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DISTINGUISHING BETWEEN TRUE AND SPURIOUS LONG MEMORY IN THE VOLATILITY OF STOCK MARKET RETURNS IN LATIN AMERICA

Renzo Pardo Figueroa y Gabriel Rodríguez

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Distinguishing between True and Spurious Long Memory in the Volatility of Stock Market Returns in Latin America

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Abstract

In this study, we investigate the long term dependence or long memory present in the volatility of the stock market returns of Peru, Brazil, Mexico, Chile, Argentina, and the S&P500. We start analyzing the form of the autocorrelation function (ACF) and the estimated spectral density. Moreover, volatility is modeled by way of FIGARCH processes that contribute additional indications of this behavior. Following a testing approach, the W statistics of Qu (2011), W_c , η_{μ} and Z_t due to Shimotsu (2006), and the statistics $t_d(1/2,1;4/5,1)$, and $mean-t_d$ of Perron and Qu (2010) are used to verify for long memory. Also we show evidence about the behavior of the long memory estimator \hat{d} for different sample sizes included in the estimation procedure. The evidence reported graphically and through the statistics suggest that the generating process of the volatility series is spurious memory, except for Chile, whose evidence of spurious memory is weak. Moreover, the graphics contain important information on the spurious memory behavior. The results of this study suggest that in reality, the long memory that is usually found in empirical studies would rather be associated with spurious memory, which could be due to the presence of structural breaks.

JEL Classification: True and Spurious Long Memory, Fractional Integration, Frequency Domain Estimator, Semiparametric, Structural Change.

Keywords: C12, C14, C22, G12.

Resumen

En este estudio, investigamos la dependencia de largo plazo o de larga memoria presente en la volatilidad de los rendimientos del mercado de valores de Perú, Brasil, México, Chile, Argentina, y el S&P500. En un primer momento se analiza el comportamiento de la ACF y la densidad espectral. Por otra parte, la volatilidad se modela por medio de procesos FIGARCH que añaden evidencia a lo observado visualmente. Para verificar la presencia de la verdadera larga memoria seguimos un enfoque de pruebas estadísticas. En este sentido, el estadístico W de Qu (2011), los estadísticos W_c , η_μ y Z_t propuestos por Shimotsu (2006), y los estadísticos $t_d(1/2,1;4/5,1)$, y $mean-t_d$ de Perron y Qu (2010) son utilizados. También mostramos evidencia sobre el comportamiento del estimador de larga memoria \hat{d} para diferentes tamaños de las muestras incluidas en el procedimiento de estimación. La evidencia reportada gráficamente y a través de los estadísticos sugieren que el proceso de generación de la serie de la volatilidad es de memoria espuria, a excepción de Chile cuya evidencia de memoria espuria es débil. Por otra parte, los gráficos contienen información importante sobre el comportamiento de la memoria falsa. Los resultados de este estudio sugieren que, en realidad, la memoria larga que se encuentra generalmente en los estudios empíricos son de memoria falsa, lo que podría deberse a la presencia de cambios estructurales.

Classificación JEL: Verdadera y Falsa Memoria Larga, Integración Fraccional, Estimator en el Dominio de las Frecuencias, Semiparamétrico, Cambio Estructural.

Palabras Claves: C12, C14, C22, G12.

Distinguishing between True and Spurious Long Memory in the Volatility of Stock Market Returns in Latin America¹

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1 Introduction

Typically, the volatility of financial series displays long-term dependence or long-memory. This property is represented in the domain of time by the behavior of its sample autocorrelation function (ACF), which presents significantly different values from zero up to a large number of lags, indicating a hyperbolic decay. In the domain of frequencies a particular behavior is also observed: the weight of low frequencies in the spectral density is greater, with rapid growth in this function observed as the frequencies approach the the origin. Several authors document this characteristic; see Taylor (1986), Ding et al. (1993), Dacorogna et al. (1993), Robinson (1994), among others.

There are several possible formalizations for this definition; see McLeod and Hipel (1978), Robinson (1994), Beran (1994) and Baille (1996), among others. We follow the definitions presented in Perron and Qu (2010). Let $\{x_t\}_{t=1}^T$ be a stationary time series with spectral density function $f_x(w)$ in the frequency w, so x_t has long memory if $f_x(w) = g(w)w^{-2d}$, for $w \Rightarrow 0$, where g(w) is a smooth variation function in a neighborhood of the origin, which means that for all real numbers t, it is verified that $g(tw)/g(w) \Rightarrow 1$ for $w \Rightarrow 0$. When d > 0, the spectral density function is growing for frequencies that are increasingly close to the origin. The rate of divergence to the infinite depends on the given value of the parameter d. On the other hand, let $\gamma_x(\tau)$ be the autocorrelation function (ACF) of x_t , so x_t has long memory if $\gamma_x(\tau) = c(\tau)\tau^{2d-1}$, for $\tau \Rightarrow \infty$, where $c(\tau)$ is a smooth variation function³. When 0 < d < 1/2 the autocorrelation function declines at a slow rate that is dependent on the value of the parameter d.

The long memory property was initially described by Hurst (1951) in a study related to the construction of a dam on the River Nile, where it was observed that the periods of greater growth and of drought on this river were not random; rather, they tended to cluster, providing evidence of greater persistence of these events over time. This long-term persistence implies that the information posted today will affect the variability of the series in the future in accordance with its degree of long memory. Hurst (1951) put forward the rescaled range (RS) as a measurement of the degree of long-memory. The RS statistic is equal to the range of the series of partial sums of the first k deviations from the original series x_t with respect to its average, divided or rescaled by its standard deviation. Greene and Fielitz (1977) apply the RS statistic to the series of asset returns listed on

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³A practical definition of long memory is to state that the sum of the autocorrelations is infinite; that is, $\lim_{T\to\infty}\sum_{j=-T}^T |\rho_j| = \infty$.

⁴These definitions in the domains of frequency and time are equivalent if a few general conditions are verified in accordance with Beran (1994).

the New York stock exchange, and most of this series displays long memory.

However, Lo (1991) showed that the RS statistic has problems in discerning long-term dependence in the presence of short-term dependence. For example, the short-term dependence present in an AR(1) process gives, as a result, values of the RS statistic that are similar to the aforementioned when the persistence is long-term. To overcome this inconsistency, Lo (1991) proposes a modification of the statistic that consists of replacing the denominator with a consistent estimator of the variance of the partial sum. The tests performed in Lo (1991) for the same assets as Greene and Fielitz (1977) discard the possibility of long memory.

For their part, Willinger et al. (1999) point out that the results of the modified RS statistic could be wrong, as this estimator is strongly biased to the rejection of the long-memory hypothesis when the degree thereof is small. Despite these disadvantages, the statistics constitute a good approximation and continue to be used in empirical references.

Moreover, the statistic known as the Hurst exponent (H), which was put forward in Mandelbrot (1972, 1975), is accepted. This statistic is equal to the limit of the ratio between the logarithm of the RS statistic and the logarithm of the sample size, when the size of the sample tends towards infinity. The values of H are quite instructive: for 0 < H < 0.5, the series is said to display antipersistence; for 0.5 < H < 1, the series shows persistence or long-memory; and if H = 0.5, the series is an independent process or is purely random.

Granger and Joyeux (1980) and Hosking (1981) introduce the first long-memory models known as fractionally integrated models I(d), denoted ARFIMA(p,d,q), which allow the parameter of integration d to take fractional values in place of 0 or 1. General procedures have been put forward for the estimation of the fractional integration parameter d. These methods are twofold: parametric and semiparametric. Yajima (1985), Fox and Taqqu (1986), and Dahlhaus (1989) developed the statistical properties of the parametric estimators. These estimators make explicit the structure of the ACF or of the spectrum, which allows a characterization of all autocorrelations and not just the decreasing hyperbolic. They use maximum likelihood methods with good asymptotic properties, but if the model is poorly specified, the estimators will be inconsistent.

The procedure proposed in Fox and Taqqu (1986) is an approximation to the maximization of the Gaussian likelihood function. This method is based on the likelihood function in the domain of frequencies originally put forward by Whittle (1962): $L_{x,T} = 1/4 \int_{-\pi}^{\pi} \left\{ \log f_x(w) + \frac{I_{x,T}(w)}{f_x(w)} \right\} dw$, where $I_{x,T}(w)$ is the periodogram of the series x_t defined as: $I_{x,T}(w) = \frac{1}{2\pi T} \left| \sum_{t=1}^{T} (x_t - \overline{x}) \exp(itw) \right|^2$. Fox and Taqqu (1986) obtain asymptotic results for certain parameters of the model and under certain restrictions on the spectral density of the process. Dahlhaus (1989) extends these results and demonstrates that the Whittle estimator is consistent and asymptotically Normal in stationary Gaussian AFIRMA processes, and also establishes conditions so that this estimator is asymptotically efficient. Nonetheless, according to Dahlhaus (1989), the behavior of the estimator in small samples is very poor. This author concludes that the exact maximum-likelihood procedures are better.

The semiparametric parameters do not require a specification of the structure of the underlying I(d) model. These estimators are based on the calculation of the periodograms and are principally of two kinds: the estimator of the logarithm of the periodogram (estimator \hat{d}) proposed by Geweke and Porter-Hudak (1983) and the local Whittle estimator (\hat{d} estimator), which was refined by Künsch (1987) and Robinson (1995b). The \hat{d} estimator is obtained on the basis of the following minimum least squares regression, using the observations belonging to the range of $j = 1, \ldots, m$,

where, $I_{x,T}(w_j)$ is the periodogram in the j-th Fourier frequency $w_j = 2\pi j/T$ (j = 1, ..., [T/2]): $\log [I_{x,T}(w_j)] = c - 2d \log \left[2\sin(\frac{w_j}{2})\right] + \epsilon_j$, where the parameter m represents the upper limit of the frequencies utilized, the frequencies close to the origin are used in order to avoid possible specification errors caused by the highest frequencies. One rule is to utilize $m = T^a$, where a = 0.5, 0.6, 0.7.

The asymptotic properties are derived in Robinson (1995a). This author proposes a modified version and shows the properties of asymptotic Normality and consistency for values of the parameter in the range -1/2 < d < 1/2. Also suggested is the elimination of the lowest frequencies by way of a truncation, in order to reduce the bias of the estimation. Nonetheless, this procedure is not recommendable in finite samples, as the reduction of the bias attained by truncating the range of frequencies does not compensate the increase in variance, as was seen in Hurbich and Beltrao (1994).

On the other hand, the \widetilde{d} estimator is obtained based on the following objective Gaussian function, which represents the discrete version of the Whittle likelihood function (1962), for $f_x(w) = Gw^{-2d}$; that is, $Q_m(G,d) = \frac{1}{m} \sum_{j=1}^m \{\log(Gw_j^{-2d}) + \frac{w_j^{2d}}{G} I_x(w_j)\}$, where m is an integer less than T. The procedure estimates d and G based on the minimization of $Q_m(G,d)$ so that: $(\widetilde{G},\widetilde{d}) = \arg\min_{G\in(0,\infty),d\in[\Delta_1,\Delta_2]} Q_m(G,d)$, where Δ_1 and Δ_2 are numbers such as $-1/2 < \Delta_1 < \Delta_2 < \infty$. The true values of the parameters are denoted by G_0 and d_0 , and by concentrating the objective function with respect to G, the following is obtained: $\widetilde{d} = \arg\min_{d\in[\Delta_1,\Delta_2]} R(d)$, where $R(d) = \log \widetilde{G}(d) - 2d\frac{1}{m} \sum_{j=1}^m \log w_j$, $\widetilde{G}(d) = \frac{1}{m} \sum_{j=1}^m w_j^{2d} I_x(w_j)$. Robinson (1995a, 1995b) showed that the two types of semiparametric estimators are asymp-

Robinson (1995a, 1995b) showed that the two types of semiparametric estimators are asymptotically Normally distributed, with the same rate of convergence, but with the variance of the estimator \tilde{d} smaller than the estimator \hat{d} ; thus, the estimator \tilde{d} is more efficient. The disadvantage is that the calculation is more laborious as it involves methods of numerical optimization. The properties of these estimators continue to be studied, with subsequent references including Velasco (2000), Phillips and Shimotsu (2004), and Shimotsu and Phillips (2006), among others.

Empirical works relating to the volatility series initially utilized GARCH and stochastic volatility models. The main disadvantage of these models is that they do not allow long memory to be generated. Therefore, to collect the empirical evidence in the volatility, models were proposed that included fractional integration in the equation of variance. For example, Baille et al. (1996) and Bollerslev and Mikkelsen (1996) consider fractionally integrated GARCH and EGARCH models (FIGARCH and FIEGARCH), respectively while Breit et al. (1998) and Harvey (1998) develop analogous stochastic volatility models. Moreover, Tse (1998) extends the GARCH model of asymmetric potential (APARCH) of Ding et al. (1993) to incorporate fractional integration. This model, known as FIAPARCH, allows an asymmetric response to the volatility of positive and negative shocks, also renders the power of returns endogenous in the equation of conditional variance that includes long-term dependence in the volatility. Tse (1998) finds this model applicable to the yen/dollar exchange rate.

Though the fractionally integrated processes reproduce long memory, there are many other processes that have this property. Granger and Ding (1996) present models of this type, among them generalized fractionally integrated models that arise from the aggregation of the data, the variant coefficient models, and non-linear models. These authors find significant variability in the estimates of parameters in the fractionally integrated models across subperiods of the series. To explain this behavior, they employ a model with two fractionally integrated process regimes

utilizing a Markov chain. This model is capable of generating a variety of forms of the ACF by producing different long-memory processes. This point is made analytically and by way of simulations in Diebold and Inoue (2001). Models are formulated with stochastic regime switching that are capable of generating long memory when a small number of regime switches occur. The model is a process with mean plus noise and is given by $x_t = \mu_t + \epsilon_t$ with $\mu_t = \mu_{t-1} + v_t$ and v_t is 0 with probability p and p0 with probability p1 and p1 with probability p2 and p3 with probability of regime switching is small (p3 with p4 of p5 with p6 with p9 with

Another model is the permanent stochastic break process (Stopbreak) of Engle and Smith (1999), which is not stationary and is given by $x_t = \mu_t + \epsilon_t$ with $\mu_t = \mu_{t-1} + \frac{\epsilon_{t-1}^2}{\gamma + \epsilon_{t-1}^2} \epsilon_{t-1}$ and where ϵ_t has Normal distribution. If $E(\epsilon_t^6) < \infty$, $\gamma \Rightarrow \infty$ when $T \Rightarrow \infty$ and $\gamma = O(T^{\delta})$ for some $\delta > 0$ then $x_t = I(1 - \delta)$.

Gourieroux and Jasiak (2001) reach very similar conclusions through an analysis of the ACF. A Markov chain model is formulated with two regimes with a very low probability of regime change capable of reproducing the ACF behavior that slowly decays in its lags. In the limit case of very low probability of regime switching, the ACF of this non-linear model converges on a non-degenerate distribution. Therefore, the hyperbolic decay rate may be the result of the non-linear dynamic of the model with infrequent regimes, instead of the linear dynamic of a fractionally integrated model.

A time series will present true long memory if the underlying process has long-term dependence or, equivalently, if it is modeled as a fractionally integrated process. It is said that the long memory of the series is spurious in nature when it is not the result of long-term dependence but of other different causes such as regime switching. Hou and Perron (2014) classify these causes of spurious memory under the name of low frequency contaminants. These contaminants include stochastic regime switching, deterministic regime switching, and deterministic trends. A short-memory process contaminated with these components will display spurious memory. Granger and Ding (1996) include temporal aggregation as a possible cause of spurious memory. Gourieroux and Jasiak (2001) suggest that common practice in financial series of rounding the indices to a number of decimals artificially introduces regime switching and could be a cause of spurious memory in the data.

Lobato and Savin (1999) identify the spurious memory generated by the non-stationarity in the series and by the aggregation of the data by way of a process that consists of dividing the series into stationary subperiods and applying a statistic for long memory in these ranges.

Granger and Hyung (2004) provide analytical and simulation arguments that show that it is a difficult problem in practice to distinguish between processes with infrequent regime switching and fractionally integrated processes. The authors demonstrate that both models can explain the absolute value of the returns well⁵. According to these authors, the detection of fractionally integrated processes utilizing traditional methods could lead to the detection of spurious memory when applied to processes with short memory containing structural break in the mean or in the trend. To correctly identify between the true long-memory process and spurious memory, different solutions have been proposed -one of which is based on the development and application of statistics. For example, the statistics proposed in Dolado et al. (2005), Shimotsu (2006), Ohanissian et al.

⁵Even though a procedure for identifying the breaks in the series allows a reduction in the evidence of long memory when the breaks are taken into account, this evidence is inconclusive, given that the statistics for structural break are significantly biased in the presence of long memory.

(2008), and Qu (2011). The new evidence tends to reject the long-memory specifications due to spurious memory in many of the time series that had been considered with long memory in the previous empirical evidence.

In this study, we investigate the long term dependence or long memory present in the volatility of the stock market returns of Peru, Brazil, Mexico, Chile, Argentina, and the S&P500. We start analyzing the form of the ACF and the estimated spectral density. Moreover, volatility is modeled by way of FIGARCH processes that contribute additional indications of this behavior. To verify the presence of true long memory we follow a testing approach. In this sense, the W statistics of Qu (2011), W_c , η_μ and Z_t due to Shimotsu (2006), and the statistics $t_d(1/2,1;4/5,1)$, and $mean-t_d$ of Perron and Qu (2010) are used. Also we show evidence about the behavior of the long memory estimator \hat{d} for different sample sizes included in the estimation procedure. The evidence reported graphically and through the statistics suggest that the generating process of the volatility series is spurious memory, except for Chile, whose evidence of spurious memory is weak. Despite the results of these statistics being mixed, all series reject the hypothesis of true long memory at least for some of the statistics. Moreover, the graphics contain important information on the spurious memory behavior. The results of this study suggest that in reality, the long memory that is usually found in empirical studies would rather be associated with spurious memory, which could be due to the presence of structural breaks.

This paper is structured as follows. Section 2 is dedicated to the methodology and presents the statistics designed to discern true and spurious long memory. The empirical results are reported in Section 3. The final Section presents the conclusions.

2 Methodology

Perron (1989) showed that unit root statistics could lead to erroneous conclusions if the true process has short memory containing breaks in the deterministic components. As underlined by Mayoral (2012), it is commonly accepted that the use of techniques for fractionally integrated processes leads to the detection of spurious fractional integration when applied to processes containing trends or breaks. The opposite effect is also documented -that conventional procedures for detecting the date of structural breaks tend to detect spurious breaks usually in half of the sample when there only exists fractional integration in the data; see Hsu (2001), Dolado et al. (2005).

Perron and Qu (2007, 2010) analyze the properties of a simple but general spurious long-memory process. This model mixes a process with short memory, with a component that accumulates realizations of a Bernoulli process. This latter component represents the stochastic regime switching component. The periodogram of this process for frequencies close to the origen has similar behavior to a process that contains a unit root and when the frequency increases, the periodogram decreases more rapidly than that corresponding to a fractionally integrated process.

There are also differences in the limit distribution of the estimator based on the logarithm of the periodogram (\hat{d}) , which was initially derived in Perron and Qu (2007). For a number of frequencies m close to $T^{1/3}$ the estimator \hat{d} is located in a vicinity of 1. When m is between $T^{1/3}$ and $T^{1/2}$, \hat{d} falls to a level that corresponds to the effect of the stationary component in the limit distribution and for values of m beyond $T^{1/2}$, \hat{d} displays a gradual fall explained by the growth in the importance of the short-memory component. Conversely, if the underlying process is fractionally integrated, the limit distribution of \hat{d} is the same regardless of the size of m with respect to the size of the sample; see Hurbich et al. (1998).

The statistics proposed in Perron and Qu (2010) are based on these properties of the estimator \hat{d} . Let $\hat{d}_{a,c}$ be the estimator \hat{d} when $m_{a,c}$ is equal to $c[T^a]$. The first step is to establish the following result, due to Hurbich et al. (1998). Under the null hypothesis of a fractionally integrated stationary process, it is verified that $\sqrt{c[T^a]}(\hat{d}_{a,c}-d_0) \Rightarrow^d N(0,\pi^2/24)$. The second step establishes the statistic t_d and its limit distribution, for 0 < a < b < 1 with a < 4/5, $t_d(a, c_1; b, c_2) =$ $\sqrt{24c_1[T^a]/\pi^2(d_{a,c_1}-d_{b,c_2})} \Rightarrow N(0,1)$. The procedure for discerning true and spurious long memory consists of assessing whether there is a smooth but significant decline in the value of \hat{d} for values of m greater than $T^{1/2}$. To this end, the statistic $t_d(1/2,1;4/5,1)$ is utilized. We must also examine whether there is a rapid decline in the value of \hat{d} for values of m between $T^{1/3}$ and $T^{1/2}$. Because the decline in this range is not monotonic, the maximum difference may not occur in $T^{1/3}$. Thus, the following two statistics are considered: the statistic sup $-t_d = \sup_{c_1 \in [1,2]} t_d(1/3, c_1; 1/2, 1)$ and $mean - t_d = mean_{c_1 \in [1,2]} t_d(1/3, c_1; 1/2, 1)$, which evaluate the maximum value and the average value of the statistic t_d in this range. The limit distributions of these statistics are not analytically available; thus, the critical values are obtained by utilizing a parametric bootstrap procedure. The exact size of these two statistics is close to 5\%, but reduces as the sample size increases. Perron and Qu (2010) do not present results related to the power of these statistics.

Ohannisian et al. (2008) propose the following statistic. Let $\hat{\mathbf{d}} = (\hat{d}^{(m_1)}, \hat{d}^{(m_2)}, ..., \hat{d}^{(m_M)})$ be the vector of estimators of the logarithm of the periodogram \hat{d} obtained utilizing up to Mlevels of aggregation, one level of aggregation from the series x_t with period m is given by $y_s^{(m)} =$ $\sum_{\tau=1}^{m} x_{m(s-1)+\tau}$ for $1 \leq s \leq T/m$. The null hypothesis is that the original series is generated by a stationary process with true long memory; that is, that the integration parameter is the same across all aggregation levels $\hat{d}^{(m_1)} = \hat{d}^{(m_2)} = \dots = \hat{d}^{(m_M)}$. Finally, the Wald statistic is given by $\hat{\mathbf{W}} = (\mathbf{T}\hat{\mathbf{d}})'(\mathbf{T}\mathbf{\Lambda}\mathbf{T}')^{-1}(\mathbf{T}\hat{\mathbf{d}}) \Rightarrow \chi^2_{(M-1)}$, where the matrix $\mathbf{\Lambda}$ is the matrix of asymptotic covariances of the distribution of $\hat{\mathbf{d}}$, and $\mathbf{T} = \mathbf{I} - (1/M)\mathbf{1}$. The exact size of this statistic is simulated by utilizing different level of aggregation, and when the matrix of theoretical and simulated covariances is utilized. The size distortion is greater for a larger number of aggregation levels and by utilizing the theoretical covariance matrix. The best performance is obtained for M=4 and utilizing a matrix of simulated covariances. In this case, the size of the statistic is between 5\% and 7\%. The power property of this statistic is evaluated by utilizing simulations of spurious memory models, which include stationary and non-stationary models of stochastic level shift, the Markov Switching model with independent regimes and identically distributed with GARCH regimes, and a white noise model with deterministic trend. The results show powers close to 100% for most of the models, but only the case when M = 12 and T = 610304 is considered; that is, a very high sample size seldom found in empirical applications.

On the other hand, Shimotsu (2006) proposes two statistics. The first of these is based on the constancy of the parameter d in subperiods of the series if the true process is I(d). The second statistic is based on the assumption that if a series is specified as I(d), then its difference of order d will be represented by a process I(0). In the first statistic, the null hypothesis is given by $H_0: d_0 = d_{0,1} = \dots = d_{0,b}$, where $d_{0,i}$ is the true value of d in the i-th subsample. The complete series and the b subsamples formed, each one with the same sample size equal to T/b, configure b+1 subperiods. Let $\widetilde{\mathbf{d}}_b = (\widetilde{d}-d_0, \widetilde{d}^{(1)}-d_0, \dots, \widetilde{d}^{(b)}-d_0)$ be the vector of estimators where \widetilde{d} is the Local Whittle semiparametric estimator. The statistic put forward is Wald-type and is given by: $W_c = 4m \left(c_{m/b}/(m/b)\right) A\widetilde{\mathbf{d}}_b (A\Omega A')^+ (A\widetilde{\mathbf{d}}_b)' \Rightarrow \chi^2_{(b-1)}$, where A is a matrix of constants, Ω is the matrix of covariances and $(A\Omega A')^+$ denotes the generalized inverse matrix of $A\Omega A'$. The

parameter m is the number of frequencies in the estimation of d and the term c_m is included to correct the bias of this estimator in finite samples; see Hurvich and Chen (2000), where it is verified that $c_m/m \Rightarrow 1$ if $m \Rightarrow \infty$.

The size and power of this statistic are obtained through simulations. The statistic W_c tends to over-reject the hypothesis of true long memory, even though this distortion is small except for b=8 and for small values of m. The power of the statistic for a mean-plus-noise model is relatively low, and this is in the range of 50% to 90%. The highest values correspond to b=8, but these do not increase significantly with the rise of m. For the stopbreak process, the power of the statistic is even weaker, particularly for low values of m. It is interesting to observe that in this case, higher values of b do not improve the power of the statistic. In the Markov Switching model, the power of the statistic improves significantly with the increase of m. Finally, in the stochastic unit root model (Gourieroux and Robert (2006)) adequate power levels of this statistic are observed (above 80%) for moderate values of b). The author recommends the use of b=4 as this specification maintains a good balance between size and power.

The second statistic uses the unit root statistic Z_t of Phillips and Perron (1988) and the statistic KPSS of Kwiatkowski et al. (1992). In this procedure, first the mean of the series is extracted and then the statistic Z_t and KPSS is applied to its difference of order d, where d is a consistent estimator of the true parameter d. Shimotsu (2006) denotes these two statistics as Z_t and η_{μ} , respectively. By way of simulations, it is shown that the size of these statistics is slightly less than 5%, but improves with the increase of m. If short memory is significantly included, the statistic Z_t has greater performance in comparison with the statistic η_{μ} , whose size is rapidly reduced with the increase of m. The power is assessed utilizing the same specifications of processes with spurious memory than for the statistic W_c . It is shown that the statistic η_{μ} has strong power against the mean-plus-noise and stopbreak processes, while the statistic Z_t has very low power. In contrast, for the Markov Switching and stochastic unit root processes Z_t have an adequate power, but the statistic η_{μ} has a very low power. The author recommends the use of both statistics Z_t and η_{μ} to improve the results against the possible processes with spurious memory.

Finally, we detail the last of the statistics, which can be useful for our objective. This statistic,

proposed by Qu (2011), is based on the Whittle likelihood function. This function is given by: $Q(G,d) = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} \text{ is the spectral density function of the } C w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \}, \text{ where } G w_j^{-2d} = \frac{1}{m} \sum_{j=1}^{m} \{ \log G w_j^{-2d} + \frac{I_j}{G w_j^{-2d}} \} \}$ process x_t with stationary long memory, $I_j = I_x(w_j)$ is the periodogram of the series in the frequency w_j and m is a lower number than the sample size T. The minimization of Q(G,d) with respect to G leads to the following expression of the likelihood function $R(d) = \log G(d) - 2d \frac{1}{m} \sum_{j=1}^{m} \log w_j$, where $G(d) = \frac{1}{m} \sum_{j=1}^{m} w_j^{2d} I_j$. The derivative of the function R(d) with respect to the parameter d is given by $\frac{\partial R(d)}{\partial d} = \frac{2G_0}{\sqrt{m}G(d)} \{m^{-1/2} \sum_{j=1}^{m} v_j (\frac{I_j}{G_0 \lambda_j^{-2d}} - 1)\}$, where $v_j = \log w_j - \frac{1}{m} \sum_{j=1}^{m} \log w_j$ is an expression in deviations of the mean and G_0 is the true value of G. The expression in brackets is the main input of the statistic. Under the null hypothesis and evaluated in d_0 , this expression is equal to $m^{-1/2} \sum_{j=1}^m v_j (\frac{I_j}{G_0 \lambda_j^{-2d_0}} - 1)$ which satisfies a central limit theorem and was demonstrated in Robinson (1995b). The proposed statistic has an expression that is very close to the previous $W = \sup_{r \in (\epsilon,1]} \left(\sum_{j=1}^m v_j^2 \right)^{-1/2} \left| \sum_{j=1}^{[mr]} v_j \left(\frac{I_j}{G(\widetilde{d}) \lambda_j^{-2\widetilde{d}}} - 1 \right) \right|$, where \widetilde{d} is the Whittle Local semiparametric estimator, m is the number of frequencies in the estimation procedure (for $m = T^{0.7}$ a good balance is achieved between size and power, and it is thus suggested in practice) and ϵ is a trimming parameter. Note that $\epsilon \in (0,1)$ establishes the minimum number of summands in the sum; thus, the value of the statistic W and its critical values are both decreasing functions of ϵ . In practice, if the sample size is small, then the use of $\epsilon = 5\%$ is recommended, but in large samples it is possible to use smaller trimmings.

This statistic diverges for two types of spurious process: the first is the model with short memory and stochastic regime switching, which was considered in Perron and Qu (2007). The second model is one that combines short memory with a trend component with smooth variation. Simulations for a set of processes are utilized to evaluate the size of the statistic. Included among these statistics are ARFIMA models with or without short memory components and fractionally integrated processes with measurement errors. The size of the statistic W is stable for different sample sizes, processes, and values of m and ϵ . There is a clear trend towards over-rejecting the null hypothesis when m is greater; that is, when $m = T^{0.75}$.

Qu (2011) also evaluates the size and power of the other statistics for sample sizes of up to 9,000 observations. The statistic $\hat{\mathbf{W}}$ shows the best size properties. The statistic W_c tends to over-reject the long-memory hypothesis primarily for the process with measurement errors (size of 11%). The other statistics $(\eta_{\mu}, Z_t \text{ and } mean - t_d)$ display sizes that decrease as the sample size increases. To evaluate the power property, six models were considered, the first five of which are dealt with in Ohannisian et al. (2008), and the sixth contains a smooth but non-monotonic trend. For all processes, except that which displays a monotonic trend, the statistic W with $\epsilon = 2\%$ displays the best properties. The power of the statistics $\hat{\mathbf{W}}$, Z_t and $mean - t_d$ is generally lower. It is noteworthy that Ohannisian et al. (2008) show that the power of the statistic $\hat{\mathbf{W}}$ is 100% for T = 610,304 but for sample sizes of less than 9,000 observations Qu (2011) shows that this statistic has power below 50%.

The statistic η_{μ} has a good performance for models with non-stationary regime switching and models with monotonic and non-monotonic trends, but its power is very low in Markov Switching processes and with stationary stochastic regime switching. For its part, the statistic W_c has a power of 70% for stationary and non-stationary regime switching processes and for Markov Switching processes with independent and identically distributed regimes, but their power is reduced for the other processes.

In summary, the properties of the statistics depend on the spurious memory-generating process and the sample size. For processes with possible stochastic regime switching and Markov Switching, the results of the statistics W and W_c are the most reliable.

3 Empirical Results

The information that we use is taken from Bloomberg. The main indices of the stock markets in Peru, Brazil, Mexico, Chile, Argentina, and United States were selected. Other countries are excluded due to their smaller sample sizes, such as Colombia, which only has data for 2001 onwards.

The series of stock market returns for each country are constructed as the differences in their end-of-day quotes $r_t = \log(P_t/P_{t-1})$. Volatility is calculated as $\log(|r_t| + 0.001)$; that is, the values of the volatility that are very negative are truncated adding a small constant of 0.001 to the argument⁶. There are other proxies of utility, such as the logarithm of squared returns, which do not alter the main results of the estimations.

⁶This constant is called "offset constant" and is much-used in the literature on stochastic volatility, following on from Kim et al. (1998), among others.

The samples for each country are detailed as follows. For Peru, there are 5,970 observations (January 2, 1990 to December 31, 2013); for Brazil, we have 5,440 observations (January 2, 1990 to December 3, 2013); for Mexico, we have 4,979 observations (January 19, 1994 to December 31, 2013); for Chile, there are 6,231 observations (January 2, 1989 to December 30, 2013) and for Argentina, we have 6,351 observations (April 11, 1988 to December 30, 2013). Solely by way of comparison, we also use information for S&P500⁷ with 7,824 observations (January 2, 1980 to December 31, 2013).

The descriptive statistics for the returns and the volatility are summarized in Table 1. The first panel shows the statistics for the series of returns, with values close to zero in the mean and the variance of all series. The standard deviation is slightly higher in the stock market returns of Brazil and Argentina in line with a broader range of values. The sign and the magnitude of the asymmetry coefficient differ from country to country, and in particular, the stock market returns of Brazil, Mexico, Argentina, and S&P500 present negative asymmetry. Also, for Brazil, Argentina and S&P500, the kurtosis coefficient is higher, the other series are leptokurtic a lesser degree. Finally, the Jarque Bera statistic strongly rejects the hypothesis of Normality of all the series covered, especially for Brazil, Argentina, and S&P500.

In the second panel, statistics are presented for the volatility series that show similar mean values, standard deviation, and extreme values. For Peru, Mexico and Chile, the mean is close to -4.8 and the standard deviation is 0.9 on average; for Brazil and Argentina, the mean is -4.4 and the standard deviation is 1.0. The statistics for the S&P500 are a little different to the rest of the indices, especially in the asymmetry coefficient which is zero, while in the other indices, this indicator is negative. A coefficient of negative asymmetry indicates that the distribution has a left tail with few low values. The excess kurtosis values are close across the volatilities and on average is -0.339, which is slightly below the value of 0 corresponding to the Gaussian distribution. Finally, in accordance with the Jarque Bera Normality statistic, all series show significant deviation from the assumption of Normality.

The first objective of this paper is to provide indications of the presence of long memory in the volatility series of stock market returns. Figures 1 to 3 provide visual evidence of long memory in all volatility series. The first panel in Figure 1 shows the spectral density⁸ of stock market returns, which gives the weight of each frequency to the series variability. That which corresponds to Peru displays an upward trend for frequencies close to the origin; in the remaining countries, the density functions are more stable and with similar values for all frequencies. In the second panel that corresponds to the volatility series, the presence of long memory is much more notable in all cases.

Figure 2 shows the ACFs of the returns up to 1000 lags, and display values within confidence bands. For Peru and Brazil, the rate of decay to zero is a little slower. In Figure 3, the behavior is clearly one of slow decay, displaying long memory in all volatility series. The persistence here is much more pronounced than in the series of returns, with values above and below zero confidence bands for greater lags.

The next exercise consists in estimating statistics for the presence of long memory and ARCH effects or autocorrelation in the volatility series. The first four columns of Table 2 display statistics for the presence of autocorrelation, which are significant in all series. These can be modeled by incorporating autoregressive or moving average components. The next four columns display the

⁷The estimation period is the same as in Qu and Perron (2013) which covers the crisis of 1987 and 2008.

⁸These results have been obtained using the Oxmetrics program. In the estimation of spectral density, the value of 20 was used for the lag truncation parameter, which is the default value in Oxmetrics.

statistics for detecting long memory, which are the RS statistics of Mandelbrot (1972) and Lo (1991) and the semiparametric estimators \hat{d} and \tilde{d} , which were presented in the previous Section. The results are similar, the RS statistics reject the absence of long memory in all series. The estimators \hat{d} and \tilde{d} show that long memory exists to different degrees in all series. The values posted by the two types of estimators are quite close to one another. The values are positive and fluctuate between 0.301 and 0.464 within the range that corresponds to the stationary processes. The value of the estimator is very similar in Peru, Chile, and S&P500, and in Mexico and Argentina, but is slightly greater for Brazil.

We also estimate ARFIMA(p,d,q) models for the volatility series. According to Stărică and Granger (2005), the ARFIMA(1,d,1) specification has good performance in terms of data fit when the series display long memory. This simple specification is also employed in Perron and Qu (2010) for the calculation of the critical values of their statistics; in consequence, this exercise may be useful in evaluating the degree of long memory in the series. Table 3 shows the results⁹. The values of the parameters are similar across all series in value and sign. The estimator \hat{d} for Peru and Chile is close to 0.35. In the other markets, this parameter is around 0.43, revealing stationary long memory in the entire volatility series. The statistics for the residuals suggests that there is heteroskedasticity that has not been captured in the ARFIMA modeling. The Jarque Bera statistic confirms the non-Normality of the residuals. This result suggests that it may be useful to include a parameter in the model that captures the presence of ARCH effects in the residuals.

Thus, the next exercise consists in estimating FIGARCH models¹⁰ for the series of returns. The volatility of the returns is determined within the model based on the equation of conditional variance. Table 4 summarizes the results. Various estimations were estimated, including variants in the equation of the mean and the variance. For Peru and Brazil, it is significant to include fractional integration in the mean, which suggests some trace of persistence in the series of returns of these markets; see also Figure 2. This fact is consistent, with which the statistics, the ACF and the spectral density report. The estimation of the FIGARCH model reports significant fractional integration in the volatility. The parameter \hat{d} in the equation of variance is close to 0.5 in all series; for Brazil, Mexico and S&P500, this parameter is greater than 0.5. It is interesting to note that when parameters are included to model the possibility of asymmetry in the volatility, the parameter d falls. In the case of Peru, the FIEGARCH specification drastically reduces the parameter \hat{d} to a value close to zero¹¹. The parameter of asymmetry is negative, which confirms the presence of leverage effects; that is, that the negative returns increase the series volatility more than the positive returns. In the case of Mexico, the FIAPARCH specification reduces the value of the parameter d to a level below 0.5, which does not eliminate the fractional integration of the volatility series but assures its stationarity. The statistics for the residuals show that they are not Normal, but the ARCH effects have been considerably reduced, as can be seen on Table 5.

The second aim of this paper is to investigate whether the indications of long memory displayed are due to the presence of fractional integration or to other causes. To identify whether this behavior in the volatility series is better represented in the volatility series by processes with true or spurious

⁹The estimation was carried out on the PC-Give module of Oxmetrics. In all cases, strong convergence was observed in the optimization procedure.

¹⁰The estimation was performed in the module G@ARCH of Oxmetrics. Student's t distribution was selected for the residuals instead of the Gaussian distribution to take into account the excess of kurtosis in the returns. Student's asymmetric t distribution was also tried, but its parameters were not statistically significant.

¹¹This can be interpreted as the fact that the introduction of non-linearities produces no long memory.

long memory, we will utilize the statistics mentioned in the previous Section. Table 6 shows the results. The two first columns show the estimated values of the statistic W of Qu (2011) with a number of frequencies $m=T^{0.7}$ and for two alternative values of the trimming ϵ equal to 2% and 5%, respectively. This statistic strongly rejects the presence of a true long memory process in the volatility series of Peru, Brazil, Argentina and S&P500. The level of significance is 5%, except for Peru in the case where $\epsilon=2\%$ (significance at 10%). Although this statistic does not succeed in rejecting the presence of long memory for Mexico's series of volatility, it is observed that the calculated values of the statistic are close to its critical values, so the non-rejection in this case is not as convincing as it is for Chile.

The statistic W_c of Shimotsu (2006) rejects the null hypothesis for Mexico with significance to 5%. This statistic also validates the result of the statistic W for Brazil, Argentina and S&P500, but not for Peru. The statistic η_{μ} , also due to Shimotsu (2006), shows evidence of spurious memory for Brazil and Mexico. The non-rejection of the statistic for the other countries is likewise not very convincing in this case, as the values are quite close to the critical points of the statistic to 10% (0.33 on average).

In the case of Shimotsu's Z_t statistic (2006), there is no rejection of the long memory hypothesis for any series, and the values are quite far from the estimated critical points of this statistic. It is interesting that this statistic shows a strong trend towards non-rejection of the null hypothesis while the statistic η_{μ} does not. Following Shimotsu (2006), the statistic η_{μ} has a high power against spurious memory models induced by random level shift components, while the power of the statistic Z_t is close to zero in these processes.

Perron and Qu's $t_d(1/2, 1; 4/5, 1)$ and $mean - t_d$ statistics (2010) are closely related to the behavior of the estimator d as a function of the range of frequencies m in the regression of the periodogram. The statistic $t_d(1/2, 1; 4/5, 1)$ evaluates whether there is a significant decline in the value of \hat{d} for values of m greater than $T^{1/2}$, while a rapid decline in the value of \hat{d} for values of m between $T^{1/3}$ and $T^{1/2}$ is the evidence of the statistic $mean - t_d$ to detect spurious memory. The results are quite informative. There is a strong rejection of the null hypothesis in favor of spurious memory for all volatility series based on the statistic $t_d(1/2, 1; 4/5, 1)$. Nonetheless, the statistic $mean - t_d$ validates this result only for the S&P500. This can be explained in Figures 4 to 9. Panels (b) show the estimator \hat{d} calculated utilizing different values of m, and therein it is clear that the decrease in the estimator \hat{d} based on a certain number of frequencies is best captured by the statistic $t_d(1/2, 1; 4/5, 1)$, which evaluates frequencies greater than $T^{0.5}$. In the frequencies closer to the origin it is not clearly observed that the values of the parameter are strongly reduced, as in these frequencies the behavior of the estimator for all series is more volatile, which does not allow the statistic $mean - t_d$ to capture the decreasing trend of the estimator. Note that the decrease in the fractional estimator starts from a value of around d > 0.5 for Brazil, Mexico, Argentina and the S&P500. But no clear structural change is observed in the behavior of the fractional parameter. This behavior explains the difficulties in the different statistics to provide conclusive results.

Panels (c) in these Figures show the behavior of the estimator d as a function of sample size; that is, the values of \hat{d} estimated by subsamples greater than 300 in size. The estimator for Mexico, Argentina, and the S&P500 is growing in sample size, including up to the whole sample, but stabilizes around values such as 0.30-0.35. Meanwhile, the estimators for Peru are decreasing from a value that is slightly higher than 0.5 but tends, rather, to stabilize for large sample sizes at values close to 0.3. For Brazil and Chile an initial decline is observed, followed by rapid growth, but the values stabilize at around 0.35.

In the case of Chile in panel (b) of Figure 7, a rapid reduction in the estimator \hat{d} is observed in the frequencies close to the origin, but they rapidly increase again. The subsequent decrease seems to occur subsequently around the frequency m=160. In this frequency, a strong decline is produced, followed by a smoother decreasing trend for the parameter. With regard to this, an attempt has been made to estimate the statistic utilizing alternative sampling periods. Table 6 shows the values of the statistics for the sample period 2005-2013, and the rejection of the null hypothesis of the statistic W can be observed, with trimming of 5%. For Mexico, Table 6 presents the results of the statistics in the alternative period 2004-2013, but in this case no rejection is observed, even though the value of the statistic W in this case is closer to its critical value at 10%.

As can be observed, the results change depending on the period utilized in the estimation. The volatility series could have periods that post true long memory and other periods that would be better explained by a spurious memory component. In order to evaluate the sensitivity of the statistic W, the following exercises are carried out, and generate Figures 10 to 13. Figures 10 and 11 show the estimated values of the statistic W with trimming of 5% and 2% respectively, and utilizing subsamples with observations up to the observation i, with i in the range of 100 up to the sample total (the end of the sample changes in each estimation). The continuous and discontinuous horizontal lines correspond to the critical values at 10% and 5%, respectively. This exercise shows the sensitivity of the statistic W when the end of the estimation period changes. Even though there is variability in the results, it is clear that Peru, Brazil and Argentina reject the hypothesis of long memory for different subsample sizes. In the case of Mexico, however, the rejection of null hypothesis occurs only if the first 700 observations that correspond approximately to the period that runs from 1994 to 1997¹² are considered, but from then on the behavior of the Mexican stock market's volatility series appears to correspond to a process with true long memory. Meanwhile, it can be seen that Chile does not reject the null hypothesis for any subsample, so in this exercise this behavior corresponding to true long memory cannot be discarded.

Figures 12 and 13 show the values of the statistic W estimated with trimming of 5% and 2% respectively, utilizing subsamples formed on changing the start of the estimation period. Each subsample is formed from the observation i and up to the end of the sample, with i in the range of 1 up to the reduced sample size at 100 observations. For Peru the hypothesis of true long memory is rejected when the complete sample is used, but when subsamples that start from observation 100 onward are considered, the values of the statistic reduce and this hypothesis can no longer be rejected. From observation 3800 approximately, the values of the statistic show again rejection. In Brazil and Argentina many sample start dates can be observed, so the long memory hypothesis is rejected. In the case of Mexico, it is seen that from observation 2800 approximately there are rejections of the hypothesis, but these are sporadic and return to the area of non-rejection. The same can be said of Chile, where we can obtain rejections of the null hypothesis if the sample starts at observation 4000 approximately, which corresponds to 2005. We consider evidence from Figures 10 and 11 as complementary to these of Figures 12 and 13.

The evidence reported graphically and through the statistics suggests that the generating process of the volatility series is spurious memory, except for the case of Chile whose evidence of long memory is weak. Despite the results of the statistics being mixed, all series reject the hypothesis of true long memory, at least for some of the statistics. Moreover, Figures 4 to 9 contain sufficient information on the spurious memory behavior, as if it were a case of true long memory,

¹²This is interesting to point out, as in 1994 the so called "Tequila" crisis arose, which had significant effects in Mexico and in the Latin American region.

ideally these graphs should not display variation in this argument.

The results of this study suggest that in reality, the long memory that is usually found is, rather, associated with spurious memory, which could be due to the presence of structural breaks more than to true long memory. The date of the structural breaks present in this series could be estimated utilizing procedures that have been developed in the literature. For example, Fernández (2005) identifies the dates of structural breaks in the volatility of stock market returns and for the period 1997-2002 find 4 dates of break for the Latin American exchanges, namely: 1997 and 1998, February 1999 and June 2000. These dates are associated with the Asian crisis.

Even though this paper takes a testing approach, there are other paths for responding to the research question and that incorporate the possibility of structural breaks to explain the long memory present in the volatility series, especially from the focus on econometric modeling. Lu and Perron (2010), Li and Perron (2013) and Xu and Perron (2014) apply a model based on non-observable components where the level-shift component (random and sporadic) is that which inserts the long-memory effect into the series. This type of component has also been incorporated into a stochastic volatility model in Qu and Perron (2013).

Humala and Rodríguez (2013) have established the stylized facts for the Peruvian exchange rate and stock market. This reference establishes a research agenda that is being followed, and this paper ranks among them. Ojeda Cunya and Rodríguez (2014), Rodríguez, Tramontana Tocto (2014), and Rodríguez (2014) use a Random Level shifts (RLS) model as in Lu and Perron (2010), Li and Perron (2013) and Xu and Perron (2014). They find that the long memory disappears once the random level shifts have been discounted. The first of these studies apply the volatility of the Peruvian stock and exchange rate market, and the other reference applies to volatility in the markets of Latin American countries. In the case of Herrera and Rodríguez (2014), the statistics $mean - t_d$ and $\sup - t_d$ are used for the volatility of the Peruvian exchange rate and stock market. The evidence does not support the rejection of the long memory hypothesis, perhaps due to the amount of data and the sampling period but also because of the lesser power of the statistics, as seen in Qu (2011).

4 Conclusions

In this study, we investigate the long term dependence or long memory present in the volatility of the stock market returns of Peru, Brazil, Mexico, Chile, Argentina, and the S&P500 index. The analysis of long memory in the series proceeds from the statistics and from the form of the ACF and the estimated spectral density. Moreover, volatility is modeled by way of FIGARCH processes that contribute additional indications of this behavior.

To verify the presence of true long memory, the W statistics of Qu (2011), W_c , η_{μ} and Z_t due to Shimotsu (2006), and the statistics $t_d(1/2, 1; 4/5, 1)$, and $mean - t_d$ of Perron and Qu (2010) are used. Also presented are graphics as in Qu (2011), which show the behavior of the long memory estimator \hat{d} for different sample sizes included in the estimation procedure.

The evidence reported graphically and through the statistics suggest that the generating process of the volatility series is spurious memory, except for Chile, whose evidence against spurious memory is weak. Despite the results of these statistics being mixed, all series reject the hypothesis of true long memory at least for some of the statistics. Moreover, the graphics contain sufficient information on the spurious memory behavior, as if the case were to be true long memory, ideally these graphs would not present variation in their argument.

The results of this study suggest that in reality, the long memory that is usually found would rather be associated with spurious memory, which could be due to the presence of structural breaks more that true long memory. Though this paper takes a testing approach, there are other routes to respond to the research question and which incorporate the possibility of structural breaks to explain the long memory present in the volatility series, especially from an econometric modeling approach that is now underway for Peru and a group of Latin American countries.

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Table 1. Summary Statistics of Series

| | Mean | SD | Max | Min | Skew | Exc. Kur | JB | | | | |
|------------------------------------|--------|-------|-------------|----------------|---------|----------|----------|--|--|--|--|
| Returns: $r_t = \log(P_t/P_{t-1})$ | | | | | | | | | | | |
| Peru | 0.002 | 0.017 | 0.144 | -0.133 | 0.519* | 8.149* | 16,788* | | | | |
| Brazil | 0.002 | 0.028 | 0.345 | -0.395 | -0.036 | 28.018* | 177,932* | | | | |
| Mexico | 0.001 | 0.016 | 0.121 | -0.143 | -0.020 | 6.633* | 9,127* | | | | |
| Chile | 0.001 | 0.012 | 0.118 | -0.077 | 0.232* | 5.382* | 6,054* | | | | |
| Argentina | 0.002 | 0.033 | 0.329 | -0.757 | -0.576* | 53.477* | 757,120* | | | | |
| S&P500 | 0.000 | 0.011 | 0.110 | -0.229 | -1.209* | 27.974* | 257,010* | | | | |
| | | Vol | atility: lo | $\log(r_t +$ | 0.001) | | | | | | |
| Peru | -4.863 | 0.949 | -1.932 | -6.908 | -0.023 | -0.382* | 36.928* | | | | |
| Brazil | -4.392 | 0.973 | -0.927 | -6.908 | -0.255* | -0.174* | 65.642* | | | | |
| Mexico | -4.802 | 0,893 | -1.937 | -6.908 | -0.164* | -0.393* | 54.411* | | | | |
| Chile | -4.992 | 0.844 | -2.128 | -6.908 | -0.156* | -0.464* | 81.209* | | | | |
| Argentina | -4.409 | 1.086 | -0.277 | -6.908 | -0.244* | -0.178* | 71.393* | | | | |
| S&P500 | -5.093 | 0.839 | -1.470 | -6.908 | 0.000 | -0.445* | 64.445* | | | | |

[&]quot;SD", "Skew", "Exc. Kur" and "JB" stand for standard deviation, skewness, excess kurtosis and Jarque Bera, respectively. * denotes significance at 1%.

Table 2. ARCH and Long Memory Statistics for Volatility Series

| | ARCH(20) | ARCH (50) | Q(20) | Q(50) | H-M R/S | Lo R/S | \widehat{d} | $\widetilde{\widetilde{d}}$ |
|-----------|----------|-----------|-------|-------|---------|--------|---------------|-----------------------------|
| Peru | 46.421 | 18.410 | 4096 | 6527 | 8.624 | 3.861 | 0.333 | 0.331 |
| Brazil | 32.140 | 14.049 | 3296 | 6516 | 8.863 | 4.152 | 0.464 | 0.404 |
| Mexico | 22.100 | 10.121 | 1662 | 2963 | 6.180 | 3.213 | 0.353 | 0.427 |
| Chile | 28.441 | 12.102 | 2024 | 2962 | 6.187 | 3.187 | 0.301 | 0.347 |
| Argentina | 46.188 | 19.601 | 4392 | 8460 | 9.260 | 4.204 | 0.402 | 0.420 |
| S&P500 | 42.339 | 19.102 | 3318 | 6870 | 7.899 | 3.988 | 0.345 | 0.430 |

Statistics are significant at 1%.

Table 3. ARFIMA Models for Volatility Series

| | Peru | Brazil | Mexico | Chile | Argentina | S&P500 |
|----------------|---------|---------|---------|---------|-----------|---------|
| \overline{d} | 0.341 | 0.405 | 0.414 | 0.346 | 0.462 | 0.461 |
| | (0.028) | (0.033) | (0.041) | (0.033) | (0.039) | (0.036) |
| μ | -4.854 | -4.350 | -4.801 | -4.981 | -4.444 | -5.082 |
| | (0.193) | (0.286) | (0.284) | (0.161) | (0.689) | (0.445) |
| ϕ | 0.229 | 0.179 | 0.336 | 0.359 | 0.252 | 0.222 |
| | (0.083) | (0.033) | (0.032) | (0.045) | (0.027) | (0.021) |
| θ | -0.409 | -0.581 | -0.683 | -0.576 | -0.662 | -0.699 |
| | (0.096) | (0.047) | (0.039) | (0.056) | (0.041) | (0.031) |
| Log Likelihood | -7,613 | -7,135 | -6,235 | -7,450 | -9,003 | -9,224 |
| AIC | 2.552 | 2.625 | 2.508 | 2.393 | 2.837 | 2.359 |
| Jarque Bera | 179.95 | 464.81 | 229.52 | 337.11 | 69.641 | 196.56 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ARCH(1) | 50.639 | 5.056 | 5.306 | 4.120 | 379.02 | 24.699 |
| | (0.000) | (0.024) | (0.021) | (0.042) | (0.000) | (0.000) |
| Q-Portmanteau | 83.376 | 69.743 | 90.270 | 65.645 | 117.850 | 95.675 |
| | (0.213) | (0.486) | (0.030) | (0.771) | (0.002) | (0.201) |

For parameters, standar desviations are given in parenthesis. For tests, p-values are given in parenthesis.

Table 4. Results for FIGARCH, FIEGARCH and FIAPARCH Models

| | Peru | | Brazil | | | xico | Chile | Argentina | S&P500 | | |
|------------------|---------|---------|---------|---------|------------|---------|---------|-----------|---------|--|--|
| Conditional Mean | | | | | | | | | | | |
| d | 0.116 | 0.113 | 0.285 | = | = | = | - | - | - | | |
| | (0.015) | (0.014) | (0.042) | | | | | | | | |
| μ | - | - | - | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | | |
| | | | | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | | |
| ϕ | - | - | 0.554 | 0.037 | 0.092 | 0.096 | 0.238 | 0.054 | - | | |
| | | | (0.038) | (0.013) | (0.014) | (0.014) | (0.013) | (0.012) | | | |
| β | 0.152 | 0.164 | -0.809 | - | - | - | - | - | - | | |
| | (0.019) | (0.018) | (0.022) | | | | | | | | |
| | | | | Condit | ional Vari | ance | | | | | |
| d | 0.477 | 0.021 | 0.565 | 0.587 | 0.505 | 0.337 | 0.438 | 0.471 | 0.546 | | |
| | (0.043) | (0.008) | (0.073) | (0.068) | (0.066) | (0.046) | (0.046) | (0.048) | (0.068) | | |
| w | - | 12.469 | 0.125 | 0.118 | 0.041 | - | 0.066 | 0.264 | - | | |
| | | (0.032) | (0.034) | (0.031) | (0.015) | | (0.013) | (0.051) | | | |
| ϕ | - | - | 0.089 | 0.091 | 0.227 | 0.253 | - | - | 0.177 | | |
| | | | (0.043) | (0.039) | (0.047) | (0.064) | | | (0.030) | | |
| β | 0.217 | 0.917 | 0.578 | 0.602 | 0.625 | 0.510 | 0.254 | 0.365 | 0.695 | | |
| | (0.049) | (0.009) | (0.085) | (0.072) | (0.072) | (0.082) | (0.053) | (0.058) | (0.052) | | |
| γ | - | -0.026 | - | - | - | 0.563 | - | - | - | | |
| | | (0.012) | | | | (0.101) | | | | | |
| θ | - | 0.432 | - | - | - | - | - | - | - | | |
| | | (0.030) | | | | | | | | | |
| δ | - | - | - | - | - | 1.765 | - | - | - | | |
| | | | | | | (0.081) | | | | | |

Standard desviations are given in parenthesis.

Table 5. Statistics for FIGARCH, FIEGARCH and FIAPARCH Models

| | Pe | eru | Br | azil | Me | xico | Chile | Argentina | S&P500 |
|----------------|---------|---------|---------|---------|---------|---------|---------|-----------|---------|
| Log Likelihood | 17893 | 17950 | 13451 | 13419 | 14528 | 14579 | 19909 | 15180 | 25592 |
| AIC | -5.992 | -6.012 | -4.942 | -4.931 | -5.833 | -5.853 | -6.388 | -4.779 | -6.540 |
| Jarque Bera | 1332 | 1332 | 647 | 569 | 1235 | 1230 | 203 | 6527 | 15352 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ARCH(1) | 0.670 | 0.048 | 1.017 | 1.176 | 0.748 | 0.200 | 0.174 | 4.898 | 17.407 |
| | (0.406) | (0.826) | (0.313) | (0.278) | (0.387) | (0.654) | (0.677) | (0.027) | (0.000) |
| Q(50) | 78.534 | 80.152 | 51.628 | 110.000 | 45.559 | 46.129 | 88.869 | 86.362 | 76.454 |
| | (0.005) | (0.003) | (0.334) | (0.000) | (0.532) | (0.590) | (0.000) | (0.001) | (0.009) |

p-values are given in parenthesis.

Table 6. Test Statistics Against Spurious Long Memory

| | $W~(\epsilon=2\%)$ | $W~(\epsilon=5\%)$ | W_c | η_{μ} | Z_t | $t_d(1/2, 1; 4/5, 1)$ | $mean - t_d$ |
|-------------------|--------------------|--------------------|--------------|--------------|--------|-----------------------|--------------|
| Peru | | | | | | | |
| (1990.01-2013.12) | 1.242^{c} | 1.242^b | 3.797 | 0.204 | -1.094 | 5.239^{a} | -0.217 |
| Brazil | | | | | | | |
| (1992.01-2013.12) | 1.483^{b} | 1.297^{b} | 8.101^{b} | 0.757^{a} | 0.096 | 8.093^{a} | 0.485 |
| Mexico | | | | | | | |
| (1994.01-2013.12) | 0.729 | 0.729 | 8.474^{b} | 0.409^{c} | -0.692 | 6.092^{a} | 0.848 |
| (2004.01-2013.12) | 0.826 | 0.826 | 1.281 | 0.236 | -1.162 | 4.889^{a} | 0.693 |
| Chile | | | | | | | |
| (1989.01-2013.12) | 0.484 | 0.484 | 0.810 | 0.272 | -1.143 | 5.163^{a} | 0.459 |
| (2005.01-2013.12) | 1.113^{c} | 1.113 | 0.869 | 0.166 | -1.365 | 4.761^{a} | -0.502 |
| Argentina | | | | | | | |
| (1988.04-2013.12) | 1.295^{b} | 1.272^{b} | 9.526^{b} | 0.257 | -1.078 | 8.235^{a} | -0.342 |
| S&P500 | | | | | | | |
| (1980.01-2010.12) | 1.481^{b} | 1.345^{b} | 12.916^{a} | 0.184 | -1.324 | 8.205^{a} | 1.053^{b} |

a,b,c denote significance at 1%, 5%, and 10% levels, respectively.

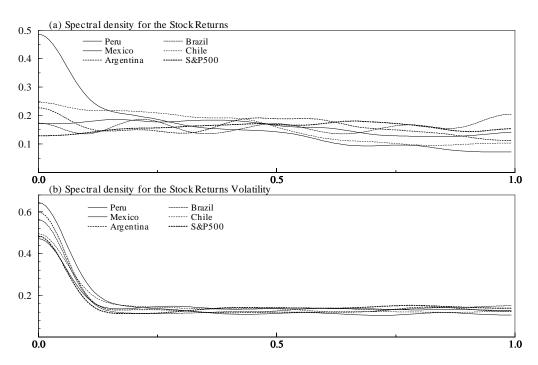


Figure 1. Spectral Density of Stock Returns and Stock Volatilities

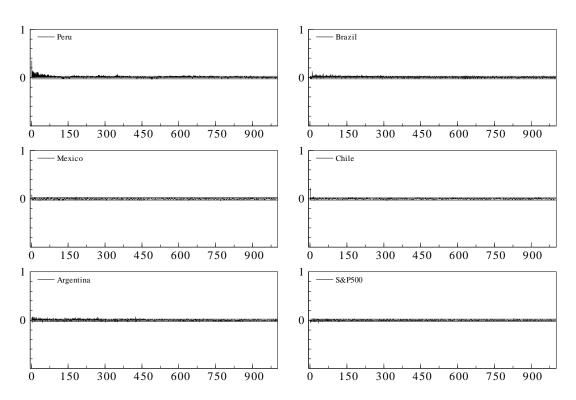


Figure 2. Autocorrelation Functions (ACF) for Stock Returns

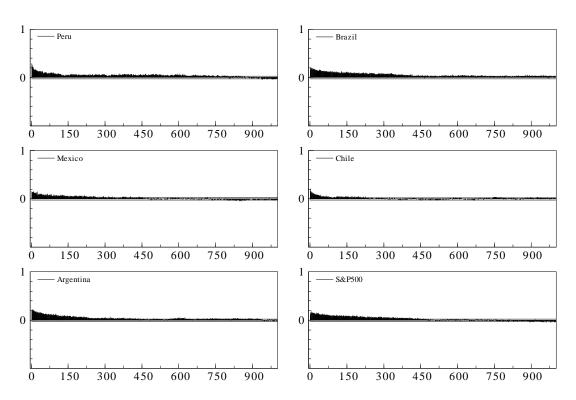
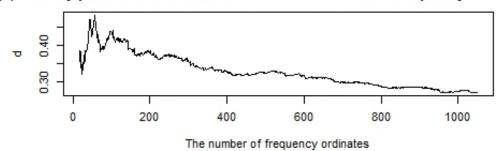


Figure 3. Autocorrelation Functions (ACF) for the Stock Returns Volatility

(a) The series: 1990-2013

(b) Memory parameter estimates with different numbers of frequency ordinates (m)



(c) Memory parameter estimates using different subsamples

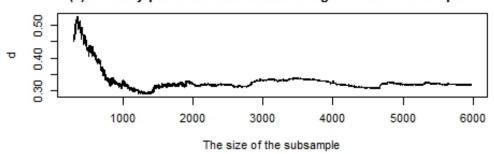
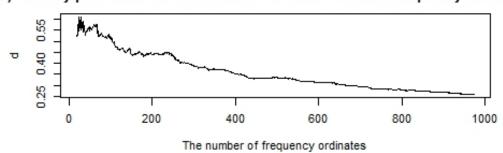


Figure 4. Results for the Stock Returns Volatility of Peru

(a) The series: 1992-2013

(b) Memory parameter estimates with different numbers of frequency ordinates (m)



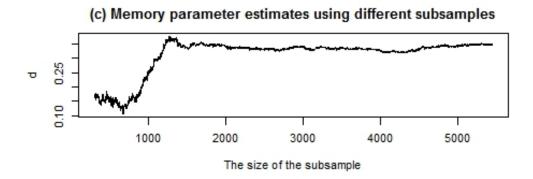
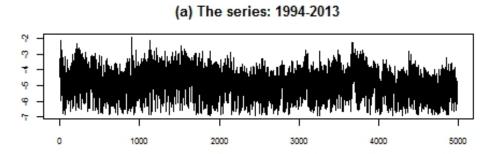
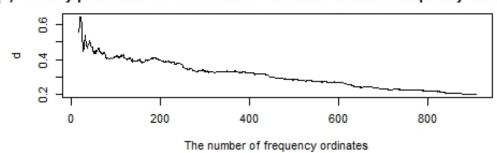


Figure 5. Results for the Stock Returns Volatility of Brazil $\,$



(b) Memory parameter estimates with different numbers of frequency ordinates (m)



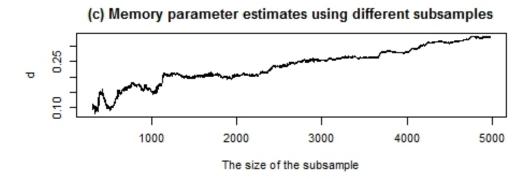
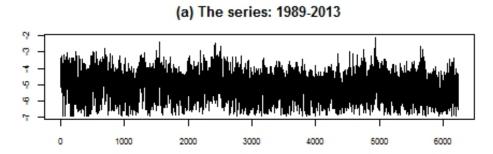
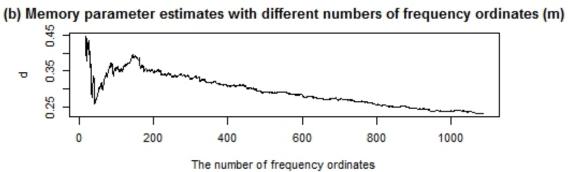


Figure 6. Results for the Stock Returns Volatility of Mexico





(c) Memory parameter estimates using different subsamples

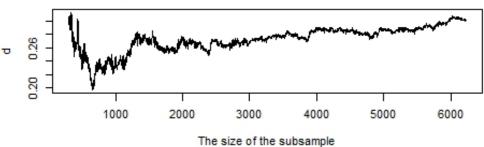
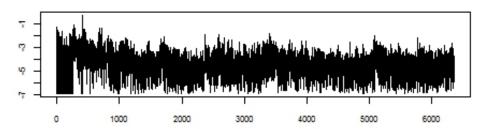
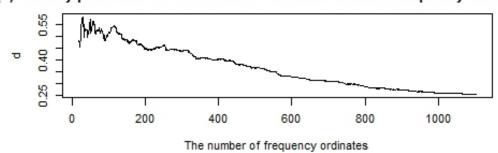


Figure 7. Results for the Stock Returns Volatility of Chile

(a) The series: 1990-2013



(b) Memory parameter estimates with different numbers of frequency ordinates (m)



(c) Memory parameter estimates using different subsamples

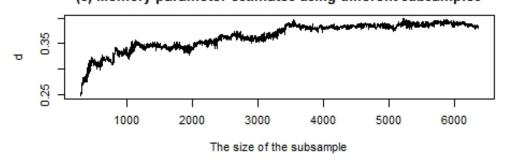
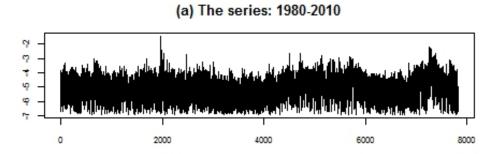
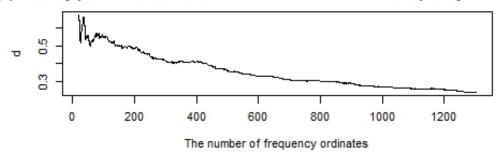


Figure 8. Results for the Stock Returns Volatility of Argentina



(b) Memory parameter estimates with different numbers of frequency ordinates (m)



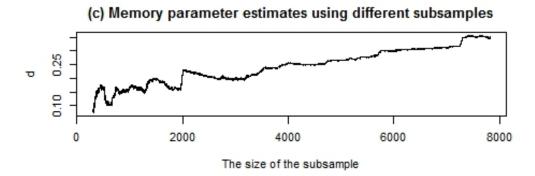
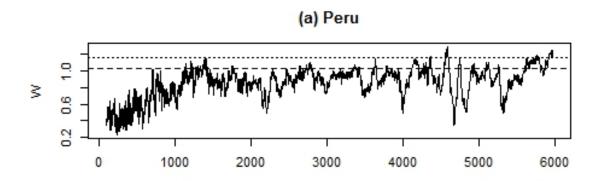
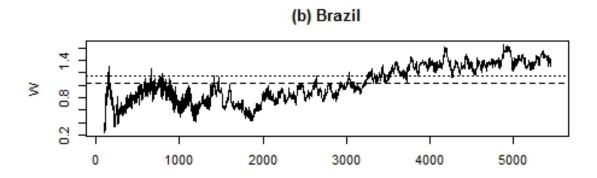


Figure 9. Results for the Stock Returns Volatility of ${\rm S\&P500}$





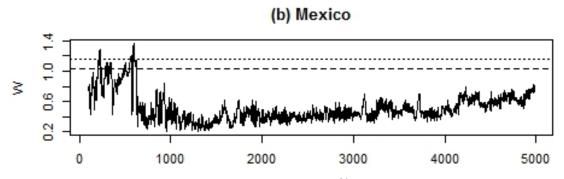
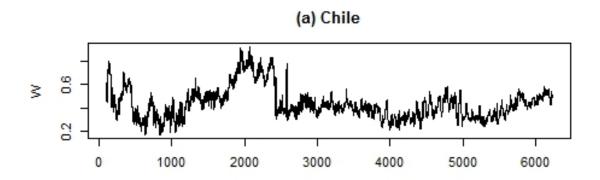
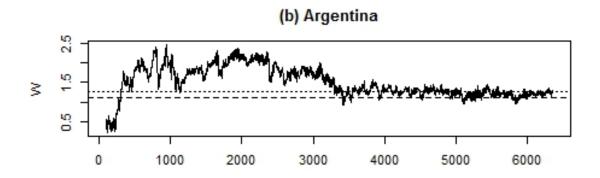


Figure 10. Sequential Values of the test W for a Trimming of 5%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.





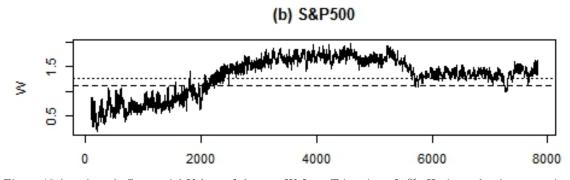
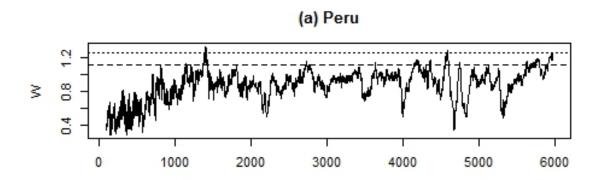
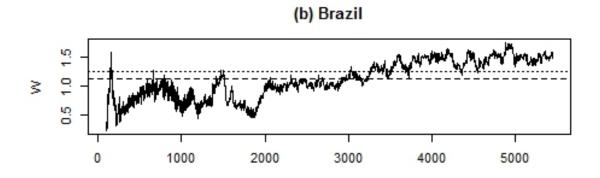


Figure 10 (continues). Sequential Values of the test W for a Trimming of 5%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.





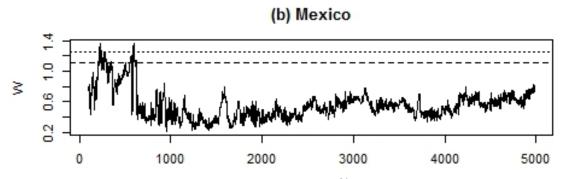
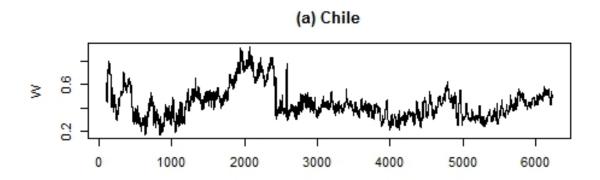
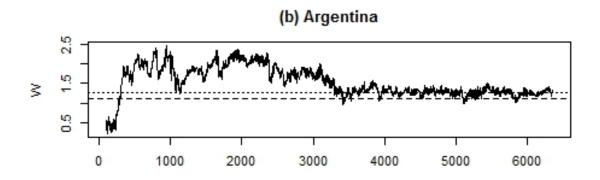


Figure 11. Sequential Values of the test W for a Trimming of 2%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.





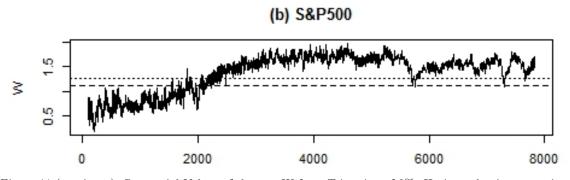
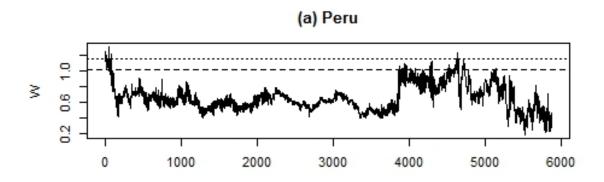
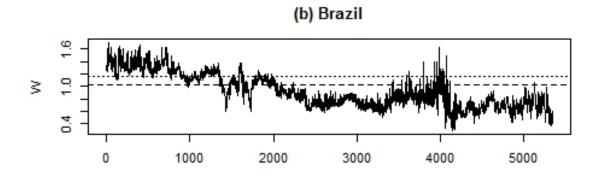


Figure 11 (continues). Sequential Values of the test W for a Trimming of 2%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.





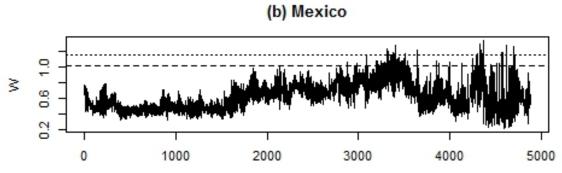
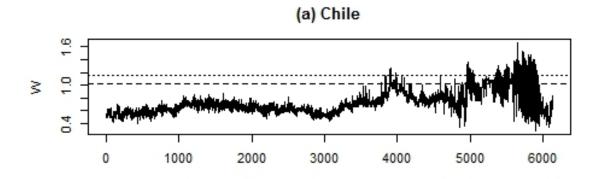
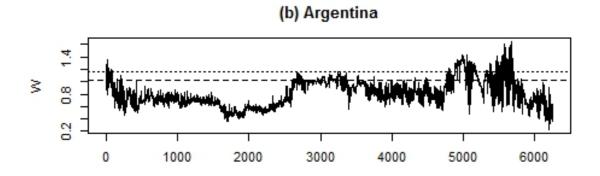


Figure 12. Sequential Values of the test W for a Trimming of 5%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.





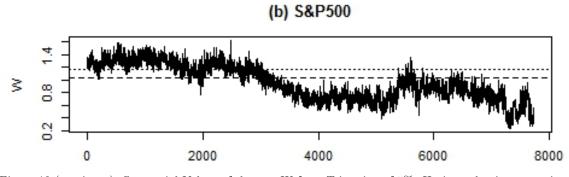
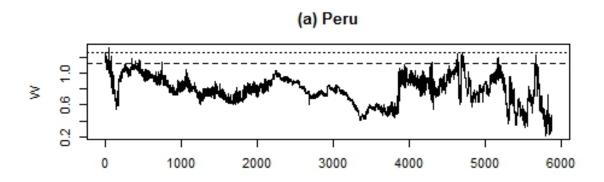
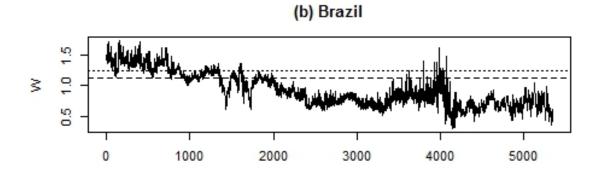


Figure 12 (continues). Sequential Values of the test W for a Trimming of 5%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.





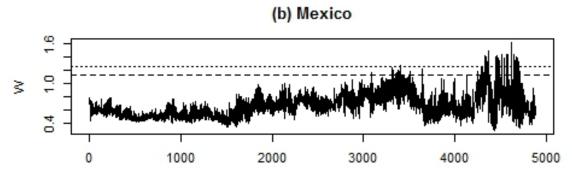
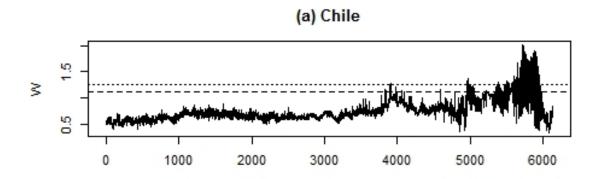
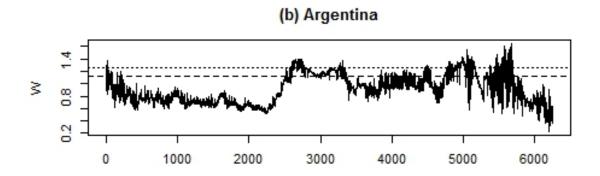


Figure 13. Sequential Values of the test W for a Trimming of 2%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.





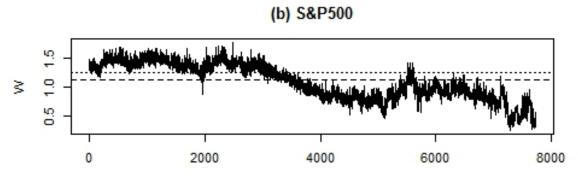


Figure 13 (continues). Sequential Values of the test W for a Trimming of 2%. Horizontal axis means size of subsamples. Horizontal solid and dotted lines indicates critical values at 10% and 5%, respectively.

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Libros

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Piero Ghezzi y José Gallardo

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