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Av. Universitaria 1801, Lima 32 – Perú.
Teléfono: (51-1) 626-2000 anexos 4950 - 4951
Fax: (51-1) 626-2874
econo@pucp.edu.pe
www.pucp.edu.pe/departamento/economia/

Encargado de la Serie: Jorge Rojas Rojas
Departamento de Economía – Pontificia Universidad Católica del Perú,
jorge.rojas@pucp.edu.pe

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Paul Bedón García
Pontificia Universidad Católica del Perú

Gabriel Rodríguez
Pontificia Universidad Católica del Perú

Abstract

An extensive family of univariate models of autoregressive conditional heteroskedasticity is applied to Peru's daily stock market returns for the period January 3, 1992 to March 30, 2012 (5053 observations) with four different specifications related to the distribution of the disturbance term. This concerns capturing the asymmetries of the behavior of the volatility, as well as the presence of heavy tails in these time series. Using different statistical tests and different criteria, the results show the following: (i) the FIGARCH (1,1)-t is the best model among all symmetric models while the FIEGARCH (1,1)-Sk is selected from the class of asymmetrical models. Also, the model FIAPARCH (1,1)-t is selected from the class of asymmetric power models; (ii) the three models capture well the behavior of the conditional volatility; (iii) the model FIEGARCH (1,1)-Sk is the one with the best performance in terms of prediction; (iv) however, the empirical distribution of the standardized residuals shows that the behavior of the tails is not well captured by either model; (v) the three models suggest the presence of long memory with estimates of the fractional parameter close to the nonstationarity region.

JEL Classification: C22, C52, C58, G12, G17.

Keywords: Univariate Autoregressive Conditional Heteroskedasticity Models, Peruvian Stock Market Returns, Volatility, Simmetries, Asymmetries, Normal, t-Student, Skewed t-Student, GED Distributions.

Resumen

Una amplia familia de modelos univariados de heterocedasticidad condicional autorregresiva se aplica a los retornos diarios del mercado de valores de Perú para el período Enero 3, 1992 a Marzo 30, 2012 (5053 observaciones) con cuatro especificaciones diferentes relacionadas con la distribución del término de error. Esto busca capturar las asimetrías del comportamiento de la volatilidad, así como la presencia de colas pesadas en estas series de tiempo. Utilizando diferentes pruebas estadísticas y diferentes criterios, los resultados muestran lo siguiente: (i) el modelo FIGARCH (1,1)-t es el mejor modelo entre todos los modelos simétricos mientras que el FIEGARCH (1,1)-Sk es seleccionado entre la clase de modelos asimétricos. Además, el modelo FIAPARCH (1,1)-t es seleccionado entre la clase de los modelos de poder asimétricos; (ii) los tres modelos capturan bien el comportamiento de la volatilidad condicional; (iii) el modelo FIEGARCH (1,1)-Sk es el que tiene el mejor desempeño en términos de predicción; (iv) sin embargo, la distribución empírica de los residuos estandarizados muestra que el comportamiento de las colas no está bien capturado por ninguno de los tres modelos; (v) los tres modelos sugieren la presencia de memoria larga pues las estimaciones del parámetro fraccional se encuentran cerca de la región no estacionaria.

Classificación JEL: C22, C52, C58, G12, G17.

Palabras Claves: Modelos de Heterocedasticidad Condicional Autoregresiva, Retornos Bursátiles Peruanos, Volatilidad, Simetrías, Asimetrías, Distribuciones Normal, t-Student, Skewed t-Student, GED.

Univariate Autoregressive Conditional Heteroskedasticity Models: An Application to the Peruvian Stock Market Returns[1]

Paul Bedón García
Pontificia Universidad Católica del Perú

Gabriel Rodríguez[2]
Pontificia Universidad Católica del Perú

1 Introduction

The Peruvian capitals market is undergoing expansion and constitutes an important part of the country's economic and financial development. This market channels a large proportion of financial intermediation, which is a relevant mean of financing the productive activities of both public and private companies; moreover, it plays a fundamental role in guiding the decisions of investors and companies, with a view to ensuring that resources are assigned more efficiently; see Bahi (2007). A set of stylized facts on the stock market returns and volatility is discussed in Humala and Rodríguez (2013): absence of autocorrelation in the returns, fat tails of the empirical distribution, asymmetries in the volatility linked with past negative returns, Normality in the aggregation, clustering of periods of volatility, slow decay in the autocorrelation function (ACF) for absolute returns (either power of the returns or monotonic transformations thereof).

On explaining the dynamic of inflation in the United Kingdom, Engle (1982) formally introduces an autoregressive conditional heteroskedasticity model (ARCH), on the basis of which a series of extensions are developed. Bollerslev (1986) presents a generalization of the ARCH (GARCH) process by allowing past conditional variances to be incorporated as regressors within the current conditional variance equation.

In the financial markets, the expected return of an asset, in equilibrium, depends on its risk, which can be measured by its variance. In this way, the conditional variance of an asset can influence the conditional mean. Engle et al. (1987) develop an extension of the ARCH model by allowing the conditional variance to be a determinant of the mean (ARCH-M).

Another specification of these volatility models corresponds to the integrated GARCH model (IGARCH); see also De Arce (2000), and Engle and Bollerslev (1986). Baillie et al. (1996) introduce a fractionally integrated generalized autoregressive conditional heteroskedasticity model (FIGARCH). Thus, a new kind of process is developed in which the shocks to conditional variance decay at a hyperbolic rate determined by the parameter of fractional differentiation, rendering the conditional variance more flexible.

The IGARCH and FIGARCH specifications are characterized by the non-stationarity of the volatility process. Nonetheless, this characteristic appears not to adequately fit the empirical properties of certain financial variables given the high degree of persistence implied by the integrated models. Thus, Davidson (2004) introduces the hyperbolic-GARCH (HYGARCH) model as a generalization of these models by assuming that the volatility process is stationary and long memory.

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[2] Address for Correspondence: Gabriel Rodríguez, Department of Economics, Pontificia Universidad Católica del Perú, Av. Universitaria 1801, Lima 32, Lima, Perú, Telephone: +511-626-2000 (4998), Fax: +511-626-2874. E-Mail Address: gabriel.rodriguez@pucp.edu.pe.

Black (1976) finds that, frequently, the changes in the returns of assets are negatively correlated with changes in their volatility. It can also be noted that negative returns predict greater volatility than positive returns of the same magnitude. This means that there is an asymmetry that is usually attributed to so-called financial leverage effects; see Engle (1995). Thus, Nelson (1991) put forward a new kind of volatility model: the exponential GARCH, or EGARCH. This type of model takes into account the leverage effects, the negative correlation between volatility and current and future returns, the inadequate restriction of the non-negativity of the variance, and the persistence of shocks.

Bollerslev and Mikkelsen (1996) propose a fractionally integrated extension of Nelson's EGARCH model (1991), known as FIEGARCH; also see Pérez and Ruiz (2009). Meanwhile, Glosten, et al. (1993) (GJR, 1993) modify the ARCH model to allow for the presence of unexpected positive and negative returns that have a different impact on the conditional variance; that is, asymmetric innovations. The GJR model allows both positive and negative innovations to produce different effects on the conditional variance and, thus, on the returns of assets (usually, the falls are longer and more sudden than the rises).

Likewise, Ding et al. (1993) put forward a generalized extension of the ARCH model, which questions the reason for assuming a linear relationship of the conditional variance based on lagged squared residuals or lagged deviation. This new model is called asymmetric power ARCH (APARCH) and allows an estimation of the long memory parameter in the volatility and the asymmetry parameter or leverage effect. Finally, Tse (1998) constructs a model by extending the APARCH model to a fractionally integrated process (FIAPARCH), incorporating the fractional process in the conditional variance.

The empirical literature is extensive and we make no pretence at an exhaustive review here. Key references include Andersen and Bollerslev (1998), Bollerslev et al. (1992), Bollerslev et al. (1994), Engle (2001), De Arce (2004), Bollerslev (2008), and Laurent et al. (2010). However, to our knowledge, there are no studies of this type for the Peruvian case.

Other authors such as Kim and Kon (1994) compare different ARCH specifications. They find that the GJR specification (1993) is the most descriptive for individual shares, while the EGARCH model is the most apt for explaining stock market indices. Engle and Ng (1993) conduct a study on the event impact curve (*"news"*). The results of the estimations suggest that the GJR model (1993) is the best parametric model against the EGARCH, which captures much of the series' asymmetry. Likewise, David (1997) prefers the EGARCH model.

Baillie and DeGennaro (1990) use a GARCH-M model to examine the relationship between the mean returns of a share portfolio and its conditional variance or standard deviation. Meanwhile, Koopman and Uspensky (2002) contrast ARCH-M volatility models with a stochastic volatility in mean (SVM) model. The authors present an empirical study on the intertemporal relationship between the share profitability index and their volatility for the United Kingdom, the United States, and Japan by finding a negative but weak relationship between the returns and their volatility in the current period. Giot and Laurent (2003) make use of an APARCH model based on an asymmetrical t-student distribution to take into account the fat tails on both sides of the distribution of the returns. Moreover, Pérez and Fernández (2006) present an application of ARCH models to Colombia's stock market returns for the period 2004 to 2006. Ávalos and Hernández (1995) make use of an ARCH model to analyze stock market returns in Mexico. López (2004) evaluates the contribution of three models from the ARCH family to model the behavior of the Mexican stock market: a symmetric GARCH model(1,1) and two asymmetric TARCH(1,1) and EGARCH(1,1) models.

In addition to stock market yields, GARCH models have been applied to study the behavior of exchange rate yields. Pozo (1992) shows that an increase in exchange rate volatility reduces commercial volume. Wang et al. (2001) establishes that the prices of many assets, including exchange rates, display periods of stability followed by strong fluctuations or interruptions. Moreover, Amigo (1997) makes use of an ARCH model to analyze whether they can adequately explain the volatility present in the Spanish exchange rate market for the period 1991-1993, finding evidence in favor of a GARCH(1,1) model.

On the other hand, Koutmos and Theodossiou (1994) analyze the predictability and properties of the weekly percentage change in the Greek exchange rate with respect to the most traded currencies in the country. The analysis is carried out using a EGARCH-M model along with an exponential distribution. Moreover, Gonzáles and Viñas (1996) examine the statistical properties of the first logarithmic differences of the daily exchange rates for the period 1890-1995 and two subperiods. The authors find that both ARCH and GARCH effects are located within the conditional variance to a significant degree. On the other hand, Engle et al. (1990) attempt to explain the causes of volatility clustering in the exchange rates through the use of a GARCH model to specify heteroskedasticity across the intra-daily market segments. Ayodeji (2009) investigates the volatility of the Naira/Dollar exchange rates in Nigeria using GARCH (1,1), GJR-GARCH(1,1), EGARCH(1,1), APARCH(1,1), IGARCH(1,1) and TS-GARCH(1,1) models. In addition, McKenzie (1998) attempts to predict the volatility of the Australian exchange rate. His results suggest that the ARCH models generate a superior prediction when the squared returns of the exchange rate series are considered. Davidson (2004) finds evidence that backs this model for the exchange rates of Asian countries in the period 1994-2000, though he points to the FIGARCH model as being favored by a series of countries. It is seen that, unlike in the securities market, the shocks of appreciation and depreciation of the yen per dollar have similar effects on future volatilities (Tse, 1998); see also Conrad et al. (2011).

In this paper, an extensive family of univariate models of autoregressive conditional heteroskedasticity is applied to Peru's daily stock market returns for the period January 3, 1992 to March 30, 2012 (5053 observations) with four different specifications related to the distribution of the disturbance term. This concerns capturing the asymmetries of the behavior of the volatility, as well as the presence of heavy tails in these time series. Using different statistical tests and different criteria, the results show the following: (i) the FIGARCH (1,1)-t is the best model among all symmetric models while the FIEGARCH (1,1)-Sk is selected from the class of asymmetrical models. Also, the model FIAPARCH (1,1)-t is selected from the class of asymmetric power models; (ii) the three models capture well the behavior of the conditional volatility; (iii) the model FIEGARCH (1,1)-Sk is the one with the best performance in terms of prediction; (iv) however, the empirical distribution of the standardized residuals shows that the behavior of the tails is not well captured by either model; (v) the three models suggest the presence of long memory with estimates of the fractional parameter close to the nonstationarity region.

The document is structured as follows. Section 2 briefly presents the models that are used in the empirical section. Section 3 displays and discusses the main empirical findings. Moreover, based on different statistical tests, the primary models for the Peruvian stock market returns are selected. Section 4 presents the main conclusions.

2 The Models

In general, $\{y_t\}$ being a series of returns, an autoregressive heteroskedasticity model can be defined as $y_t = x_t'\beta + \epsilon_t$, $\epsilon_t \mid \Omega_{t-1} \sim f(0, \sigma_t^2)$ and $\sigma_t^2 = g[\sigma_{t-1}^2(\theta), \sigma_{t-2}^2(\theta), ...; \epsilon_{t-1}(\theta), \epsilon_{t-2}(\theta), ...; \upsilon_{t-1}, \upsilon_{t-2}, ...]$, where x_t is a vector $k \times 1$ of endogenous and exogenous explanatory variables included in the set of information Ω_{t-1}, β a vector $k \times 1$ of unknown parameters, $f(.)$ is a function of density, $g(.)$ is a linear or non-linear functional form, and υ_t is a vector of predetermined variables included in Ω_t. The conditional variance is a linear or non-linear function of the lagged values of σ_t, and ϵ_t and of predetermined variables ($\upsilon_{t-1}, \upsilon_{t-2}, ...$) included in Ω_{t-1}.

Engle (1982) defined an ARCH process as $\epsilon_t = z_t \sigma_t$, where z_t is an independent and identically distributed process with $E(z_t) = 0$ and $Var(z_t) = 1$. Moreover, it is assumed that ϵ_t is not serially correlated, has a mean 0 and a conditional variance equal to σ_t^2 changing over time with the equation of variance being $\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$. In order for the ARCH(q) process to be well defined σ_t^2, $\forall t$ has to be positive. The conditions of sufficiency to assure the positivity of the variance are given by $w > 0$ and $\alpha_i \geq 0$ for $i = 1, ..., q$. An alternative way of describing the ARCH(q) process, according to Degiannakis and Xekalaki (2004), is given by: $\sigma_t^2 = w + \alpha(L)\epsilon_t^2$, where L represents the lag operator and $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + ... + \alpha_q L^q$.

In Bollerslev's GARCH model (1986), it is found that $\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$ where, using the lag operator L, the GARCH model(p,q) can be written as: $\sigma_t^2 = w + \alpha(L)\epsilon_t^2 + \beta(L)\epsilon_t^2$ which reduces the number of estimated parameters by imposing restrictions so that the conditional variance is positively defined: $w > 0$, $\alpha_i \geq 0$ for $i = 1, ..., q$ and $\beta_i \geq 0$ for $i = 1, ..., p$ and where $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + ... + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + ... + \beta_q L^q$.

The ARCH-M model of Engle et al. (1987) proposes that $y_t = x_t'\beta + \phi(\sigma_t^2) + \epsilon_t$, where $\phi(\sigma_t^2)$ represents the risk premium. The ARCH-M model is frequently used in financial time series where the expected risk depends on its return. The estimated coefficient of this risk helps to analyze the risk-return trade-off.

Nelson's EGARCH model (1991) is formulated in terms of the logarithm of conditional variance. Following Degiannakis and Xekalaki (2004), the conditional variance of the EGARCH(p,q) model is represented by $\log(\sigma_t^2) = w + \sum_{i=1}^{q} \pi_i g(\frac{\epsilon_{t-i}}{\sigma_{t-i}})$, where $\pi \equiv 1$. In turn, the model incorporates the asymmetrical relationship between the squared returns and the shifts in the volatility, rendering $g(\epsilon_t/\sigma_t)$ a linear combination of $\mid \epsilon_t/\sigma_t \mid$ and ϵ_t/σ_t. Thus, we have $g(\epsilon_t/\sigma_t) = \gamma_1(\epsilon_t/\sigma_t) + \gamma_2(|\epsilon_t/\sigma_t| - E|\epsilon_t/\sigma_t|)$ where γ_1 and γ_2 are constant. Let us note that $z_t = \epsilon_t/\sigma_t$ and $E(|\epsilon_t/\sigma_t|) = \sqrt{2/\pi}$. The innovation of the equation $\log(\sigma_t^2)$ will be positive (negative) when the magnitude of z_t is larger (smaller) than its expected value. As Degiannakis and Xekalaki (2004) point out, a natural parametrization is to model the conditional variance as an autoregressive moving average model:[3] $\log(\sigma_t^2) = w + [(1 + \alpha(L)][1 - \beta(L)]^{-1} g(z_{t-1})$.

The GJR (1993) model specifies both the positive and negative asymmetry of the innovations through the incorporation of a dummy variable: $\sigma_t^2 = w + \sum_{i=1}^{q}(\alpha_i \epsilon_{t-i}^2) + \sum_{i=1}^{q}(\gamma_i S_{t-i}^- \epsilon_{t-i}^2) + \sum_{j=1}^{p}(\beta_j \sigma_{j-i}^2)$, where γ_i for $i = 1, ..., q$ are parameters that have to be estimated, S_t^- is a dummy variable that takes the value of 1 when $\epsilon_{t-i} < 0$ and takes the value of 0 if $\epsilon_{t-i} > 0$. In other words, it recognizes the presence of "good" ($\epsilon_{t-i} > 0$) and "bad" ($\epsilon_{t-i} < 0$) news by assuming that the impact of ϵ_t^2 on the conditional variance is different if ϵ_t is positive or negative.

In the APARCH model of Ding et al. (1993) it is found that $\sigma_t^\delta = w + \sum_{i=1}^{q} \alpha_i (|\epsilon_{t-i}^2| - \gamma_i \epsilon_{t-i})^\delta + \sum_{j=1}^{p}(\beta_j \sigma_{j-i}^\delta)$, where $\delta > 0$ and $-1 < \gamma_i < 1 \; \forall \; i = 1, ..., q$. Moreover, $w > 0, \delta \geq 0$ and $\beta_j \geq 0$,

[3] Or similarly: $\log(\sigma_t^2) = w + (1 + \sum_{i=1}^{q} \alpha_i L^i)(1 - \sum_{j=1}^{q} \beta_j L^j)^{-1}[\gamma_1(\varepsilon_t/\sigma_t) + \gamma_2(|\varepsilon_t/\sigma_t| - E|\varepsilon_t/\sigma_t|)]$.

$j = 0, ..., p$. As detailed by Degiannakis and Xekalaki (2004), this model imposes a Box-Cox (1964) power transformation of the conditional standard deviation process and of the absolute asymmetric innovations. Within this expression, δ assumes the role of the Box-Cox transformation of σ_t while γ_i reflects the leverage effect. Moreover, this model has the peculiarity of including another seven ARCH models as special cases: (i) the Engle's ARCH model (1982) when $\delta = 2$, $\gamma_i = 0$ $(i = 1, ..., p)$ and $\beta_j = 0$ $(j = 1, ..., p)$; (ii) the Bollerslev's GARCH model (1986) when $\delta = 2$, $\gamma_i = 0$ $(i = 1, ..., p)$; (iii) the GARCH model of Taylor (1986) and Schwert (1990) when $\delta = 1$, $\gamma_i = 0$ $(i = 1, ..., p)$; (iv) the GJR (1993) when $\delta = 2$; (v) the Zakoian's TARCH model (1994) when $\delta = 1$; (vi) the Higgins and Bera's NARCH model (1992) when $\gamma_i = 0$ $(i = 1, ..., p)$ and $\beta_j = 0$ $(j = 1, ..., p)$; (vii) the log-ARCH of Geweke (1996) and Pantula (1986) when $\delta \Rightarrow 0$.

The IGARCH model seeks to estimate the conditional variance of the financial time series in the event that this is integrated, I(1). This model was put forward by Engle and Bollerslev (1986): $\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$, for $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1$. Similarly, using the lag operator, we have $\sigma_t^2 = w + \alpha(L)\epsilon_{t-i}^2 + \beta(L)\sigma_{t-j}^2$, for $\alpha(L) + \beta(L) = 1$. The IGARCH model is based on a GARCH model(p,q) whose conditional variance displays a high degree of persistence, where the polynomial $\alpha(L) + \beta(L) = 1$ has $r > 0$ roots and $\max(p, q) - r$ roots outside the unit circle.

In the FIGARCH model of Baillie et al. (1996), the specification is $\phi(L)(1 - L)^d \epsilon_t^2 = w + [1 - \beta(L)]v_t$, where $\phi(L) \equiv [1 - \alpha(L) - \beta(L)](1 - L)^{-d}$, $0 < d < 1$ and $v_t = \epsilon_t^2 - \sigma_t^2$. The process $\{v_t\}$ is interpreted as the innovations for the conditional variance. Thus, the conditional variance of the process is defined as: $\sigma_t^2 = w[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\epsilon_t^2 = w[1-\beta(L)]^{-1} + \lambda(L)\epsilon_t^2$. Bollerslev and Mikkelsen's FIEGARCH model (1996) is defined as $\log(\sigma_t^2) = w + \phi(L)^{-1}(1-L)^{-d}[1+\alpha(L)]g(z_{t-1})$.

Similarly, Tse (1998) suggests the FIAPARCH model where the conditional variance is expressed as $\sigma_t^\delta = w + \{1 - [1 - \beta(L)]^{-1}\phi(L)(1 - L)^d\}(|\epsilon_t| - \gamma\epsilon_t)^\delta$.

Davidson (2004) introduces the HYGARCH model as a generalization of the IGARCH and the FIGARCH models. The HYGARCH model is given by $\sigma_t^2 = w[1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1}\phi(L)[1 + \alpha[(1 - L)^d]]\}\epsilon_t^2$. The HYGARCH model nests the FIGARCH model when $\alpha = 1$, and the process is stationary when $\alpha < 1$.

3 Empirical Results

3.1 The Data

The stock market returns consist of 5053 daily observations on the General Index of the Lima Stock Exchange (IGBVL) for the period January 3, 1992 to March 30, 2012. Moreover, in the volatility analysis of the stock market returns, there may be a presence of "day-of-the-week"effects; that is, effects related to the days on which stock markets open (Monday) and close (Friday) that can affect market volatility; see Alberg et al. (2008). Thus, dummy variables are introduced in the regression analysis. Many studies have documented the presence of these effects on financial markets; see Cross (1973), French (1980), Alexakis and Xanthakis (1995) and Peña, (1995), among others.

Figure 1 displays the stock market returns (Top Panel). The series exhibits periods of high and low volatility (clustering), representing a clear sign of the presence of ARCH effects. The middle Panel displays the ACF of the returns while the last Panel shows the ACF of the squared returns. This Figure presents clear evidence of long memory.

The unconditional distribution of the stock market returns is shown in Figure 2 (Top Panel), and is compared with the Normal density. Its peak is higher (solid line) than the Normal density (dotted line). Moreover, it has fatter tails which can be seen on Figure 2 (middle Panel and lower Panel). In addition, the skewness (-0.139) and the kurtosis (10.571) -located above the values of 0 and 3, respectively, for a symmetric distribution, highlight this characteristic. This is an indicator of the presence of an asymmetric distribution with heavy tails.

The estimations of the models consist of two equations: one for the mean, which is specified as ARMA(p,q), for $p, q = 0, 1, 2$ and another for the variance, which is specified as ARCH(0,1), GARCH(1,1), EGARCH(1,1), GJR(1,1), APARCH(1,1), IGARCH(1,1), FIGARCH(1,1), FIEGARCH(1,1), FIAPARCH(1,1), HYGARCH(1,) and ARCH(0,1)-M. The objective is to find, firstly, the mean equation, and secondly, the best model for volatility within the ARCH specification. All models are estimated using four different specifications relative to the distribution of the disturbance term: Normal, t-Student, Skeweed, and generalized error distribution (GED).

To select the best models, the following statistics are used: (i) the LM-ARCH statistic to check for the presence of ARCH effects on the residuals of the models; (ii) four information criteria: Akaike (AIC), Schwartz (BIC), Hannan-Quinn (HQ) and Shibata (SH); (iii) the Engle and Ng's diagnostic statistic (1993) that investigates possible specification errors in the conditional variance equation. To test for the presence of leverage effects, the Sign Bias (SB) statistic is used**,** which examines the impact on the conditional variance due to the positive or negative innovations not predicted by the estimated model; the Negative Sign Bias (NSB) statistic, which focuses on the impacts of the negative innovations on the conditioned variance; and the Positive Sign Bias (PSB) statistic, which estimates the effect of the positive innovations. Finally, the joint statistic (JT), which indicates the benefits of the volatility model with respect to the three statistics aforementioned. These statistics test whether the negative or positive shocks on the conditional variance depend on their size and how they affect conditional volatility; (iv) the adjusted Pearson goodness-of-fit statistic, which compares the empirical distribution of the innovations with the theoretical. In order to carry out this process, it is necessary to classify the residuals in cells according to their magnitude. For observations *i.i.d.*, Palm and Vlaar (1997) show that the null hypothesis of a correct distribution is limited between a $\chi^2_{(r-1)}$ and a $\chi^2_{(r-k-1)}$ where k is the number of estimated parameters; (v) the Residual-Based Diagnostic (RBD) statistic for detecting conditional heteroskedasticity suggested by Tse (2002); (vi) the Q statistic on the standardized residuals, and the squared standardized residuals.

3.2 Results[4]

With respect to the mean equation, different specifications were tested out, and the best was found to be an AR(1) process. Moreover, with respect to the dummy variables linked to two weekdays (Monday and Friday), in most of the estimations these variables are statistically significant. In general their signs are negative, reflecting the fact that the returns and the volatility are, on average, lower on those days, especially on Friday.

Starting with the ARCH(1,1) model and using the logarithm of likelihood, we find that the best performing model is between the ARCH(1)-t and the ARCH(1)-Sk. Nonetheless, the asymmetry coefficient of the t-Sk distribution is not statistically significant, so we reject this model. Under the

[4]The number of estimated models jointly to the different specifications of the distribution of the disturbance term give rise to a large number of Tables. The complete set of these tables is available upon request. In this paper, we only include the most important.

four criteria of information, the ARCH(1,1)-t is better than the ARCH(1,1)-Sk model. On the other hand, the four models reject the null hypothesis of no ARCH effects, so a specification of this type for modeling the volatility of stock market returns does not seem adequate. The SB, NSB and PSB statistics are not significant in all situations; that is, the models would be correctly incorporating the positive and negative innovations. Nonetheless, the JT is not significant only in the ARCH(1)-N, with the rest of the models giving bad results with relation to the modeling of shocks on the conditional variance. The Q statistics applied to standardized squared residuals display a rejection of the null hypothesis. Finally, the Pearson Chi-Square goodness-of-fit statistic suggests that the ARCH(1)-t and ARCH(1)-Sk models do not reject the null hypothesis. In conclusion, the best model in this group would be the ARCH(1)-t.

The results for the GARCH(1,1) family show that the asymmetry parameter of the Sk specification is not significant. Observing the four information criteria, we find that the best model is the GARCH(1,1)-t. The four models account for ARCH effects (the null hypothesis of no ARCH effects is not rejected). The RBD statistic with several lags helps us to analyze the presence of conditional heteroskedasticity in the time series, and we observe that the GARCH-t and GARCH-Sk specifications are not appropriate, while the two remaining do not present problems of this kind. Moreover, we analyze the presence of leverage effects by way of the SB, NSB, PSB and JT statistics, and it is seen that the effect of negative shocks on the conditional variance (NSB) are greater than the positive shocks (PSB) while the null hypothesis of the Joint Test (JT) is not rejected by all specifications. The statistics show that the asymmetric effect of the innovations is being captured to a large extent. In turn, the Q statistic applied to the standardized squared residuals does not reject the null hypothesis of no serial correlation to 1% of significance in the four distributions. Moreover, the P statistic (with different numbers of cells) rejects the null hypothesis of a correct specification (both p-values) in the GARCH(1,1)-N and GARCH (1,1)-GED models, while the remaining models do not reject the hypothesis. In summary, combining all criteria used, the best model would be the GARCH(1,1)-t.

With respect to the EGARCH(1,1) model, the parameters β_1 and θ_1 and θ_2 are significant, assuming the four distributions with the exception of the coefficient α_1, which shows statistical significance only for EGARCH(1,1)-GED and EGARCH(1,1)-Sk. Following the logarithm of likelihood, we find that the best performing model is the EGARCH(1,1)-Sk. Nonetheless, if we analyze the four information criteria, the EGARCH(1,1)-t displays a smaller BIC, while the EGARCH(1,1)-Sk displays a smaller AIC, SH and HQ. None of the four models show ARCH effects (the null hypothesis of the ARCH effects is not rejected). The RBD statistic indicates that the four specifications are appropriate. The EGARCH models adequately capture the non-symmetric effects of shocks on the conditional variance. Adding together the results of the Q statistic and the P statistic, we can conclude that the EGARCH(1,1)-Sk model is the best.

In the case of the GJR specification, the parameters α_1 and β_1 and γ_1 are significant by assuming the four distributions. Analyzing the logarithm of likelihood, we find that the best performing model is the GJR(1,1)-Sk. However, if we analyze the four information criteria, we find that the best model is the GJR(1,1)-t. The four models do not find evidence of ARCH effects. The RBD statistic tells us that the GJR-GED specification is not appropriate, while the rest of the models have some problems with heteroskedasticity. The negative shocks on the conditional variance (NSB) are slightly greater than the positive shocks (PSB). Adding together the result with the Q statistic and the P statistic, we find that the best model for this specification is the GJR(1,1)-Sk.

In the case of the APARCH (1,1) specification, the parameters α_1, β_1 and γ_1 and δ_2 are significant by assuming the four distributions, and a high degree of persistence in variance is observed. As is the case with many models, the asymmetry coefficient of the APARCH(1,1)-Sk is insignificant and small, and as such this model cannot be representative. Following the logarithm of likelihood, we find that the best performing model is the APARCH(1,1)-Sk. If we analyze the four information criteria, the APARCH(1,1)-t is better than the APARCH(1,1)-Sk in the BIC and the HQ, while in the AIC and SH they are indifferent. The four models show evidence of an absence of ARCH effects. The null hypothesis of the RBD statistic is not rejected in all cases, so the specifications are adequate. The negative shocks on the conditional variance are more significant or greater than the positive shocks. In turn, the Q statistic on the standardized squared residuals show similar results to the other models. Finally, the P statistic establishes that the APARCH(1,1)-Sk model does not reject the null hypothesis. The results allow the APARCH(1,1)-t model to be selected.

The estimation of the IGARCH(1,1) models show that the parameters α_1 and β_1 are significant by assuming the four distributions. Following the logarithm of likelihood, we find that the best performing model is the IGARCH(1)-Sk. Under the four information criteria, the IGARCH(1,1)-t is the best, being indistinct from the AIC and the SH criteria. The four models have problems with respect to the ARCH effects remaining in the residuals. The RBD statistic establishes a correct specification for all models, above all in the RBD(2). There appears to be good modeling of the asymmetry of innovations. The Q statistic shows no evidence of autocorrelation in the residuals of the four models (at 1.0%). The P statistic allows the IGARCH(1,1)-N and IGARCH(1,1)-GED models to be discarded. In consequence, we can select the IGARCH(1,1)-t model.

The evidence of long memory between the stylized facts of the stock market returns suggests the estimation of fractional models. The estimation of the FIGARCH(1,1) models suggests that the parameters α_1 and β_1 are insignificant by assuming the four distributions. Observing the logarithm of likelihood, we find that the best model is the FIGARCH(1,1)-Sk, but the parameter of asymmetry is insignificant. The four information criteria, however, suggest evidence in favor of the FIGARCH(1,1)-t model. The four models show an absence of ARCH effects in the residuals, while the RBD statistic suggests that the four specifications are appropriate. The statistics based on the sign suggest that models of this kind capture well the behavior of the shocks on the conditional variance. The residuals do not show signs of autocorrelation in accordance with the Q statistic. The P statistic allows us to discard the FIGARCH(1,1)-N and FIGARCH(1,1)-GED models. The conclusion is the selection of the FIGARCH(1,1)-t model.

With respect to the estimations of the FIEGARCH(1,1) models, the parameters α_1, β_1 are insignificant, unlike the θ_1 and θ_2 by assuming the four distributions. Following the logarithm of likelihood, we find that the best performing model is the FIEGARCH(1,1)-Sk. At the level of the four information criteria, the FIEGARCH(1,1)-Sk model continues to exceed the FIEGARCH(1,1)-t. Moreover, three of the four models do not reject the null hypothesis of the ARCH effects, with the FIEGARCH(1,1)-t displaying problems. The RBD statistic does not reject the null hypothesis of a correct specification in each model, so problems of heteroskedasticity would not be of concern. The statistics based on the signs suggest that the FIEGARCH(1,1)-Sk model is correctly incorporating the positive and negative innovations at a distance from the model. The best model in the group would be the FIEGARCH(1,1)-Sk.

As regards the estimations of the FIAPARCH(1,1) models, the parameters α_1 and β_1 are insignificant, and the opposite occurs with the parameters γ_1 and δ. The logarithm of likelihood shows that the best-performing model is the FIAPARCH(1,1)-Sk, but the four information criteria establishes that the best model is the FIAPARCH(1,1)-t. Both models are seen to be superior to the other two. The four models does not reject the null hypothesis of no ARCH effects. The RBD statistic suggests that the four specifications are correct. The asymmetric effect of the innovations is relatively well captured by the four specifications. There is no evidence of autocorrelation in the residuals, while the P statistic rejects the null hypothesis of a correct specification (both p-values) in the FIAPARCH(1,1)-N and FIAPARCH (1,1)-GED models. The best model would be the FIAPARCH(1,1)-t.

In the case of the HYGARCH(1,1) estimations, the parameters α_1, β_1 and HY are not significant by assuming their four distributions. According to the logarithm of likelihood, we find that the best performing model is the HYGARCH(1,1)-Sk. Nonetheless, if we analyze the four information criteria we find that the best model is the HYGARCH(1,1)-t. The four models provide evidence for the absence of ARCH effects. The RBD statistic indicates that the specification is appropriate, and suggests that the leverage effects are adequately captured. There is no evidence of autocorrelation in the residuals according to the Q statistic. Moreover, the P statistic does not reject the null hypothesis of a correct specification (both p-values) in the HYGARCH(1,1)-t and HYGARCH(1,1)-Sk models. In this case, the HYGARCH(1,1)-t model is selected.

Finally, estimations of the ARCH-M (0,1) models are performed, where the parameters are significant by assuming the four distributions. As with many previous models, the asymmetry coefficient of the ARCH(1)-M-Sk is insignificant and small, and so this model is not representative. Following the logarithm of likelihood, we find that the best performing models are both the ARCH(1)-M-t and the ARCH(1)-M-Sk, which have the lowest values. However, if we analyze the four information criteria, the ARCH(1)-M-t is better than the ARCH(1)-M-Sk. The four models provide evidence of ARCH effects in the residuals. The positive shocks on the conditional variance are greater than the negative shocks. The Q statistic shows clear evidence of autocorrelation in the residuals estimated by the four models. They suggest a better performance of the ARCH(1)-M-Sk and ARCH(1)-M-t models. According to the criteria utilized, the model selected is ARCH(1)-M-t. It is important to mention that though we selected this model as a representative of the ARCH(1)-M family, the different statistics suggest a poor performance of this type of models. This is unsurprising, given that it concerns simpler ARCH models, only that the mean is modeled by including volatility.

3.2.1 Selection of Models

Given that the dependent variable changes in the different estimated models, the selection criteria of the models is applied in three different groups. The first group, whose dependent variable is σ_t^2, is comprised of ARCH, GARCH, GJR, IGARCH, FIGARCH, HYGARCH and ARCH-M models. The second group, whose dependent variable is the $\log(\sigma_t^2)$, is comprised of EGARCH and FIEGARCH models, while the last group, whose dependent variable is σ_t^δ, will be comprised of APARCH and FIAPARCH models.

Following the maximum likelihood criterion, the best model from the first group is the FIGARCH(1,1)-t. The best model in the second group is the FIEGARCH(1,1)-Sk, while in the last group the FIAPARCH(1,1)-t would be the representative. Moreover, analyzing the information criteria, these support the previous findings. It is important to note that the models selected belong to the group of fractional integration; that is, we have evidence of a long-memory process in the volatility.

Within the first group, the models that do not reject the null hypothesis of no ARCH effects are GARCH(1,1)-t, GJR(1,1)-t, IGARCH(1,1)-t, FIGARCH(1,1)-t and HYGARCH(1,1)-t. In this sense, according to this criteria, the ARCH(1)-t and ARCH-M(1)-t models are discarded. Both the second and the third group provide evidence of the absence of ARCH effects in the residuals when the FIEGARCH(1,1)-Sk, APARCH(1,1)-t and FIAPARCH(1,1)-t models are used.

Utilizing the RBD statistic, it is observed that only the FIGARCH(1,1)-t, HYGARCH(1,1)-t and IGARCH(1,1)-t models appear to correct the problem of conditional heteroskedasticity in the estimated residuals. On the other hand, the SB, NSB, PSB and JT statistic show the presence of leverage effects, which is equivalent to stating that these models largely capture the asymmetric effects of positive and negative innovations in the variance of stock market returns. With respect to the P statistic, this reveals that the empirical distribution of the innovations is adjusted to the theoretical distribution in all the models in the three groups analyzed.

Based on the above-mentioned, we find that the best three models are the FIGARCH(1,1)-t, FIEGARCH(1,1)-Sk and FIAPARCH(1,1)-t, in each of the three groups analyzed, respectively. Figure 3 shows some interesting aspects. The conditional variance obtained from the three models show very similar to the squared residuals behavior which is a good indicator of adjustment of each of the models. On the other hand, the empirical density of the standardized residuals compared to underlying distribution used in the estimates (t-student, Student-t and Skewed Student-t, respectively) still shows significant differences. The qq-plot confirms this: the behavior of the tails of the distribution of stock returns is not well captured by either model. Other ongoing research is looking to capture this aspect.

3.2.2 Prediction

A final indicator for evaluating the performance of the selected models is the use of an out-of-sample prediction exercise. In the experiment, a $h = 1, 2, ..., 15$ horizon is assumed; that is, the model is estimated by leaving 15 observations for the prediction experiment, and then the 15 forward predictions of the stock market returns are estimated, based on the FIGARCH(1,1)-t, FIEGARCH(1,1)-sk and FIAPARCH(1,1)-t models.

To assess the predictive performance of the three models, the following measurements are used: (i) the Mean Squared Error (MSE), which is the sum of the squared prediction errors for each of the observations divided by the number observations; (ii) the Median Squared Error (MedSE), which is the median of the squared errors; (iii) the Mean Error (ME), which is the average error; (iv) the Mean Absolute Error (MAE), which is the average error when the signs are not taken into account; (v) the Root Mean Squared Error (RMSE), which is the square root of the MSE; (vi) the Theil Inequality Coefficient (TIC), which stabilizes between 0 and 1, where 0 indicates a perfect fit.

The values of the MSE and MAE are generally affected by the presence of atypical values (outliers), while the Median Squared Error (MedSE) has the advantage of reducing these effects. The FIEGARCH model reports a value of 0.417 for the MedSE, reinforcing its selection. In turn, the TIC measures the degree of difference between a temporary series of estimated values and their corresponding values observed, if the value of the coefficient is 0, the projection would be identical to reality. The FIEGARCH model has the lowest TIC coefficient with 0.343.

With respect to the Mean Error (ME), this indicates the average deviation of the predictions; positive values reflect an overvaluation, while the negative values reflect the reverse. The FI-

GARCH, FIEGARCH and FIAPARCH models post average errors of -0.6608, -0.3861 and -0.5819, respectively; that is, the predictive capacity underestimates the genuine values, FIEGARCH being that which best models the data.

According to Alberg et al. (2008), the advantage of using different prediction measurements is the robustness of selecting the optimum model. The estimations are ranked using the following measurements: MSE, MedSE, ME, MAE, RMSE, and TIC. The results (see Table 2) show that the FIEGARCH(1,1)-sk model provides the best out-of-sample predictions. The measurements of relative dispersion around the central trend are lower for the conditional variance. It is found that the confidence intervals at 95% around the stock market returns better fit this model.

The conditional variance forecast for the FIEGARCH model has the lowest values for the horizon of the 15 observations forecast. With the exception of the first forecast, where the FIAPARCH(1,1)-t model presents a lower variance, for the remaining observations there are less observations; see Figure 4.

3.3 Conclusions

An extensive family of univariate models of autoregressive conditional heteroskedasticity is applied to Peru's daily stock market returns for the period January 3, 1992 to March 30, 2012 (5053 observations) with four different specifications related to the distribution of the disturbance term. This concerns capturing the asymmetries of the behavior of the volatility, as well as the presence of heavy tails in these time series.

Different criteria and statistics are utilized for the process of selecting the best models. Given the different nature of the dependent variable, the models have been selected separately. Finally, the selected models are the FIGARCH(1,1)-t, the FIEGARCH(1,1)-Sk, and the FIAPARCH(1,1)-t in each of the groups divided according to the structure of the dependent variable. Making a predictive comparison, the most satisfactory model is the FIEGARCH(1,1)-Sk. The selection is interesting as it reflects the following aspects: (i) it is a model that captures the asymmetries and thus the leverage effects; (ii) it is a fractionally integrated model, which allows the evidence of long memory to be captured in the volatility of stock market returns; (iii) the distribution of the disturbance term is skeweed, which allows us to approximate the behavior of the structure of the disturbance term.

It is important to emphasize the long-memory aspect in the time series analyzed. The three models allow an estimate of the fractional parameter $\widehat{d} = 0.467, 0.495, 0.467$, respectively. The three estimations are close to the frontier of the stationarity (0.5), and the three values indicate strong evidence of long memory. This result can be interpreted as strong evidence in favor of fractionally integrated models. Nonetheless, as the literature has pointed out, this behavior may be contaminated by the presence of sporadic or random level shifts; see Diebold and Inoue (2001), Mikosch and Stărică (2004a, 2004b), among others. From the standpoint of the application of statistics, see Perron and Qu (2010) and Qu (2011). From the standpoint of modeling, see Lu and Perron (2010), Li and Perron (2013), and Xu and Perron (2014). Recent applications and research underway for the Peruvian and Latin American cases include Ojeda Cunya and Rodríguez (2014), Rodríguez and Tramontana Tocto (2014), Rodríguez (2014), Herrera and Rodríguez (2014), and Pardo and Rodríguez (2014).

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Table 1. Selection of Models

	Symetric Models						
Criterion	ARCH(1)-t	GARCH(1,1)-t	GJR(1,1)-t	IGARCH(1,1)-t	FIGARCH(1,1)-t	HYGARCH(1,1)-t	ARCH-M(1,1)-t
Log-Lik	-8270.6	-7984.8	-7981.3	-7985.9	-7962.4	-7962.2	-8263.2
AIC	3.276	3.163	3.162	3.163	3.155	3.155	3.273
BIC	3.285	3.173	3.174	3.172	3.166	3.168	3.284
SH	3.276	3.163	3.162	3.163	3.155	3.155	3.273
HQ	3.279	3.167	3.166	3.166	3.159	3.159	3.277
	Asymetric and Power Asymetric Models						
	EGARCH(1,1)-Sk	FIEGARCH(1,1)-Sk	APARCH(1,1)-t	FIAPARCH(1,1)-t			
Log-Lik	-7982.5	-7961.7	-7980.8	-7958.7			
AIC	3.163	3.156	3.162	3.154			
BIC	3.178	3.171	3.175	3.168			
SH	3.163	3.155	3.162	3.154			
HQ	3.168	3.161	3.167	3.159			

Table 1 (Continuation). Selection of Models

Symetric Models (σ_t^2)														
	ARCH(1)-t		GARCH(1,1)-t		GJR(1,1)-t		IGARCH(1,1)-t		FIGARCH(1,1)-t		HYGARCH(1,1)-t		ARCH-M(1,1)-t	
Criterion	Valor	p-value	Valor	p-value	valor	p-value	valor	p-value	valor	p-value	valor	p-value	valor	p-value
ARCH 1-2	77.879	0.0000	1.710	0.180	1.408	0.244	1.126	0.324	0.147	0.862	0.247	0.781	58.340	0.000
ARCH 1-5	54.896	0.000	1.816	0.106	1.808	0.107	1.854	0.098	0.392	0.854	0.431	0.827	42.307	0.000
ARCH 1-10	34.800	0.000	1.801	0.055	1.728	0.068	1.998	0.029	0.699	0.725	0.724	0.702	27.988	0.000
RBD(2)	-265.71	1.000	18.47	0.000	11.087	0.004	0.877	0.644	1.059	0.588	-3.061	1.000	3571.9	0.000
RBD(5)	-69.983	1.000	25.09	0.000	11.561	0.041	11.043	0.050	-1.694	1.000	-0.870	1.000	-97.37	1.000
RBD(10)	-38.345	1.000	36.85	0.000	19.741	0.031	22.719	0.011	5.350	0.866	5.344	0.867	-82.97	1.000
SB	1.3119	0.189	0.105	0.916	0.041	0.967	0.059	0.952	0.314	0.752	0.318	0.749	1.240	0.214
NSB	0.8785	0.379	1.497	0.134	0.791	0.428	1.128	0.259	1.108	0.267	1.258	0.208	1.169	0.242
PSB	1.4015	0.161	1.112	0.265	0.721	0.470	1.433	0.151	1.461	0.143	1.339	0.180	1.853	0.063
JT	7.1067	0.068	6.640	0.084	2.156	0.540	6.186	0.102	7.512	0.057	7.616	0.054	9.467	0.023

Asymetric ($\log \sigma_t^2$) and Power Asymetric Models (σ_t^δ)								
	EGARCH(1,1)-Sk		FIEGARCH(1,1)-Sk		APARCH(1,1)-Sk		FIAPARCH(1,1)-t	
Criterion	Valor	p-value	Valor	p-value	valor	p-value	valor	p-value
ARCH 1-2	5.179	0.005	0.967	0.380	1.974	0.138	0.180	0.835
ARCH 1-5	2.518	0.027	0.709	0.615	1.958	0.081	0.487	0.785
ARCH 1-10	1.981	0.031	0.885	0.545	1.787	0.057	0.718	0.707
RBD(2)	-31.089	1.000	-0.501	1.000	-20.059	1.000	-5.310	1.000
RBD(5)	4.216	0.518	2.125	0.831	5.968	0.309	-1.257	1.000
RBD(10)	14.01	0.172	9.632	0.473	13.255	0.209	4.324	0.931
SB	0.262	0.792	0.142	0.886	0.036	0.970	0.156	0.875
NSB	1.283	0.199	1.345	0.178	0.969	0.332	0.761	0.446
PSB	0.761	0.446	0.847	0.396	0.659	0.509	1.032	0.301
JT	3.093	0.377	3.931	0.268	2.320	0.508	2.487	0.477

Table 1 (Continuation). Selection of Models

Criterion	Symetric Models													
	ARCH(1)-t		GARCH(1,1)-t		GJR(1,1)-t		IGARCH(1,1)-t		FIGARCH(1,1)-t		HYGARCH(1,1)-t		ARCH-M(1,1)-t	
	Valor	p-value	Valor	p-value	valor	p-value	valor	p-value	valor	p-value	valor	p-value	valor	p-value
Q(5)	282.08	0.000	9.2774	0.025	9.209	0.026	9.502	0.023	1.971	0.578	2.168	0.538	217.231	0.000
Q(10)	451.36	0.000	18.017	0.021	17.149	0.028	19.749	0.011	6.880	0.549	7.0888	0.527	355.031	0.000
Q(20)	737.83	0.000	26.573	0.087	26.316	0.092	27.191	0.075	14.961	0.664	15.427	0.632	623.811	0.000
Q(50)	1063.6	0.000	59.099	0.130	60.797	0.101	57.604	0.161	38.354	0.838	38.709	0.828	905.241	0.000
Criterion	Asymetric and Power Asymetric Models													
	EGARCH(1,1)-Sk		FIEGARCH(1,1)-Sk		APARCH(1,1)-t		FIAPARCH(1,1)-t							
	Valor	p-value	Valor	p-value	valor	p-value	valor	p-value						
Q(5)	12.510	0.005	3.537	0.315	9.961	0.018	2.440	0.486						
Q(10)	19.667	0.011	8.832	0.356	17.869	0.022	7.0586	0.530						
Q(20)	26.602	0.086	18.961	0.394	26.804	0.082	15.186	0.649						
Q(50)	52.687	0.297	42.232	0.707	60.576	0.105	39.306	0.810						

Table 1 (Continuation). Selection of Models

	Symetric Models													
Criterion	ARCH(1)-t		GARCH(1,1)-t		GJR(1,1)-t		IGARCH(1,1)-t		FIGARCH(1,1)-t		HYGARCH(1,1)-t		ARCH-M(1,1)-t	
	Valor	p-value	Valor	p-value	valor	p-value	valor	p-value	valor	p-value	valor	p-value	valor	p-value
P(40)	29.644	0.860	32.509	0.759	44.415	0.254	33.254	0.728	45.397	0.222	42.706	0.314	32.066	0.776
		0.586		0.392		0.043		0.405		0.035		0.048		0.413
P(50)	41.271	0.775	45.625	0.610	42.023	0.749	46.575	0.572	41.845	0.755	42.122	0.746	38.539	0.858
		0.502		0.285		0.383		0.289		0.390		0.337		0.580
P(60)	54.884	0.627	41.823	0.955	49.089	0.817	41.965	0.954	56.452	0.570	52.509	0.711	48.662	0.829
		0.365		0.816		0.509		0.838		0.246		0.339		0.567

	Asymetric and Power Asymetric Models							
Criterion	EGARCH(1,1)-Sk		FIEGARCH(1,1)-Sk		APARCH(1,1)-t		FIAPARCH(1,1)-t	
	Valor	p-value	Valor	p-value	valor	p-value	valor	p-value
P(40)	35.993	0.607	43.418	0.288	42.706	0.314	43.529	0.284
		0.142		0.023		0.048		0.030
P(50)	35.017	0.933	44.635	0.650	45.268	0.625	53.600	0.302
		0.608		0.181		0.226		0.047
P(60)	46.383	0.883	46.715	0.876	48.425	0.835	51.939	0.730
		0.539		0.48		0.496		0.323

Table 2. Evaluation of Forecasts

Meaures of Evaluation	FIGARCH(1,1)-t		FIEGARCH(1,1)-Sk		FIAPARCH(1,1)-t	
	Mean	Variance	Mean	Variance	Mean	Variance
Mean Squared Error (MSE)	0.807	0.846	0.806	0.567	0.807	0.750
Median Squared Error (MedSE)	0.473	0.633	0.473	0.417	0.487	0.679
Mean Error (ME)	-0.001	-0.660	-0.004	-0.386	0.007	-0.581
Mean Absolute Error (MAE)	0.798	0.836	0.797	0.677	0.797	0.785
Root Mean Squared Error (RMSE)	0.898	0.919	0.898	0.753	0.898	0.866
Theil Inequality Coefficient (TIC)	0.895	0.370	0.892	0.342	0.903	0.360

Table 2 (Continuation). Evaluation of Forecasts

Horizonte	FIGARCH(1,1)-t		FIEGARCH(1,1)-Sk		FIAPARCH(1,1)-t	
	Mean	Variance	Mean	Variance	Mean	Variance
1	0.234	0.974	0.251	1.037	0.231	0.949
2	0.151	1.077	0.153	1.056	0.142	1.042
3	0.116	1.182	0.117	1.086	0.106	1.139
4	0.107	1.269	0.107	1.117	0.097	1.218
5	0.105	1.256	0.105	0.975	0.095	1.194
6	0.045	1.368	0.058	1.167	0.038	1.301
7	0.104	1.437	0.104	1.185	0.094	1.364
8	0.104	1.495	0.104	1.201	0.094	1.416
9	0.104	1.548	0.104	1.218	0.094	1.462
10	0.104	1.511	0.104	1.053	0.094	1.413
11	0.045	1.602	0.058	1.265	0.038	1.501
12	0.104	1.654	0.104	1.278	0.094	1.548
13	0.104	1.699	0.104	1.294	0.094	1.587
14	0.104	1.741	0.104	1.315	0.094	1.622
15	0.104	1.693	0.104	1.137	0.094	1.564

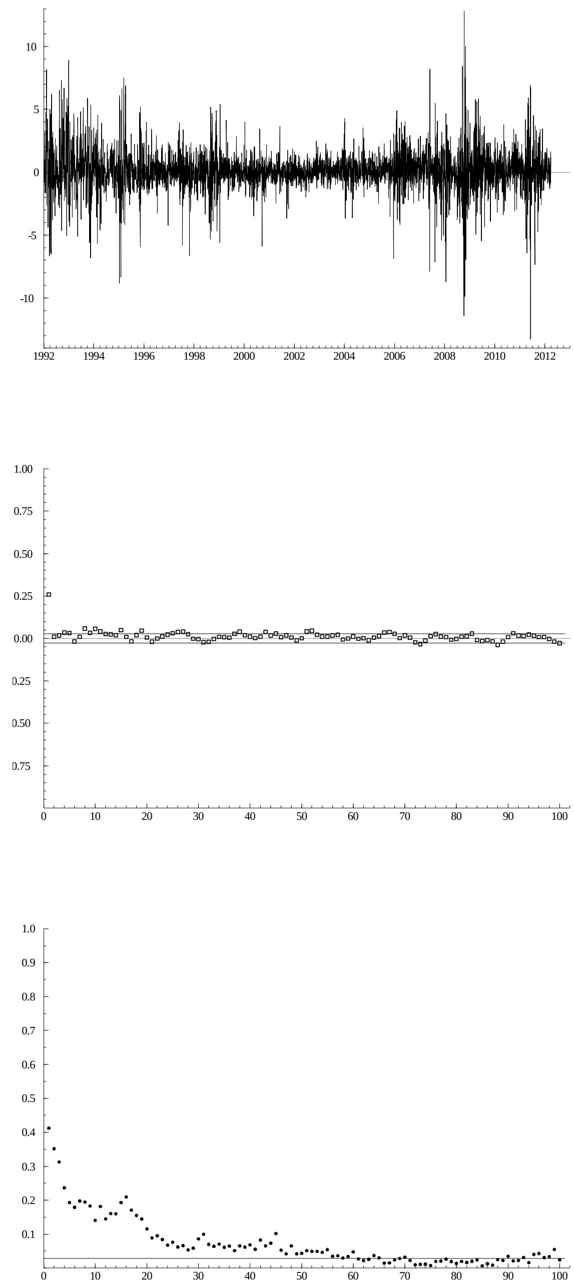

Figure 1. From Top to Bottom: Daily Stock Returns, ACF of Daily Stock Returns and ACF of Daily Stock Squared Returns

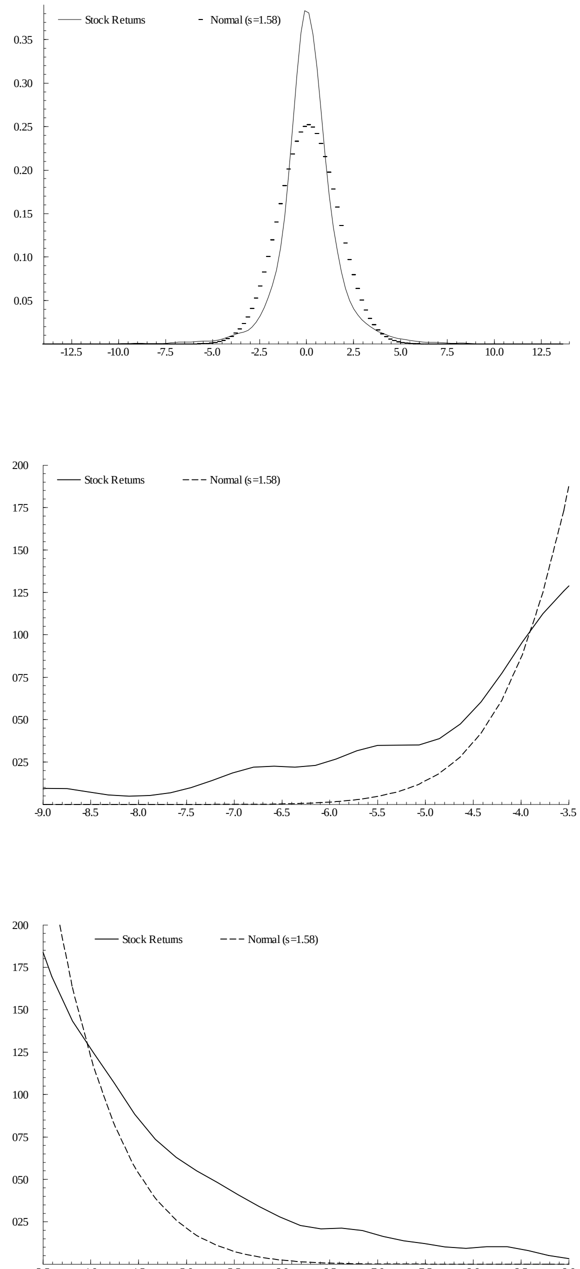

Figure 2. From Top to Bottom: Density Function of Returns, Left Tail Density of Returns and Rigth Tail Density of Returns

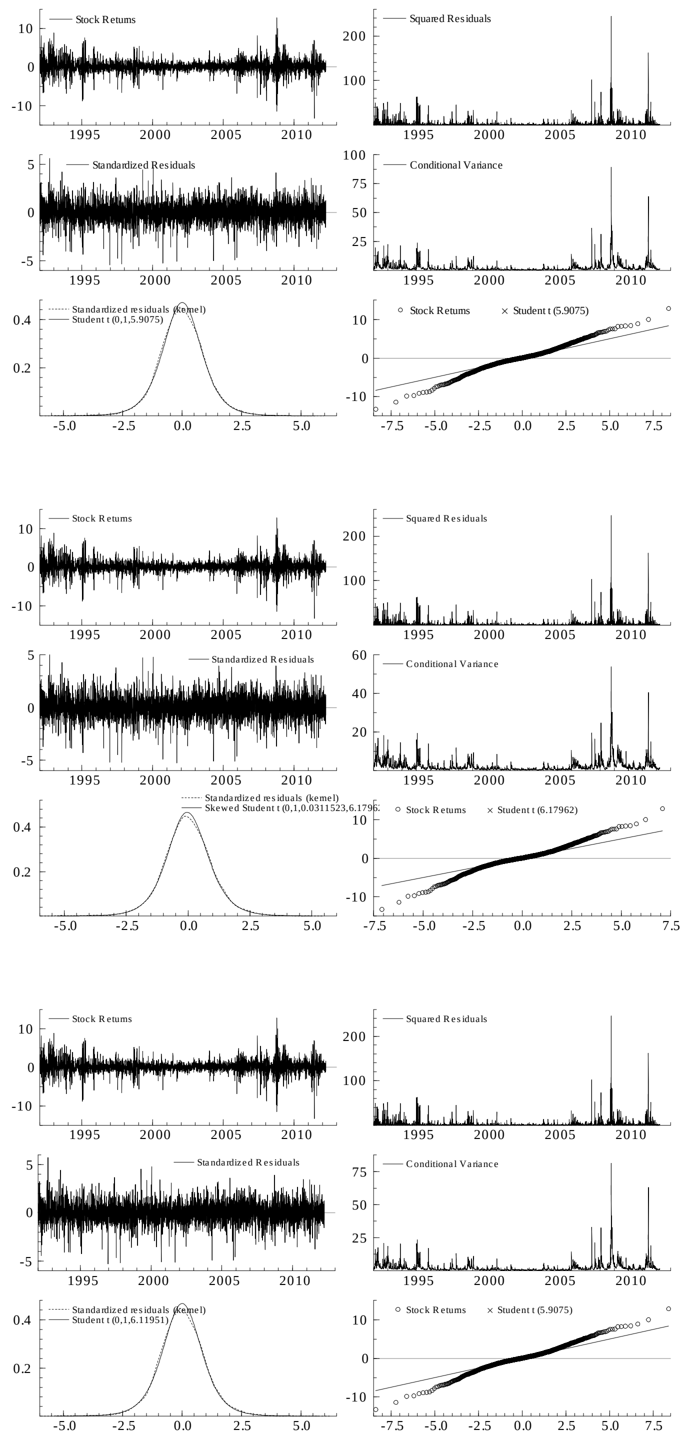

Figure 3. From Top to Bottom: Results of FIGARCH(1,1)-t, FIEGARCH (1,1)-Sk, FIAPARCH(1,1)-t. In each Panel (from left to rigth): Stock Returns, Squared Residuals, Standarized Residuals, Conditional Variance, Kernel of Standarized Residuals vs Density Function (t or Sk), qq-Plot.

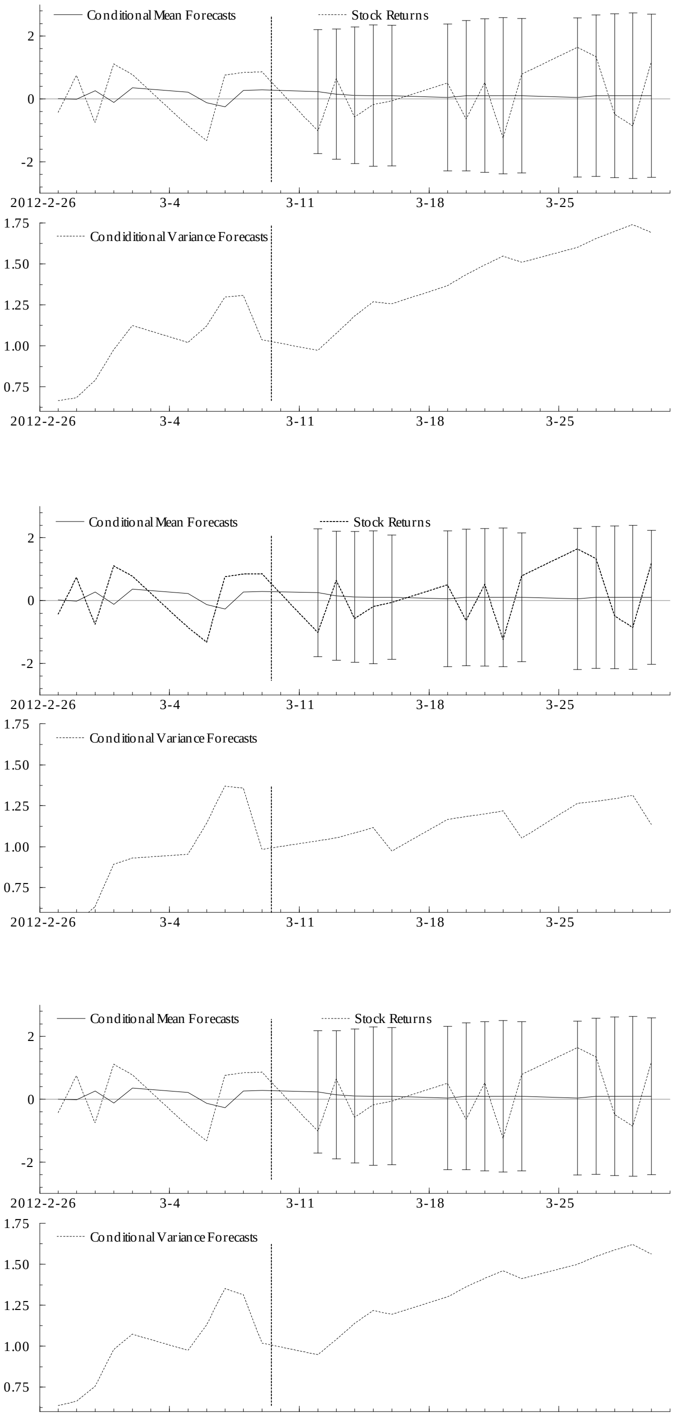

Figure 4. From Top to Bottom: Forecasts of FIGARCH(1,1)-t, FIEGARCH (1,1)-Sk, FIAPARCH(1,1)-t. In each Panel: Conditional Mean Forecasts, Conditional Variance Forecasts

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Departamento de Economía - Pontificia Universidad Católica del Perú
Av. Universitaria 1801, Lima 32 – Perú.
Telf. 626-2000 anexos 4950 - 4951
http://www.pucp.edu.pe/economia