
	φ	0.148	0.008	0.000
CGARCH (using τ_t from Bai and Perron)	β_1	0.055	0.069	0.420
	β_2	0.020	0.393	0.960
	ρ	0.460	0.060	0.000
	φ	0.098	0.066	0.139
CGARCH (using smoothed estimate of τ_t)	β_1	0.065	0.036	0.072
	β_2	0.021	0.182	0.908
	ρ	0.520	0.001	0.000
	φ	0.192	0.041	0.000

Table 5. Comparison of Forecasts ($\widehat{y}_{t+\tau|t}$)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
Stock Market						
$T_{out} \epsilon [1, 550]$						
RLS	0.49	4.35	11.94	31.94	142.01	620.40
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)
ARFIMA(0,d,0)	0.80	6.86	19.72	58.01	271.13	927.40
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ARFIMA(1,d,1)	0.80	6.87	19.84	58.91	283.42	1002.74
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$T_{out} \epsilon [1, 1866]$						
RLS	0.50	5.51	16.68	54.61	308.78	1148.30
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)
ARFIMA(0,d,0)	0.92	9.60	30.12	97.58	464.32	1511.35
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ARFIMA(1,d,1)	0.93	9.88	31.22	102.06	493.11	1630.22
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Numbers are the MSFE; p-values of the MCS are reported in parentheses; * denotes that the model belongs to the 5% of the MCS of Hansen et al. (2011) comparing between all models.

Table 5 (continuation). Comparison of Forecasts ($\widehat{y}_{t+\tau|t}$)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Forex Rate Market			
			$T_{out} \epsilon [1, 400]$			
RLS	0.14	1.39	4.43	22.20	182.41	828.16
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(0.04)	(0.00)
ARFIMA(0,d,0)	0.30	3.57	11.69	39.03	143.32	265.29
	(0.00)	(0.00)	(0.00)	(0.00)	(0.04)	(0.00)
ARFIMA(1,d,1)	0.30	3.55	11.60	38.68	141.16	258.78
	(0.00)	(0.00)	(0.00)	(0.00)	(1.00*)	(1.00*)
			$T_{out} \epsilon [1, 1856]$			
RLS	0.18	2.25	7.57	28.64	168.40	810.32
	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)	(1.00*)
ARFIMA(0,d,0)	0.38	5.08	16.79	56.62	287.53	999.98
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ARFIMA(1,d,1)	0.38	5.12	16.95	57.29	291.85	1019.83
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Numbers are the MSFE; p-values of the MCS are reported in parentheses; * denotes that the model belongs to the 5% of the MCS of Hansen et al. (2011) comparing between all models.

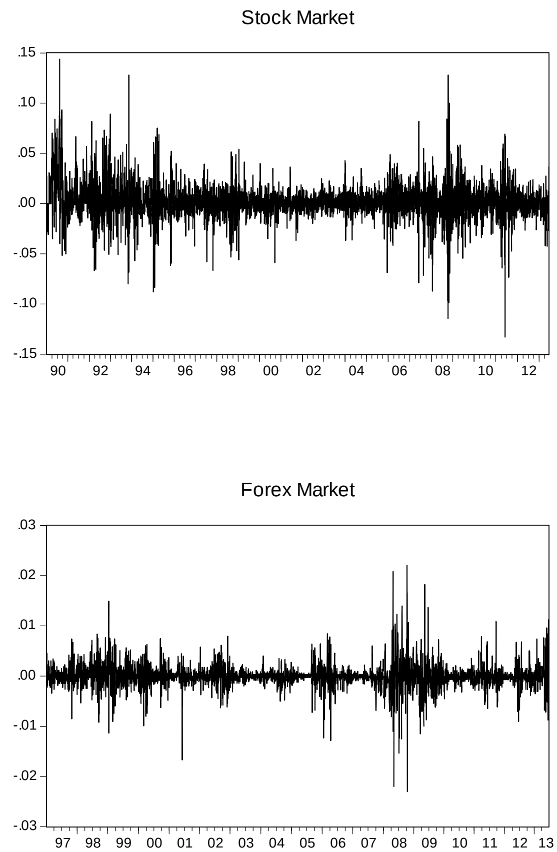

Figure 1. Stock (Top panel) and Forex Rate (Bottom panel) Returns

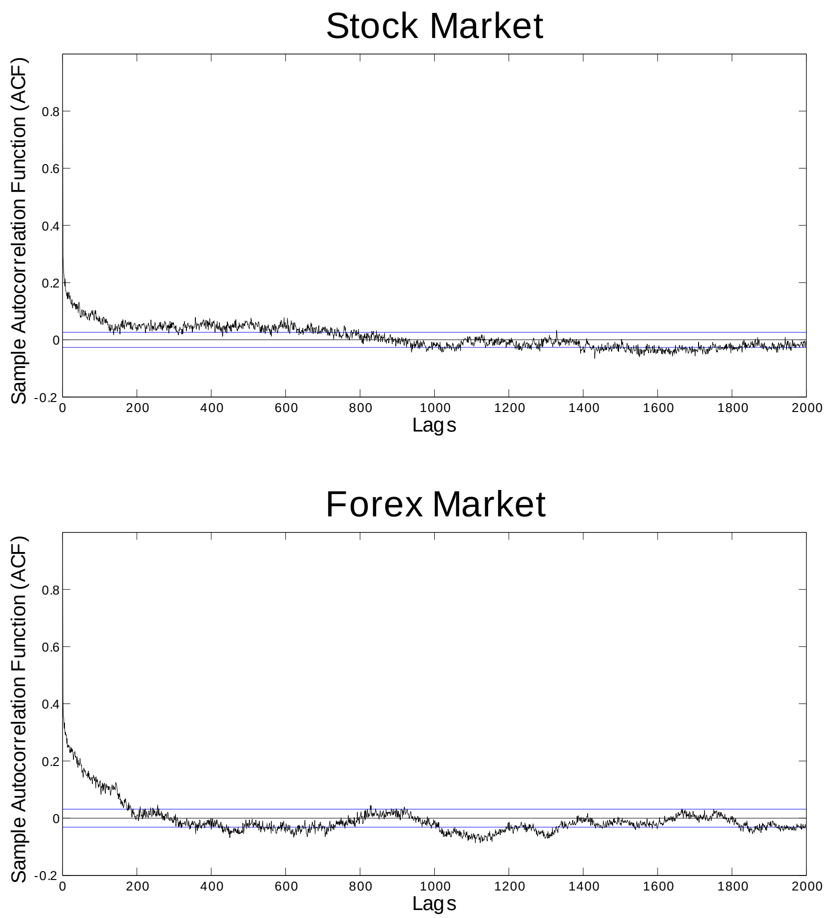

Figure 2. Sample ACF of Stock (Top) and Forex Rate (Bottom) Returns Volatilities

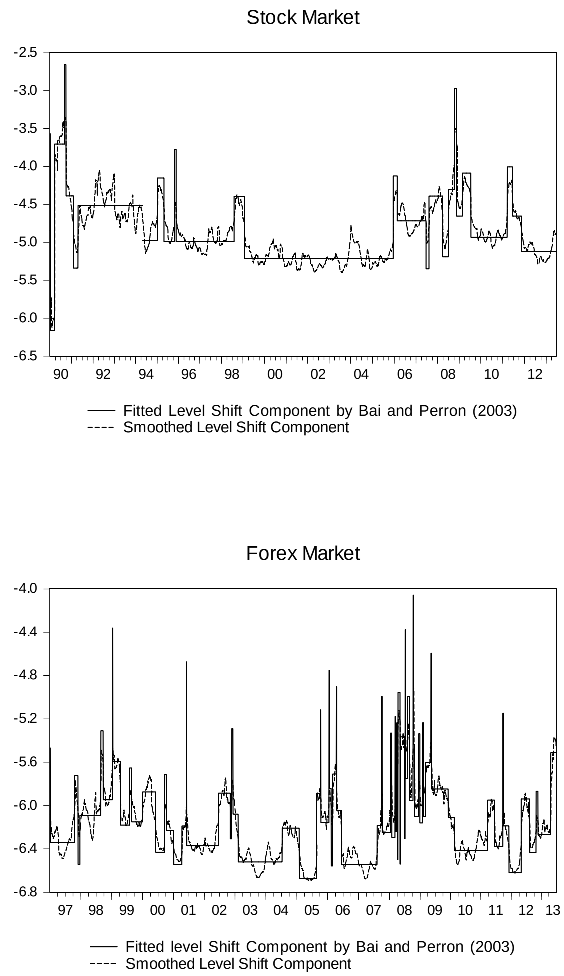

Figure 3. Level Shift Component (τ_t) estimated by Bai and Perron (2003) and Smoothed Level Shift Component

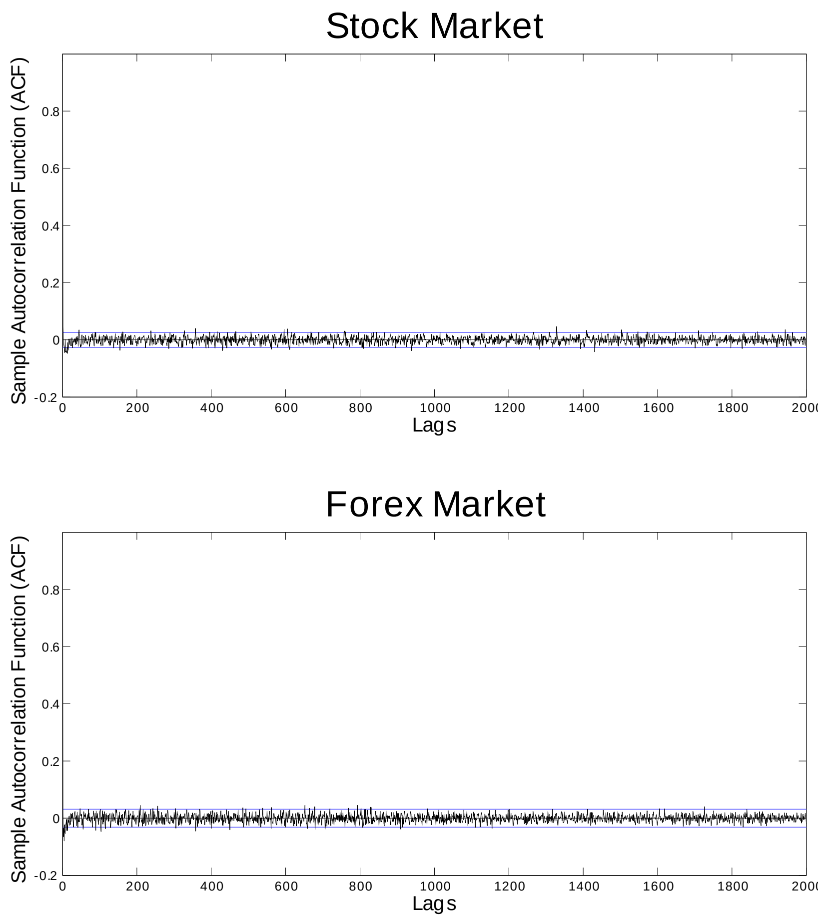

Figure 4. Sample ACF of Residuals of the RLS Model for Stock (Top) and Forex Rate (Bottom) Markets

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