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Abstract

Following Xu and Perron (2014), we applied the extended RLS model to the daily stock market returns of Argentina, Brazil, Chile, Mexico and Peru. This model replaces the constant probability of level shifts for the entire sample with varying probabilities that record periods with extremely negative returns; and furthermore, it incorporates a mean reversion mechanism with which the magnitude and the sign of the level shift component will vary in accordance with past level shifts that deviate from the long-term mean. Therefore, four RLS models are estimated: the basic RLS, the RLS with varying probabilities, the RLS with mean reversion, and a combined RLS model with mean reversion and varying probabilities. The results show that the estimated parameters are highly significant, especially that of the mean reversion model. An analysis is also performed of ARFIMA and GARCH models in the presence of level shifts, which shows that once these shifts are taken into account in the modeling, the long memory characteristics and GARCH effects disappear. Our forecasting analysis confirms that the RLS models are more accurate than other classic long-memory models.
KeyWords: Random Level Shifts Model, Volatility, Long Memory, GARCH, Latin-American Stock Markets, Varying Probabilities, Mean Reversion, Forecasting.
JEL Classification: C22, C52.

Resumen

Siguiendo el trabajo de Xu y Perron (2014), en este documento se aplica el modelo extendido de cambios de nivel aleatorios (RLS) a los retornos diarios de los mercados bursátiles de Argentina, Brasil, Chile, Mexico y Perú. A diferencia del modelo RLS básico, en este modelo se usan probabilidades cambiantes asociadas a periodos de retornos extremadamente negativos y además se incorpora un mecanismo de reversión a la media el cual depende de los cambios de nivel pasados y de las desviaciones de la media de largo plazo. Así, se estiman cuatro modelos de cambios de nivel aleatorios: el modelos RLS básico, el modelo RLS con probabilidades variantes, el modelo RLS con reversión a la media y finalmente, el modelo RLS que combina los dos aspectos ya mencionados. Los resultados muestran que los coeficientes estimados son significativos, en especial cuando se usa el modelo RLS con reversión a la media. Asimismo, se realizan estimaciones de modelos ARFIMA y GARCH a las series de volatilidad a las cuales se le ha sustraído el componente de cambios de nivel. Los resultados, muestran que una vez que dichos componentes son tomados en cuenta, las características de larga memoria y efectos GARCH desaparecen. Finalmente, un análisis de predicción es proporcionado el cual confirma que los modelos RLS son más eficientes que otros modelos clásicos de larga memoria.
Palabras Claves: Modelo con Cambios de Nivel Aleatorios, Volatilidad, Larga Memoria, GARCH, Mercados Bursátiles de América Latina, Probabilidades Variantes, Reversión a la Media, Predicción.
Classificación JEL: C22, C52.

Modeling Latin-American Stock Markets Volatility: Varying Probabilities and Mean Reversion in a Random Level Shifts Model[1]

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1 Introduction

Typically, the volatility of financial series exhibits long-term dependence or long memory. This property is represented in the domain of time by the behavior of its autocorrelation function (ACF), which presents significantly different values from zero up to a large number of lags, indicating hyperbolic decay. In the domain of frequencies a peculiar behavior can also be observed which is given by the higher weight of the low frequencies in the spectral density, and a rapid growth in this function can be observed as the frequencies approach the origin. Several authors document this characteristic; see Taylor (1986), Ding et al. (1993), Dacorogna et al. (1993), Robinson (1994), among others.

Formally, the long memory property is defined in a time series that has an ACF that slowly decays in its lags or, equivalently, if its spectral density function has an infinite value at the zero frequency. There are several possible formalizations for this definition; see McLeod and Hipel (1978), Robinson (1994), Beran (1994) and Baille (1996), among others. We follow the definitions presented in Perron and Qu (2010). Let $\{x_t\}_{t=1}^{T}$ be a stationary time series with spectral density function $f_x(\omega)$ at frequency ω, so x_t has long memory if $f_x(\omega) = g(\omega)\omega^{-2d}$, for $\omega \to 0$, where $g(\omega)$ is a function of smooth variation in a vicinity of the origin, which means that for all real numbers t, it is verified that $g(t\omega)/g(\omega) \to 1$ for $\omega \to 0$. When $d > 0$, the spectral density function increases for frequencies increasingly close to the origin. The divergent infinite rate depends on the value of parameter d. On the other hand, let $\gamma_x(\tau)$ be the ACF of x_t, so x_t has long memory if $\gamma_x(\tau) = c(\tau)\tau^{2d-1}$, for $\tau \to \infty$, where $c(\tau)$ is a function of smooth variation. When $0 < d < 1/2$ the ACF decreases at a slow rate of decay that depends on the value of parameter d[3].

Granger and Joyeux (1980) were the first to formulate the notion of fractional integration in terms of an infinite filter corresponding to the expansion of $(1-L)^d$, where L is the lag operator. Hosking (1981) generalizes the autoregressive integrated moving average processes by allowing the degree of integration d to take fractional values, and this model is known as ARFIMA(p,d,q). The fractional integration processes have long memory when $0 < d < 0.5$. On the other hand, if $-0.5 < d < 0.5$ the series is stationary. Geweke and Porter-Hudak (1983), based on a linear regression of the log-periodogram with a deterministic regressor, show that the asymptotic distribution of the long memory parameter d has a Normal distribution; see also Robinson (1995).

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[3] These definitions in the domain of frequency and time are equivalent if certain general conditions are verified, in accordance with the findings in Beran (1994).

Baillie, Bollerslev and Mikkelsen (1996) propose the FIGARCH model. In this model, the presence of the fractional integration allows an explanation and a representation of the temporary dependences in the volatility of the financial markets. Bollerslev and Mikkelsen (1996) propose the FIEGARCH model. In both cases, the fractional parameter is significant and asymmetries are identified in the series.

Ding, Granger and Engle (1993) find that the ACF of the absolute value of the returns is greater than the correlation of the returns. Thus, the specification $|r_t|^d$ exhibits a large correlation between very distant lags, especially when $d = 1$. Through the Monte Carlo method it is shown that the ARCH model with squared returns and absolute value returns possess the long-memory property. Finally, the authors propose the APARCH model (asymmetric power GARCH) which allows an estimation of the long-memory parameter in the volatility and the asymmetric power.

Lobato and Savin (1998) apply a semiparametric test, which is robust when there is weak dependence, to detect the presence of long memory in the daily returns on the S&P500 market and in squared returns. For the level of stock returns the null hypothesis of short memory is not rejected, while for the squared stock returns and the absolute value of the returns the null hypothesis is rejected. However, the authors argue that the results obtained may be spurious: due to the non-stationarity of the series in the squared stock returns, and due to aggregation in the absolute value of the returns. Finally, by dividing the sample into two periods and taking January 1973 as the breakpoint, no evidence is found of structural change causing the long memory.

At present, the literature argues that it could be structural changes that bring about long memory. Thus, the estimates and conclusions on financial returns and their modeling as long memory would be biased. In studies such as that of Perron (1989, 1990) it is shown that when a stationary process is contaminated with structural breaks, the sum of the autoregressive coefficients is biased to the unit. Teverovsky and Taqqu (1997), using the daily returns of the Center for Research in Security Prices (CRSO) for the period 1962-1987, present a method for distinguishing between the effects of level shifts and the effects of long memory.

Gourieroux and Jasiak (2001) evaluate the relationship between the presence of infrequent breaks and long memory based on the correlogram estimation instead of estimating the fractional parameter. The authors find that non-linear time series with infrequent breaks could have long memory. Therefore, these series and not the fractionally integrated processes with *i.i.d.* innovations would cause the hyperbolic decay of the autocorrelogram.

On the other hand, Diebold and Inoue (2001) argue that long memory and structural changes are related through the following models: the simple mixture permanent stochastic breaks of Engle and Smith's (1999) and the Markov-Switching model of Hamilton (1989). The authors show analytically that stochastic regime shifts are easily confused with long memory, even asymptotically, provided that the probabilities of structural breaks are small. The Monte Carlo simulations attest to the relevance of the finite samples theory, and make it clear that the confusion is not only a theoretical matter, but a real possibility in empirical economic and financial applications.

Granger and Hyung (2004), for their part, show that the slow decay in the ACF and other properties of the fractionally integrated models are caused by occasional breaks. Analytically, the authors show that not taking the breaks into consideration causes the presence of long memory in the ACF and that the fractional parameter estimated with the Geweke and Porter-Hudak method (1983) is biased. Then, empirically, the structural break model and the fractional integration models are compared to analyze the absolute value of the daily S&P500 stock returns from 1928 to 2002. The results show that the presence of long memory could be caused by not having taken the breaks in the series into consideration.

Mikosch and Stărică (2004a) provide the theoretical base to explain the stylized facts observed in the logarithm of returns: the long-range dependence in the volatility and the integrated GARCH (IGARCH) if it assumed that the date is not stationary. The simulations allow an appreciation that the time series with changing unconditional variance produce estimates of the long-memory parameter d that could be erroneously interpreted as evidence of long memory under the assumption of stationarity. There is evidence that the characteristic of long-range dependence is caused by feasible structural changes in the logarithm of stock returns.

Mikosch and Stărică (2004b) propose a goodness-of-fit test that shows the similarity between the spectral density of a GARCH process and the logarithm of stock market returns. Applying the test to the S&P 500 data from 1928 to 1991, changes are detected in the structure of data related to changes in the unconditional variance. These changes would induce long-range dependence in the ACF of absolute value stock market returns; see also Granger and Stărică (2005).

A recent study on the analysis of long memory and level shifts, or structural shifts, is that of Perron and Qu (2010). The authors present a method to distinguish between long memory and level shifts using the ACF, the periodogram and the fractional integration parameter d. Perron and Qu (2010) propose a simple mixture model that combines a short memory process and a component that reflects the level shifts, determined by an occurrence variable related to a Bernouilli process. Applying this method to the log-squared returns of four indices (S&P 500, NASDAQ, AMEX and Dow Jones), they conclude that the model that best describes the volatility of the returns is that which considers a short memory process with random level shifts.

Lu and Perron (2010) and Li and Perron (2013) use the model with random level shifts (RLS) to model the volatility of stock market and exchange rate returns, respectively. Empirical studies applied to financial series in Latin America are scanty. The RLS model has recently been applied by Ojeda Cunya and Rodríguez (2014) to explain stock market and exchange rate volatility in Peru, and by Rodríguez and Tramontana Tocto (2014) to analyze the behavior of stock market volatilities in a sample of Latin American countries. In this study, we follow the expanded RLS model of Xu and Perron (2014) applied to the volatility of stock market returns of Argentina, Brazil, Chile, Mexico and Peru by taking two aspects into account: (a) replacing the constant probability of level shifts for the entire sample with varying probabilities that record periods with extremely negative returns; and (b) incorporating a mean reversion mechanism with which the magnitude and the sign of the level shift component will vary in accordance with past level shifts that deviate from the long-term mean. Four RLS models are estimated: basic RLS, RLS with varying probabilities, RLS with mean reversion, and a combined RLS model with mean reversion and varying probabilities. The results show that the estimated parameters are highly significant, especially that of the mean reversion model. An analysis is also performed of ARFIMA and GARCH models in the presence of level shifts, which shows that once these shifts are taken into account in the modeling, the long memory characteristics and GARCH effects disappear. Our prediction analysis confirms that the RLS models are more accurate than other classic long-memory models.

The paper is structured as follows. Section 2 presents the basic RLS model and describes the two modifications to that model, and describes the estimation method. Section 3 presents the data and the results of the estimation of the different models. Moreover, a comparison with the ARFIMA(p,d,q), GARCH and CGARCH models is presented. Section 4 shows the prediction results, while Section 5 discusses the main conclusions.

2 Methodology

This Section presents the basic RLS model that considers a constant probability of level shifts. Then, the two extensions to this model are presented and the details of the estimation algorithm are briefly described.

2.1 The Basic RLS Model

We use a simple mixture model, which is a combination of a short memory process and a level shift component that depends on a Binomial distribution. Following Lu and Perron's notation (2010), the RLS is specified as follows:

$$\begin{aligned} y_t &= a + \tau_t + c_t, \\ \tau_t &= \tau_{t-1} + \delta_t, \\ \delta_t &= \pi_t \eta_t, \end{aligned} \tag{1}$$

where a is a constant, τ_t is the level-shift component, c_t is the short-memory component, and π_t is a Binomial variable, which takes the value of 1 with probability α and the value of 0 with probability $(1-\alpha)$. In this way, following the third expression in (1), when π_t assumes the value of 1, a random level shift η_t occurs with a distribution $\eta_t \sim i.i.d.\ N(0, \sigma_\eta^2)$. The short-memory process (in its general form) is defined by the process $c_t = C(L)e_t$, with $e_t \sim i.i.d.\ N(0, \sigma_e^2)$ and $E|e_t|^r < \infty$ for values $r > 2$, where $C(L) = \sum_{i=0}^{\infty} c_i L^i$, $\sum_{i=0}^{\infty} i|c_i| < \infty$ and $C(1) \neq 0$. Moreover, it is assumed that π_t, η_t and c_t are mutually independent. Based on the results of Lu and Perron (2010) and Li and Perron (2013), even when it would be useful to consider the component e_t as a random variable (noise), in this paper we model this component as an AR(1) process, that is, $c_t = \phi c_{t-1} + e_t$[4].

Note that the process δ_t can be described as $\delta_t = \pi_t \eta_{1t} + (1-\pi_t)\eta_{2t}$, with $\eta_{it} \sim i.i.d. N(0, \sigma_{\eta_i}^2)$ for $i = 1, 2$ and $\sigma_{\eta_1}^2 = \sigma_\eta^2$, $\sigma_{\eta_2}^2 = 0$. The first-differences model, with the aim of eliminating the autoregressive process of the level shift component, depends solely on the Binomial process: $\Delta y_t = \tau_t - \tau_{t-1} + c_t - c_{t-1} = c_t - c_{t-1} + \delta_t$, and moving to the state-space form, the mean and transition equations are obtained, respectively: $\Delta y_t = c_t - c_{t-1} + \delta_t$, $c_t = \phi c_{t-1} + e_t$. In matrix form $\Delta y_t = H X_t + \delta_t$ and $X_t = F X_{t-1} + U_t$ are obtained, where $X_t = [c_t, c_{t-1}]$, $F = \begin{bmatrix} \phi & 0 \\ 1 & 0 \end{bmatrix}$, $H = [1, -1]'$. In this case, the first row of the matrix F shows the coefficient ϕ of the autoregressive part of the short-memory component. Moreover, U is a Normally distributed vector of dimension 2 with mean 0 and variance: $Q = \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & 0 \end{bmatrix}$. In comparison with the standard state-space model, the important difference in the current model is that the distribution of δ_t is a mixture of two Normal distributions with variance σ_η^2 and 0, occurring with probabilities α and $1-\alpha$, respectively[5].

[4] We opted for an AR(1) specification but if the coefficient ϕ is statistically insignificant, $c_t = e_t$. Estimates with longer lags for the AR process showed no significance of the respective parameters. This is consistent with the statements by the RLS model because if the persistence or long memory in the volatility of the series analyzed is mainly explained by rare or sporadic level shifts, then the short-memory component contains little persistence or it is a noise. This justifies c_t is modeled as a noise or maximum as an AR (1) process.

[5] Note that this model can be extended to model the short-memory component as an ARMA(p,q) process. However, the estimates show no statistical significance beyond an AR(1) process.

The model described above is a special version of the models included in Wada and Perron (2006) and Perron and Wada (2009). In this case, there are only shocks that affect the level of the series, and the restriction is imposed that the variance of one of the components of the mixture of distributions is zero. The basic input for the estimation is the increase in the states through the realizations of the mixture at time t so that the Kalman filter can be used to construct the likelihood function, conditional to the realizations of the states. The latent states are eliminated from the final expression of the likelihood by summing over all the possible realizations of the states. In consequence, despite its fundamental differences, the model takes a structure that is similar to that of Hamilton's Markov-Switching model (1994)[6]. Let $Y_t = (\Delta y_1, ..., \Delta y_t)$ be the vector of observations available at time t and denote the vector of parameters by $\theta = [\sigma^2_\eta, \alpha, \sigma^2_e, \phi]$. Adopting the notation used in Hamilton (1994), $\mathbf{1}(.)$ represents a vector of ones of dimension (4×1), the symbol $\odot$ denotes element-by-element multiplication, $\widehat{\xi}^{ij}_{t|t-1} = vec(\widetilde{\xi}_{t|t-1})$ with the (i, j)th element of $\widetilde{\xi}_{t|t-1}$ being $\Pr(s_{t-1} = i, s_t = j|Y_{t-1}; \theta)$ and $\omega_t = vec(\widetilde{\omega}_t)$ with the (i, j)th element of $\widetilde{\omega}_t$ being $f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}; \theta)$ for $i, j \in \{1, 2\}$. Thus, we have $s_t = 1$ when $\pi_t = 1$, that is, a level shift occurs. Using the same notation as Lu and Perron (2010), the logarithm of the likelihood function is $\ln(L) = \sum_{t=1}^{T} \ln f(\Delta y_t|Y_{t-1}; \theta)$, where $f(\Delta y_t|Y_{t-1}, \theta) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}, \theta) \Pr(s_{t-1} = i, s_t = j|Y_{t-1}, \theta) \equiv \mathbf{1}'(\widehat{\xi}_{t|t-1} \odot \omega_t)$. By applying rules of conditional probabilities, Bayes's rule and the independence of s_t with respect to past realizations, we obtain $\widetilde{\xi}^{ki}_{t|t-1} = \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1}; \theta)$. The evolution of $\widehat{\xi}_{t|t-1}$ can be expressed as:

$$\begin{bmatrix} \widetilde{\xi}^{11}_{t+1|t} \\ \widetilde{\xi}^{21}_{t+1|t} \\ \widetilde{\xi}^{12}_{t+1|t} \\ \widetilde{\xi}^{22}_{t+1|t} \end{bmatrix} = \begin{bmatrix} \alpha & \alpha & 0 & 0 \\ 0 & 0 & \alpha & \alpha \\ 1-\alpha & 1-\alpha & 0 & 0 \\ 0 & 0 & 1-\alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} \widetilde{\xi}^{11}_{t|t} \\ \widetilde{\xi}^{21}_{t|t} \\ \widetilde{\xi}^{12}_{t|t} \\ \widetilde{\xi}^{22}_{t|t} \end{bmatrix}, \tag{2}$$

which is equal to $\widehat{\xi}_{t+1|t} = \Pi \widehat{\xi}_{t|t}$ with $\widehat{\xi}_{t|t} = \frac{(\widehat{\xi}_{t|t-1} \odot \omega_t)}{\mathbf{1}'(\widehat{\xi}_{t|t-1} \odot \omega_t)}$. Note that thus far the model includes the probabilities of level shift (α) as constant. Thus, once the specific estimate of α is obtained, a possible change is the use of a smoothed estimate of the level shift component τ_t. However, in the present context of abrupt structural shifts, the conventional smoothers may perform poorly. In place of this, we use the model proposed by Bai and Perron (1998, 2003) to obtain the dates on which the level shifts occur, as well as the means (averages) within each segment. Indeed, we use the estimation of α to obtain an estimate of the number of level shifts, and the Bai and Perron method (1998, 2003) to obtain estimates of the break dates that globally minimize the following sum of squared residuals: $\sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [y_t - \mu_i]^2$, where m is the number of breaks, T_i $(i = 1, 2, ...; m)$ are the break dates $T_0 = 0$, and $T_{m+1} = T$ and μ_i $(i = 1, 2, ..., m + 1)$ are the means (averages) inside each regime, which can be estimated once the date breaks have been estimated or known . This method is efficient and can handle a large number of observations; see Bai and Perron (2003) for further details[7].

[6] In comparison with Hamilton's Markov-Switching model (1989), this model does not limit the magnitude of the level shifts, so any number of regimes is possible. Moreover, the probability 0 or 1 does not depend on past events, unlike the Markov model.

[7] Note that because the model permits consecutive level shifts, we set the minimum length of a segment at only

In consequence, the conditional likelihood function for Δy_t corresponds to the following Normal density: $\widetilde{\omega}_t^{ij} = f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}} |f_t^{ij}|^{-1/2} \exp(-\frac{v_t^{ij\prime}(f_t^{ij})^{-1/2} v_t^{ij}}{2})$, where v_t^{ij} is the prediction error and f_t^{ij} is its variance, and these terms are defined as: $v_t^{ij} = \Delta y_t - \Delta y_{t|t-1}^i = \Delta y_t - E[\Delta y_t | s_t = i, Y_{t-1}; \theta]$, and $f_t^{ij} = E(v_t^{ij} v_t^{ij\prime})$. The best predictions for the state variable and its respective conditional variance in $s_{t-1} = i$ are $X_{t|t-1}^i = F X_{t-1|t-1}^i$, and $P_{t|t-1}^i = F P_{t-1|t-1}^i F' + Q$, respectively.

The mean equation is $\Delta y_t = H X_t + \delta_t$, where the error δ_t has a mean 0 and a variance that can take values $R_1 = \sigma_\eta^2$ with probability α or $R_2 = 0$ with probability $(1-\alpha)$. Thus, the prediction error is $v_t^{ij} = \Delta y_t - H X_{t|t-1}^i$ and its variance is $f_t^{ij} = H P_{t|t-1}^i H' + R_j$. In this way, given that $s_t = j$ and $s_{t-1} = i$ and using updating formulas, $X_{t|t}^i = X_{t|t-1}^i + P_{t|t-1}^i H'(H P_{t|t-1}^i H' + R_j)^{-1}(\Delta y_t - H X_{t|t-1}^i)$ and $P_{t|t-1}^{ij} = P_{t|t-1}^i - P_{t|t-1}^i H'(H P_{t|t-1}^i H' + R_j)^{-1} H P_{t|t-1}^i$ are obtained. In order to reduce the dimensionality problem in the estimation, Lu and Perron (2010) use the re-collapsing procedure posed by Harrison and Stevens (1976). In so doing, $\widetilde{\omega}_t^{ij}$ is unaffected by the history of the states before time $t-1$. We have four possible states corresponding to $S_t = 1$ when $(s_t = 1, s_{t-1} = 1)$, $S_t = 2$ when $(s_t = 1, s_{t-1} = 2)$, $S_t = 3$ when $(s_t = 2, s_{t-1} = 2)$ and $S_t = 4$ when $(s_t = 2, s_{t-1} = 2)$ and the matrix Π is defined as (2). Taking the definitions of $\widetilde{\omega}_t$, $\widehat{\xi}_{t|t}$, $\widehat{\xi}_{t+1|t}$, the set of conditional probabilities and the one-period forward predictions, the same structure as a version of Hamilton's Markov model (1994) is obtained. However, the EM algorithm cannot be used. This is because the mean and the variance in the conditional density function are non-linear functions of the parameters θ and of past realizations $\{\Delta y_{t-j}; j \geq 1\}$. Likewise, the conditional probability of being in a determined regime $\widehat{\xi}_{t|t}$ is inseparable from the conditional densities $\widetilde{\omega}_t$. For further details, see Lu and Perron (2010), Li and Perron (2013), and Wada and Perron (2006).

2.2 Extensions to the Basic RLS Model

As pointed out in Xu and Perron (2014) and in the results of Ojeda Cunya and Rodríguez (2014) and Rodríguez and Tramontana Tocto (2014), level shifts usually occur in clusters in certain periods of time related to financial crisis and, in the case of the exchange rate, with exchange rate intervention measures by the Central Bank [8]. This phenomenon of clustering indicates that level shifts are not *i.i.d.*, but that the probability of these shifts varies in accordance with economic, political, and social conditions in the country.

Following on from the notation used in Xu and Perron (2014), the probability of level shift is defined as $p_t = f(p, x_{t-1})$, where p is a constant and x_{t-1} are the covariables that help to better predict the probability of level shifts. According to the study by Martens, van Dijk and de Pooter (2004), there is a strong relationship between current volatility and past returns, also known as the leverage effect. This effect will be modeled through the *news impact curve* proposed by Engle and Ng (1993) as follows: $\log(\sigma_t^2) = \beta_0 + \beta_1 \mathbf{1}(r_{t-1} < 0) + \beta_2 |r_{t-1}| \mathbf{1}(r_{t-1} < 0)$, where σ_t^2 represents the volatility and $\mathbf{1}(A)$ is the indicator function that takes the value of one when the event A occurs. Given that our objective in this part of the study is not to model the volatility but the probability of level shifts, the variable x_{t-1} will not be represented by past returns (r_{t-1}). Instead, extreme past returns that are below a threshold κ will be used. Therefore, we will employ the returns that belong to 1%, 2.5% and 5% of the distribution of the returns $(\kappa = 1.0\%, 2.5\%, 5.0\%)$. Thus, the

one observation.

[8] This is investigated in Gonzáles Tanaka, Ojeda Cunya and Rodríguez (2015).

probability of level shifts will be given by:

$$f(p, x_{t-1}) = \left\{ \begin{array}{ll} \Phi(p + \gamma_1 \mathbf{1}\{x_{t-1} < 0\} + \gamma_2 \mathbf{1}\{x_{t-1} < 0\} |x_{t-1}|) & \text{for } |x_{t-1}| > \kappa \\ \Phi(p) \quad \text{other cases,} & \end{array} \right\}, \tag{3}$$

where $\Phi(.)$ is a function of Normal accumulated distribution, with which we ensure that $f(p, x_{t-1})$ is between 0 and 1.

The second observation of the afore-mentioned studies is that level shifts occur around a mean; that is, each time a level shift occurs and the volatility of the series increases, a similar change occurs in the opposite direction, which makes the mean of the volatility remain at a given value. This process of mean reversion is modeled as follows: $\eta_{1t} = \beta(\tau_{t|t-1} - \overline{\tau}_t) + \widetilde{\eta}_{1t}$, where $\widetilde{\eta}_{1t}$ is distributed Normally with mean 0 and variance σ^2_η, $\tau_{t|t-1}$ is the estimated level shift component at time t, and $\overline{\tau}_t$ is the mean of all level-shift components estimated from the start of the sample to time t. The process of mean reversion occurs when $\beta < 0$ and this parameter will represent the velocity at which the volatility returns to its mean. The final model combines the two stated characteristics, giving us four models to estimate.

2.3 Estimation Method

The estimation method is based on the work of Xu and Perron (2014), which is an extension of the basic RLS model by Lu and Perron (2010) and Li and Perron (2013). The first difference compared with the basic model is that the vector of parameters is different: $\theta = [\sigma^2_\eta, p, \sigma^2_e, \phi, \gamma_1, \gamma_2, \beta]$[9]. The second important difference is that, given the probability of level shifts is now varying, the equation (2) is replaced by:

$$\begin{bmatrix} \widetilde{\xi}^{11}_{t+1|t} \\ \widetilde{\xi}^{21}_{t+1|t} \\ \widetilde{\xi}^{12}_{t+1|t} \\ \widetilde{\xi}^{22}_{t+1|t} \end{bmatrix} = \begin{bmatrix} p_{t+1} & p_{t+1} & 0 & 0 \\ 0 & 0 & p_{t+1} & p_{t+1} \\ (1-p_{t+1}) & (1-p_{t+1}) & 0 & 0 \\ 0 & 0 & (1-p_{t+1}) & (1-p_{t+1}) \end{bmatrix} \begin{bmatrix} \widetilde{\xi}^{11}_{t|t} \\ \widetilde{\xi}^{21}_{t|t} \\ \widetilde{\xi}^{12}_{t|t} \\ \widetilde{\xi}^{22}_{t|t} \end{bmatrix}. \tag{4}$$

Therefore, the conditional likelihood function for Δy_t follows the Normal density: $\widetilde{\omega}^{ij}_t = f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}} |f^{ij}_t|^{-1/2} \exp(-\frac{v^{ij\prime}_t (f^{ij}_t)^{-1/2} v^{ij}_t}{2})$, where v^{ij}_t is the prediction error and f^{ij}_t is its variance and is defined as: $v^{ij}_t = \Delta y_t - \Delta y^{ij}_{t|t-1} = \Delta y_t - E[\Delta y_t | s_t = i, s_{t-1} = j, Y_{t-1}, \theta]$ and $f^{ij}_t = E(v^{ij}_t v^{ij\prime}_t)$. Note that $\Delta y^{ij}_{t|t-1}$ depends only on the information contained in $t-1$. The predictions for the variable of state and its respective conditional variance to $s_{t-1} = i$ are: $X^i_{t|t-1} = F X^i_{t-1|t-1}$ and $P^i_{t|t-1} = F P^i_{t-1|t-1} F' + Q$. Our mean equation is $\Delta y_t = H X_t + \delta_t$, where the error δ_t has zero mean and a variance that can take values $R_1 = \sigma^2_\eta$ or values $R_2 = 0$, so our prediction error will be $v^{ij}_t = \Delta y_t - H X^i_{t|t-1}$ and will be associated with a variance $f^{ij}_t = H P^i_{t|t-1} H' + R_j$. Then, given $s_t = j$ and $s_{t-1} = i$ and using the updating formula we get: $X^{ij}_{t|t} = X^i_{t|t-1} +$

[9] This vector of parameters corresponds to the model that contains the two extensions. In the case of the RLS model only with varying probabilities, the vector of parameters is $\theta = [\sigma^2_\eta, p, \sigma^2_e, \phi, \gamma_1, \gamma_2]$, while in the case of the RLS model with mean reversion, the vector of parameters is $\theta = [\sigma^2_\eta, p, \sigma^2_e, \phi, \beta]$.

$P^i_{t|t-1}H'(HP^i_{t|t-1}H'+R_j)^{-1}(\Delta y_t - HX^i_{t|t-1})$, $P^{ij}_{t|t} = P^i_{t|t-1} - P^i_{t|t-1}H'(HP^i_{t|t-1}H'+R_j)^{-1}HP^i_{t|t-1}$. As in Perron and Wada (2009) we will reduce the estimation problem by using the re-collapsing process proposed by Harrison and Stevens (1976): $X^i_{t|t} = \frac{\sum_{i=1}^2 \Pr(s_{t-1}=i,s_t=j|Y_t,\theta)X^{ij}_{t|t}}{\Pr(s_t=j|Y_t,\theta)} = \frac{\sum_{i=1}^2 \widetilde{\xi}^{ij}_{t|t}X^{ij}_{t|t}}{\sum_{i=1}^2 \widetilde{\xi}^{ij}_{t|t}}$ and $P^i_{t|t} = \frac{\sum_{i=1}^2 \Pr(s_{t-1}=i,s_t=j|Y_t,\theta)[P^{ij}_{t|t}+(X^i_{t|t}-X^{ij}_{t|t})(X^i_{t|t}-X^{ij}_{t|t})']}{\Pr(s_t=j|Y_t,\theta)} = \frac{\sum_{i=1}^2 \widetilde{\xi}^{ij}_{t|t}[P^{ij}_{t|t}+(X^j_{t|t}-X^{ij}_{t|t})(X^j_{t|t}-X^{ij}_{t|t})']}{\sum_{i=1}^2 \widetilde{\xi}^{ij}_{t|t}}$.

For the model of mean reversion, certain modifications are necessary. The prediction error v^{ij}_t of the previous expressions is no longer Normally distributed with mean 0 and variance that depends on the value of the state, but is modeled as: $y_t = a + c_t + \tau_t$, $\Delta y_t = \tau_t - \tau_{t-1} + c_t - c_{t-1}$, $\tau_t - \tau_{t-1} = \pi_t[\beta(\tau_{t|t-1} - \overline{\tau}_t) + \widetilde{\eta}_{1t}] + (1-\pi_t)\eta_{2t}$. Moreover, $\widetilde{\omega}^{ij}_t = f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}}|f^{ij}_t|^{-1/2}\exp(-\frac{\widetilde{v}^{ij\prime}_t (f^{ij}_t)^{-1/2}\widetilde{v}^{ij}_t}{2})$, $\widetilde{v}^{ij}_t = \left\{ \begin{matrix} v^{11}_t - \beta(\tau^{11}_{t|t-1} - \overline{\tau}^{11}_t) \\ v^{12}_t \\ v^{21}_t - \beta(\tau^{21}_{t|t-1} - \overline{\tau}^{21}_t) \\ v^{22}_t \end{matrix} \right\}$ and $f^{ij}_t = E(\widetilde{v}^{ij}_t \widetilde{v}^{ij\prime}_t) = HP^i_{t|t-1}H' + R_j$. Further details appear in Xu and Perron (2014).

3 Empirical Results

3.1 The Data

To apply and estimate the parameters of the models set out above we use five daily time series: those corresponding to the IGBVL index (Peru), from 03/01/1990 to 13/06/2013 (5832 observations), the MERVAL index (Argentina) from 04/08/1988 to 13/06/2013 (6142 observations), the IBOV (Brazil) from 02/01/1992 to 13/06/2013 (5303 observations), the IPSA (Chile) from 02/01/1989 to13/06/2013 (6098 observations) and the MEXBOL (Mexico) from 19/01/1994 to 13/06/2013 (4841 observations). The returns are calculated as $r_t = \ln(P_t) - \ln(P_{t-1})$, where P_t are the values presented for the five indices. Following recent literature (see Lu and Perron (2010), Li and Perron (2010), Xu and Perron (2010), among others), we model log-absolute returns[10]. When returns are zero or close to it, the log-absolute transformation implies extreme negative values. Using the estimation method described in Section 2.1, these outliers would be attributed to the level shifts component and thus bias the probability of shifts upward. To avoid this inconvenient, we bound absolute returns away from zero by adding a small constant, i.e., we use $y_t = \log(|r_t| + 0.001)$, a technique introduced to the stochastic volatility literature by Fuller (1996). The results are robust to alternative specifications, for example using another value for this so-called offset parameter, deleting zero observations, or replacing them by a small value. Another important comment is the fact that we use daily returns as opposed to realized volatility series constructed from intra-daily high-frequency data which has recently become popular. It is true that realized volatility series are less noisy measure of volatility. However, it is problematic in the current context for he following

[10] Using this measure has two advantages: (i) it does not suffer from a non-negativity constraint as do, for example, absolute or squared returns. Actually, it is a similar argument as used in the EGARCH(1,1) model proposed by Nelson (1991): the dependent variable is $\log(\sigma^2_t)$ in order to avoid the problems of negativity when the dependent variable is σ^2_t as in the standard GARCH models and other relatives models; (ii) there is no loss relative to using square returns in identifying level shifts since log-absolute returns is a monotonic transformation. It is true that log-absolute returns are quite noisy but it is not problematic since the algorithm used is robust to the presence of noise.

reasons: (i) such series are typically available for short span. Given the fact that the level shifts will be relatively rare, it is imperative to have a long span of data in order to made reliable estimates of the probability of occurrence of the level shifts; (ii) such series are available only for specific assets as opposed to market indices. Because the goal of the RLS model is to allow for particular events affecting overall markets, using specific asset would confound such market-wide events with idiosyncratic ones associated with the particular asset used; (iii) we are interested to re-evaluate the adequacy of ARFIMA and GARCH models applied to daily returns when taking into account the possibility of level shifts. Therefore, it is important to have estimates of these level shifts for squared daily returns which are equivalent to those estimated using log-absolute returns.

Table 1 shows the main descriptive statistics of the returns and the volatility, and Figure 1 illustrates the behavior of the returns. It can be seen that these series move around a mean close to zero and exhibit clusters in their distribution in time. This backs the modeling of varying probabilities put forward in the previous section. For volatility, the asymmetry is very small and ranges from -0.259 to -0.027. The kurtosis in all series is very close to 3 (2.578-2.827). Further details on the stylized facts in Peru's foreign exchange and stock markets can be found in Humala and Rodríguez (2013).

On the other hand, Figure 2 shows the ACF of the volatility series. The persistent behavior of the ACF is clearly observed, a stylized fact frequently found in the empirical literature that suggests the existence of long memory or long range dependence.

3.2 Results of the Estimations

The estimations of the basic RLS model (Basic RLS), the model RLS with varying probabilities (Threshold κ% RLS), the RLS model with mean reversion (Mean reversion RLS) and the modified RLS with the two extensions (Modified RLS) are shown in Tables 2, 3, 4 and 5, respectively.

The results presented in Table 2 show values that are slightly different to those that can be appreciated in Rodríguez and Tramontana Tocto (2014) given the changes made to the sample of Argentina and Brazil to remove very extensive non-trading periods in the respective markets due to the problems of high inflation undergone by those countries. What is observed is that for Chile, Mexico and Peru, the autoregressive parameter of the short-memory component (ϕ) is significant, and is not for Argentina and Brazil, which means that this series is modeled with a Normally distributed short-memory process with 0 and variance σ_e^2. This distinction is important because this rule will be followed in the coming models where the modifications for the basic RLS are introduced[11].

Given the estimates of the level-shift probabilities (α) and the number of observations, the number of breaks can be calculated in each of the markets. Thus, we have 49, 53, 49, 29 and 25 breaks or level shifts for Argentina, Brazil, Chile, Mexico and Peru, respectively[12]. Figure 3 shows the smoothed level component estimated by the algorithm (τ_t) along with estimates of the level-shift component estimated using the method of Bai and Perron (1998, 2003). The shifts or jumps in the mean seem to describe the behavior of the series well; see Rodríguez and Tramontana Tocto (2014) for further explanation.

For Table 3, the parameters introduced in the estimation are γ_1 and γ_2, which refer to the leverage or the news effect, represented by the past returns in the volatility. Moreover, to estimate

[11] For Mexico, the parameter ϕ is significant at only 10%, due to which it is used only for the model with varying probability and not for the mean reversion or modified models, where it is insignificant.

[12] The results are the same as Rodríguez and Tramontana (2014), except for Argentina where the sample was reduced a little.

each series three threshold levels have been applied for the extremely negative returns; that is, a given value κ is taken, under which 1%, 2.5% and 5% of the returns are found. With respect to the values of p, the estimated values in the varying probability model can be changed into constant probabilities to make a comparison with the probabilities of the basic RLS model "α". The estimates of α for Argentina are 0.004, 0.008 and 0.008 for 5%, 2.5% and 1% as thresholds, respectively. For Brazil, the estimates are 0.005, 0.008, and 0.008 for the three threshold levels. For Chile, these values are 0.031, 0.013 and 0.018 for the thresholds 5%, 2.5% and 1%. For Mexico, the values are 0.029, 0.018 and 0.017. Finally, for Peru the estimates of p allow the probabilities of 0.0037, 0.0037 and 0.0045 to be obtained for the three respective thresholds. As can be seen, the probability estimates obtained based on the estimation p give very similar results to those obtained with the basic RLS model, as shown in Table 2, except for Chile.

The estimations are positive values for γ_1 and γ_2, with which the sense of the model is achieved and the probability of level shift increases with extremely negative news. We observe that the coefficient γ_2 is highly significant for Brazil, Mexico and Peru. For Argentina, the significant coefficient is γ_1, which indicates that the new information or news has an important impact on the probability of level shifts. For Chile's volatility series, both γ_1 and γ_2 are not significant, unless γ_2 in the estimation with a threshold of 5%. Moreover, from estimates of p_t the implicit probabilities can be deduced, which allow the infrequency of level shifts to be affirmed. Another notable fact that can be seen in Table 3 is that as the threshold decreases from 5% to 1%, the value of γ_2 decreases and becomes more significant; that is, the standard error decreases, and conversely, the γ_1 acquires greater value and its significance reduces.

Table 4 shows the model when the mean reversion mechanism is introduced. As we have seen in the estimations, the parameter β is negative and significant, which confirms that a mean reversion process exists in the five series. It can also be noted that the value of the probability of level shifts is greater than in the basic RLS model. Moreover, the standard error of the level-shift component decreases in relation to the basic RLS model, which owes to the fact that β absorbs much of the volatility captured by the other model, which reduces in importance and even comes to be insignificant, as is the case of Peru. The value of the mean reversion parameter is higher (in absolute value) for Peru, at almost double what is observed for other countries.

Table 5 shows the estimations of the modified RLS model. We observe that for Argentina, Brazil and Peru, the estimates in Tables 3 and 4 are maintained; that is, only the parameter γ_2 is significant for Brazil and Peru, only γ_1 is significant for Argentina, and the parameter β is significant and negative for the three countries and is, as before, more negative for the case of Peru. In Chile and Mexico a change to the estimated parameters is noted, given that for the first, in Table 3 neither γ_1 nor γ_2 are significant; however, in Table 5 we see that both parameters are significant. For Mexico, in Table 3 the parameter that was not significant was γ_1; conversely, with the combined model this coefficient becomes significant.

To back up our numerical results, graphs were prepared that show the relationship between the estimated level shifts and high volatility events in the market. Figure 4 shows the extreme past returns ($\kappa = 1.0\%$) and the level shift component[13,14]. We observe that the periods of turbulence on the markets corresponds with increases in the value of the level shifts component, which in the

[13] The level shift components estimated by the four models are very similar. We can opt to chart one of them. However, Figure 3 shows the four level shifts estimates. The evidence is clear that all are very similar, which gives rise to the superposition of the lines.

[14] Note that on modeling the probabilities as changing, it is not possible to use the algorithm of Bai and Perron (1998, 2003).

volatility series translates as periods with higher mean value. The greater (more negative) level shifts almost always coincide with jumps in the smoothed component of the level shift. These events are associated with domestic or foreign financial turbulence, as well as political elections and social demands in the countries. A more detailed explanation can be found in Rodríguez and Tramontana Tocto (2014) and Ojeda Cunya and Rodríguez (2014).

3.3 Effect of Level Shifts on Long Memory and ARFIMA Models

Figure 5 shows the ACF of the residuals of each of the four estimated RLS models. These results are obtained as the difference between the volatility series and the smoothed level component estimated by the algorithm[15]. What is observed is a behavior that is totally different from that seen in Figure 2. Now there is no trace of persistence or long memory in the ACF of the different series. Note, in addition, that the four models allow the same conclusion to be obtained[16].

As in the work of Ojeda Cunya and Rodríguez (2014) and Rodríguez and Tramontana Tocto (2014), here also an analysis involving the ARFIMA (0,d,0) and ARFIMA (1,d,1) models is presented, applied to the volatility and the short-memory process[17]. The results are set out in Table 6. In the case of the ARFIMA (0,d,0), the fractional parameter of the volatility is seen to fluctuate between 0.152 and 0.221 and is significant for all countries, which means that the series exhibits long-memory behavior. However, on assessing the short-memory component, which reflects the inclusion of the level shifts, we see that in four of the five countries the parameter $\widehat{d}$ becomes negative, which indicates that the series no longer has long memory and that the autocorrelations decay rapidly without past shocks having a prolonged effect. For the Peruvian series the parameter $\widehat{d}$ is positive, but is very close to zero and is insignificant, which shows that its value is zero and that it does not have long memory.

The ARFIMA (1,d,1) model is also evaluated for volatility and short-memory process. In the five countries it can be noted that the parameter $\widehat{d}$, in comparison with the previous model, increases in magnitude, reflecting an even greater persistence (the values fluctuate between 0.31 and 0.42). Moreover, the parameters ϕ and θ of the autoregressive and mobile average processes, respectively, are all significant. On evaluating the short-memory process, it can be observed that the parameter $\widehat{d}$ is highly anti-persistent. On the other hand, the coefficient of the autoregressive component increases to a significant value very close to one in the five countries. The parameter of the moving-averages component is very small for the five countries and is insignificant for Chile and Mexico. Therefore, on evaluating the long-memory component in series that already include level shifts (short-memory process), it can be seen that the series no longer displays this characteristic and is anti-persistent or short-memory.

3.4 Effect of Level Shifts in GARCH, FIGARCH and CGARCH Models

In this section we estimate and evaluate the GARCH, FIGARCH and CGARCH models in the level shifts scenario. These models were applied to the volatility without introducing level shifts, and to the volatility once the level shifts were introduced in the form of the component τ_t. The

[15] For the basic RLS model the difference between the volatility and the level shifts component obtained using the Bai and Perron method (1998, 2003) can be used. The results are essentially unvariable. See Ojeda and Rodríguez (2014) and Rodríguez and Tramontana (2014).

[16] In the Figure, the different lines are superimposed given the extreme similarity between them.

[17] Given the afore-mentioned similarity, in this and the upcoming cases we opt to use the level shift component from the RLS model with mean reversion.

GARCH $(1,1)$ model was modeled as follows:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \\ \sigma_t^2 &= \mu + \beta_r \widetilde{r}_{t-1}^2 + \beta_\sigma \sigma_{t-1}^2, \end{aligned} \tag{5}$$

where ϵ_t is *i.i.d.* t-Student with mean 0 and variance 1. Based on the study of Baillie et al. (1996), we will now write this model in the form of an ARMA(m,p) in $\widetilde{r}_t$: $\phi(L)\widetilde{r}_t = \mu + [1-\beta_\sigma(L)]\nu_t$, with $m \equiv \max\{p,q\}$, $\phi(L) = [1-\alpha(L)-\beta(L)]$ and $\nu_t \equiv \widetilde{r}_t - \sigma_t^2$. From this equation the FIGARCH (p,d,q) model can be defined by

$$\phi(L)(1-L)^d \widetilde{r}_t = \mu + [1-\beta_2(L)]\nu_t, \tag{6}$$

where the parameter d represents the velocity of the decay in the autocorrelations and is included in the interval $]0,1[$.

The CGARCH model is specified as follows:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \\ (\sigma_t^2 - n_t) &= \beta_r(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_\sigma(\sigma_{t-1}^2 - n_{t-1}), \\ n_t &= \mu + \rho(n_{t-1} - \mu) + \varphi(\widetilde{r}_{t-1}^2 - \sigma_{t-1}^2), \end{aligned} \tag{7}$$

where the important coefficients are β_r and β_σ for the GARCH model, d for the FIGARCH, and ρ for the CGARCH. The parameter μ is a constant to which n_t converges, which represents the variable and long term component of the volatility. Therefore, the second equation of (7) represents the transitory component of the volatility that tends toward zero, and the third part represents the permanent component. The parameter ρ measures the persistence of the shocks in the permanent component of the equation (7), while this persistence is measured by $(\beta_r + \beta_\sigma)$ in the equation (5).

On the other hand, a CGARCH model is estimated but by incorporating the level shifts as follows [18]:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \\ (\sigma_t^2 - n_t) &= \beta_r(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_\sigma(\sigma_{t-1}^2 - n_{t-1}), \\ n_t &= \mu + \rho(n_{t-1} - \mu) + \varphi(\widetilde{r}_{t-1}^2 - \sigma_{t-1}^2) + \widehat{\tau}_t \gamma_i. \end{aligned} \tag{8}$$

As can be seen in the results presented in Table 7, the estimated parameters when the level shifts are not included clearly reflect a long-memory component and the existence of GARCH effects for the five countries. However, when the level shifts are included in the modeling, the results are completely reverted. In the case of the GARCH estimations, it is observed that the variance of the five countries is highly persistent as $\beta_r + \beta_\sigma$ is close to the unit. In effect, the half-life implied by the estimates is 86, 77, 27, 99 and 43 days for Argentina, Brazil, Chile, Mexico, and Peru, respectively. However, when level shifts are included the effects are not persistent.

In the case of the estimations of the FIGARCH models, though the sum $\beta_r + \beta_\sigma$ is of limited relevance or implies limited persistence, this characteristic is assumed by the estimate of the fractional parameter d. The estimates of this parameter are around 0.5, that is, they do not only imply

[18] The system (8), unlike the studies by Ojeda and Rodríguez (2014 and Rodríguez and Tramontana (2014), is no longer estimated with the dummies corresponding to the regimes that produce the level shifts, but that only the smoothed component $\widehat{\tau}_t$ is used. The coefficients γ_i are estimated along with the other parameters of the CGARCH and reflect the magnitude of the level shifts.

long memory but are on the edge of stationarity/non-stationarity. The value of this parameter is substantially less or insignificant when the level-shift component is included.

In the case of the CGARCH model estimation, the values of the parameter ρ are very close to the unit in all the cases analyzed. In these conditions, the half-life of this parameter implies an effect that lasts around 86, 77, 40, 231 and 173 days for each of the five countries analyzed. Observing these estimates, it can be affirmed that the markets of Mexico and Peru display the greatest impact to shocks. However, once the level shift component is introduced, the parameter ρ decreases. The most significant reduction is seen in the series of Peru, which moves from a coefficient ρ of 0.996 to 0.673, implying that the lags that influence the current volatility go from 173 days to just 2 days. The second change in importance is noted in the series of Mexico, where the parameter ρ goes from 0.997 to 0.823, implying that the mean life of past shocks goes from 230 days to just 3 days. In the other countries, the impact is between 40 and 82 days, and 1 and 6 days when the level shifts are included in the modeling.

4 Forecasting

The construction of the prediction of volatility in time $t+h$ is determined on the basis of the study undertaken by Varneskov and Perron (2014): $\widehat{y}_{t+\tau|t} = y_t + HF^{\tau}[\sum_{i=1}^{2}\sum_{j=1}^{2} \Pr(s_{t+1} = j) \Pr(s_t = i|Y_t) X_{t|t}^{ij}]$, where $E_t(y_{t+\tau}) = \widehat{y}_{t+\tau|t}$ is the prediction of the volatility for $t+h$, conditioned for the information up to time t. The matrices F and H are determined as in Section 2. The prediction horizons are $\tau = 1, 5, 10, 20, 50$ and 100.

To measure effectiveness in the prediction we use the criteria of the mean squared forecast error (MSFE), proposed by Hansen and Lunde (2006), which is presented as: $MSFE_{\tau,i} = \frac{1}{T_{out}} \sum_{t=1}^{T_{out}} (\overline{\sigma}_{t,\tau}^2 - \overline{y}_{t+\tau,i|t})^2$, where T_{out} is the number of predictions, $\overline{\sigma}_{t,\tau}^2 = \sum_{s=1}^{\tau} y_{t+s}$, and $\overline{y}_{t+\tau,i|t} = \sum_{s=1}^{\tau} \widehat{y}_{t+\tau,i|t}$, with i being the models to be compared. The evaluations are undertaken based on the 10% of the Model Confidence Set (MCS) proposed by Hansen et al. (2011). This model allows not just one model to be selected because if the data is not sufficiently informative, several models may be inside the confidence set.

The predictions have been calculated from the first day of trading in 2006 in the five countries, which ensures that the sample with which the predictions are compared contains periods of high volatility, such as those caused by the crises of 2007-2008 and 2011. Moreover, for the models with varying probability and the modified (varying probability with mean reversion) only the threshold of 1% ($\kappa = 1\%$) is used.

Different measures of volatility are used to evaluate the performance of the random level shift models. The first is that which is used throughout this paper; that is, the logarithm of absolute value of the returns. The predictions are devised through the equation for $\widehat{y}_{t+\tau|t}$. As in the studies of Ojeda Cunya and Rodríguez (2014) and Rodríguez and Tramontana Tocto (2014), for this specification of volatility comparisons are made with the four models employed here using ARFIMA (0,d,0) and ARFIMA (1,d,1).

The second measure of volatility used is squared returns. In this case we increase the number of rival models to include GARCH and FIGARCH. Given that the variable used here up to this point is $y_t = \ln(|r_t| + 0.001)$, we need to make some transformations to obtain the squared returns. The complete details are included in Lu and Perron (2010).

The results set out in Table 8 correspond to the logarithm of the absolute value of the returns as a measure of volatility. It can be seen that for Argentina, the RLS with mean reversion pertains to the MCS by just 10%. This result is consistent with the sampling in Tables 3, 4 and 5, since for Argentina, the parameter γ_2 is not significant at any threshold and the parameter γ_1 is only significant to 10% under the threshold of 1%, due to which the most robust model will be RLS with mean reversion.

For the other four countries the results are more varied, given that there is no model that prevails in all horizons. For Brazil, the best model in the three first horizons is the basic RLS, while in the following three the best model is the modified RLS. Nonetheless, for the horizons $\tau = 5, 10, 20$, the RLS with mean reversion is also included among the models belonging to the MCS. For Chile, the results are even more diverse, as there is no single model that clearly prevails. What can be ascertained in this case is that the RLS model with mean reversion pertains to the MCS for four of the six horizons analyzed. In the predictions for Mexico the winning model is the RLS with mean reversion and for Peru the best models are the basic RLS, RLS with mean reversion, and modified RLS.

Despite the different results given, the winning models for all countries are the RLS, and it is thus possible to affirm that the models with random level shifts produce better predictions than the classic long-memory models such as the ARFIMA.

The results for the volatility represented as the squared returns are set out in Table 9. For Argentina, unlike Table 8, the RLS model with mean reversion is no better than the other models for any prediction horizon; however, this model and the others in the random level shift group pertain to 10% of MCS in most horizons. On the other hand, for Brazil, the RLS models with mean reversion and the modified are better for almost all horizons, except when the prediction is calculated 100 periods ahead, but it still pertains to 10% of the MCS. For the other countries analyzed the same applies; that is, though the RLS models are not the best predictors in all horizons, in most cases they pertain to 10% of MCS, so they can be said to compete with the traditional models for estimating and predicting volatility such as GARCH and FIGARCH.

5 Conclusions

Typically, the volatility of financial series displays long-term dependency or long memory. This property is represented in the domain of time by the behavior of its sample ACF that exhibits values that are significantly different from zero up to a large number of lags, indicating hyperbolic decay. However, new literature has stressed that the presence of long memory could be caused by the presence of infrequent or random level shifts. In this vein, Lu and Perron (2010) and Li and Perron (2013) use the RLS model to model the volatility of stock market and exchange rate returns, respectively. In this model there is a short memory component and a random level shift component. The estimations show that once these level shifts are taken into account, no evidence of long memory is found. Recently, Xu and Perron (2014) extended the RLS model to include probabilities of time-varying level shifts and a mean reversion mechanism in the volatility.

Empirical studies applied to financial series in Latin America are scanty. The RLS model was recently applied by Ojeda Cunya and Rodríguez (2014) to explain stock market and exchange rate volatility in Peru, and by Rodríguez and Tramontana Tocto (2014) to analyze the behavior of stock market volatilities in a sample of Latin American countries. In this study we follow Xu and Perron (2014), who extend the RLS model applied to the volatility of exchange rate returns in Argentina, Brazil, Chile, Mexico, and Peru by factoring in the two above-mentioned extensions. Four RLS models are estimated: basic RLS, RLS with varying probabilities, RLS with mean reversion, and a combined RLS model with mean reversion and varying probability. The results show that the parameters estimated are highly significant, especially that of mean reversion. Moreover, an analysis of ARFIMA and GARCH in the presence of level shifts is conducted, which shows that once these changes are included in the modeling, the long-memory characteristics and GARCH disappear. Our prediction analysis confirms that the RLS models are more precise than the other classic models, which include long memory.

References

[1] Bai, J. and P. Perron (1998), "Estimating and Testing Linear Models with Multiple Structural Changes", *Econometrica* **66**, 47-78.

[2] Baillie, R. T., T. Bollerslev and H. O. Mikkelsen (1996), "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics* **74**, 3-30.

[3] Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics* **31**, 307-327.

[4] Bollerslev, T. and H. O. Mikkelsen (1996), "Modeling and Pricing Long Memory in Stock Market Volatility", *Journal of Econometrics* **73**, 151-184.

[5] Diebold, F. and A. Inoue (2001), "Long Memory and Regime Switching", *Journal of Econometrics* **105**, 131-159.

[6] Engle, R. F. (1982), "Autoregressive Conditional Heteroskedasticity with Estimated of Variance of United Kingdom Inflation", *Econometrica* **50**, 987-1008.

[7] Engle, R. F. and A. D. Smith (1999), "Stochastic Permanent Breaks", *Review of Economics and Statistics* **81**, 553-574.

[8] Gonzales Tanaka, J. C., J. A. Ojeda Cunya and G. Rodríguez (2015), "Peruvian Forex Market Volatility and Official Interventions of the Central Bank," manuscript, Department of Economics, Pontificia Universidad Católica del Perú.

[9] Gourieroux, C. and J. Jasiak (2001), "Memory and Infrequent Breaks", *Economics Letters* **70**, 29-41.

[10] Granger, C. W. J. and N. Hyung (2004), "Occasional Structural Breaks and Long Memory with an Application to the S&P 500 Absolute Stock Returns", *Journal of Empirical Finance* **11**, 399-421.

[11] Granger, C. W. J. and R. Joyeux (1980), "An Introduction to Long Memory Time Series Models and Fractional Differencing", *Journal of Time Series Analysis* **1**, 15-39.

[12] Hamilton, J. D. (1989), "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", *Econometrica* **57**, 357-384.

[13] Hansen, P. R. and A. Lunde (2006), "Consistent Ranking of Volatility Models", *Journal of Econometrics* **131**, 97-121.

[14] Hansen, P. R., A. Lunde and J. M. Nason (2011), "The Model Confidence Set", *Econometrica* **79**, 453-497.

[15] Harrison, P. J. and C. F. Stevens (1976), "Bayesian Forecasting", *Journal of the Royal Statistical Society, Series B* **38,** 205-247.

[16] Hosking, J. R. M. (1981), "Fractional Differencing", *Biometrika* **68**, 165-176.

[17] Humala, A., and G. Rodríguez (2013), "Some Stylized Facts of Returns in the Stock and Foreign Exchange Markets in Peru", *Studies in Economics and Finance* **30(2)**, 139-158.

[18] Li, Y. and P. Perron (2013), "Modeling Exchange Rate Volatility with Random Level Shifts", Working paper, Department of Economics, Boston University.

[19] Lobato, I. N. and N. E. Savin (1998), "Real and Spurious Long Memory Properties of Stock-market Data", *Journal of Business and Economics Statistics* **16**, 261-268.

[20] Lu, Y. K. and P. Perron (2010), "Modeling and Forecasting Stock Return Volatility Using a Random Level Shift Model", *Journal of Empirical Finance* **17**, 138-156.

[21] Martens, M., D. van Dijk and M. de Pooter (2004), "Modeling and Forecasting S&P 500 Volatility: Long Memory, Structural Breaks and Nonlinearity", Tinbergen Institute discussion paper.

[22] Mikosch, T. and C. Stărică (2004a), "Nonstationarities in Financial Time Series, the Long-range Effect Dependence, and the IGARCH Effects", *Review of Economics and Statistics* **86**, 378-390.

[23] Mikosch, T. and C. Stărică (2004b), "Changes of Structure in Financial Time Series and the GARCH model", *REVSTAT-Statistical Journal* **2**, 42-73.

[24] Nelson, C. R. and C. I. Plosser (1982), "Trends and Random Walks in Macroeconomic Time Series", *Journal of Monetary Economics* **10**, 139-162.

[25] Ojeda Cunya , J. A. and G. Rodríguez (2014), "Application of a Random Level Shift Model to the Volatility of Peruvian Stock and Forex Returns", Working Paper 383, Departamento de Economía, Pontificia Universidad Católica del Perú.

[26] Perron, P. (1989), "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis", *Econometrica* **57**, 1361-1401.

[27] Perron, P. (1990), "Testing for a Unit Root in a Time Series Regression with a Changing Mean", *Journal of Business and Economic Statistics* **8**, 153-162.

[28] Perron, P. and Z. Qu (2010), "Long-Memory and Level Shifts in the Volatility of Stock Market Return Indices", *Journal of Business and Economic Statistics* **28**, 275-290.

[29] Perron, P. and T. Wada (2009), "Let's Take a Break: Trends and Cycles in U. S. Real GDP", *Journal of Monetary Economics* **56**, 749-765.

[30] Rodríguez, G. and R. Tramontana Tocto (2014), "Application of a Random Level Shift Model to the Volatility of Latin-American Stock Returns", Working Paper 385, Departamento de Economía, Pontificia Universidad Católica del Perú.

[31] Wada, T. and P. Perron (2006), "An Alternative Trend-Cycle Decomposition using a State Space Model with Mixtures of Normals: Specifications and Applications to International Data", Working paper, Department of Economics, Boston University.

[32] Varneskov, R.T. and P. Perron (2014), "Combining Long Memory and Level Shifts in Modeling and Forecasting the Volatility of Asset Returns", Boston University Working Paper.

[33] Xu, J. and P. Perron (2014), "Forecasting Return Volatility: Level Shifts with Varying Jump Probability and Mean Reversion", *International Journal of Forecasting* **30,** 449-463.

Table 1. Summary Descriptive Statistics of Returns and Volatility Series

	Media	SD	Maximum	Minimum	Skewness	Kurtosis	Sample
				Returns			
Argentina	0.002	0.032	0.330	-0.757	-0.862	62.476	6142
Brazil	0.002	0.028	0.345	-0.395	-0.039	30.609	5303
Chile	0.001	0.012	0.118	-0.077	0.182	8.696	6096
Mexico	0.001	0.016	0.122	-0.143	-0.019	9.595	4839
Peru	0.001	0.017	0.143	-0.132	0.519	11.094	5831
				Volatility			
Argentina	-4.397	1.060	-0.277	-6.908	-0.235	2.808	6142
Brazil	-4.383	0.975	-0.927	-6.908	-0.259	2.827	5303
Chile	-4.993	0.845	-2.128	-6.908	-0.151	2.538	6096
Mexico	-4.797	0.895	-1.937	-6.908	-0.161	2.604	4839
Peru	-4.858	0.951	-1.931	-6.907	-0.027	2.622	5831

Table 2. Estimates of the Basic RLS Model

	σ_η	α	σ_e	ϕ
Argentina	0.679^a	0.008^c	0.937^a	
(SD:1.060)	(0.189)	(0.004)	(0.009)	
Brazil	0.425^a	0.010^c	0.881^a	
(SD:0.975)	(0.118)	(0.006)	(0.009)	
Chile	0.612^a	0.008^c	0.778^a	0.080^a
(SD:0.845)	(0.150)	(0.004)	(0.007)	(0.014)
Mexico	0.520^a	0.006^c	0.830^a	0.025^c
(SD:0.895)	(0.157)	(0.004)	(0.009)	(0.015)
Peru	0.875^a	0.0045^a	0.842^a	0.115^a
(SD:0.9511)	(0.128)	(0.0016)	(0.008)	(0.015)

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5%,10% level, respectively.

Table 3. Estimates of the RLS Model with Time Varying Probabilities (Threshold $\kappa\%$)

Threshold ($\kappa\%$)	σ_η	p	σ_e	ϕ	γ_1	γ_2
			Argentina			
5%	0.609^a	-2.661^a	0.938^a		1.190^a	3.451
	(0.124)	(0.505)	(0.009)		(0.373)	(11.269)
2.5%	0.568^a	-2.408^a	0.937^a		-0.576^b	21.266
	(0.140)	(0.441)	(0.009)		(0.247)	(119.356)
1%	0.604^a	-2.432^a	0.937^a		1.368^c	4.301
	(0.123)	(0.379)	(0.009)		(0.744)	(12.790)
			Brazil			
5%	0.336^a	-2.584^a	0.881^a		-0.814^c	51.645
	0.089	0.666	(0.009)		(0.460)	(619.496)
2.5%	0.318^c	-2.421^b	0.881^a		2.753	0.210^a
	(0.184)	(1.034)	(0.009)		(4.490)	(0.075)
1%	0.357^a	-2.394^a	0.882^a		5.271	0.472^a
	(0.075)	(0.487)	(0.009)		(44.173)	(0.118)
			Chile			
5%	0.239^b	-1.859^a	0.777^a	0.078^a	1.727	0.864^a
	(0.097)	(0.714)	(0.008)	(0.014)	(1.134)	(0.317)
2.5%	0.426^b	-2.230^a	0.778^a	0.079^a	1.155	0.633
	(0.217)	(0.715)	(0.008)	(0.015)	(1.138)	(1.755)
1%	0.362^b	-2.094^a	0.777^a	0.079^a	2.070	1.046
	(0.157)	(0.704)	(0.008)	(0.014)	(2.424)	(1.484)

Table 3 (continued)

Threshold (κ%)	σ_η	p	σ_e	ϕ	γ_1	γ_2
			Mexico			
5%	0.153^b	-1.899^b	0.830^a	0.026^c	3.104	1.269^a
	(0.062)	(0.800)	(0.009)	(0.015)	(10.918)	(0.367)
2.5%	0.202^a	-2.099^a	0.830^a	0.026^c	5.526	0.424^c
	(0.039)	(0.538)	(0.009)	(0.015)	(38.846)	(0.256)
1%	0.245^a	-2.107^a	0.830^a	0.027^c	5.252	0.428^a
	(0.063)	(0.549)	(0.009)	(0.015)	(16.108)	(0.058)
			Peru			
5%	0.790^a	-2.690^a	0.839^a	0.111^a	1.160^a	0.516^a
	(0.075)	(0.075)	(0.008)	(0.015)	(0.375)	(0.176)
2.5%	0.835^a	-2.679^a	0.841^a	0.115^a	1.143^a	0.318^c
	(0.141)	(0.403)	(0.008)	(0.015)	(0.437)	(0.183)
1%	0.833^a	-2.607^a	0.840^a	0.113^a	1.164	0.157^a
	(0.172)	(0.436)	(0.009)	(0.016)	(0.900)	(0.031)

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 4. Estimates of the RLS Model with Mean Reversion

	σ_η	p	σ_e	ϕ	β
Argentina	0.215^a	0.036^b	0.935^a		-0.164^a
	(0.063)	(0.018)	(0.009)		(0.008)
Brazil	0.375^b	0.010	0.880^a		-0.185^a
	(0.177)	(0.008)	(0.009)		(0.017)
Chile	0.079^a	0.486^a	0.771^a	0.043^b	-0.039^a
	(0.008)	(0.012)	(0.008)	(0.017)	(0.000)
Mexico	0.102^c	0.049	0.826^a		-0.147^a
	(0.057)	(0.030)	(0.009)		(0.009)
Peru	0.105	0.031^b	0.833^a	0.084^a	-0.332^a
	(0.109)	(0.015)	(0.009)	(0.019)	(0.024)

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 5. Estimates of the Modified RLS Model (Threshold at 1%)

	σ_η	p	σ_e	ϕ	γ_1	γ_2	β
Argentina	0.231^a	-1.924^a	0.934^a		0.304^b	5.252	-0.208^a
	(0.086)	(0.727)	(0.009)		(0.122)	(63.078)	(0.025)
Brazil	0.353^a	-2.379^a	0.882^a		6.008	1.991^c	-0.012^a
	(0.080)	(0.521)	(0.009)		(22.749)	(1.10)	(0.000)
Chile	0.080^a	-0.064^a	0.771^a	0.043^b	0.014^a	0.061^a	-0.040^a
	(0.008)	(0.004)	(0.008)	(0.017)	(0.000)	(0.004)	(0.000)
Mexico	0.110^b	-1.596^b	0.826^a		0.559^a	0.162^a	-0.129^a
	(0.048)	(0.719)	(0.009)		(0.167)	(0.008)	(0.011)
Peru	0.119	-1.816^a	0.832^a	0.081^a	-2.205	0.219^c	-0.319^a
	(0.109)	(0.479)	(0.009)	(0.020)	(11.441)	(0.126)	(0.029)

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 6. Estimated Parameters of $ARFIMA(0, d, 0)$ and $ARFIMA(1, d, 1)$ models

	d	AR	MA	d	AR	MA	d	AR	MA
		Argentina			Brazil			Chile	
Volatility	0.178			0.155			0.181		
	(0.000)			(0.000)			(0.000)		
	0.429	0.252	-0.620	0.409	0.174	-0.579	0.340	0.356	-0.565
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
c_t	-0.068			-0.069			-0.046		
	(0.000)			(0.000)			(0.000)		
	-0.864	0.911	-0.112	-0.919	0.943	-0.115	-0.902	0.896	-0.021
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.495)
			Mexico				Peru		
Volatility		0.152				0.221			
		(0.000)				(0.000)			
		0.412	0.338	-0.682		0.336	0.210	-0.384	
		(0.000)	(0.000)	(0.000)		(0.000)	(0.018)	(0.000)	
c_t		-0.053				0.016			
		(0.000)				(0.173)			
		-0.997	0.944	0.014		-0.775	0.866	-0.070	
		(0.000)	(0.000)	(0.647)		(0.000)	(0.000)	(0.020)	

p-values are reported in parentheses.

Table 7. Estimated Parameters of $GARCH$, $FIGARCH$ and $CGARCH$ models

	β_r	β_σ	ρ	φ	d
		Argentina			
GARCH	0.114	0.878			
	(0.000)	(0.000)			
FIGARCH	0.115	0.519			0.521
	(0.098)	(0.000)			(0.000)
CGARCH (volatility)	-0.017	-0.487	0.992	0.117	
	(0.340)	(0.496)	(0.000)	(0.000)	
CGARCH (short memory process)	-1.182	2.064	0.890	1.274	
	(0.821)	(0.694)	(0.000)	(0.807)	
		Brazil			
GARCH	0.095	0.896			
	(0.000)	(0.000)			
FIGARCH	0.085	0.591			0.582
	(0.037)	(0.000)			(0.000)
CGARCH (volatility)	-0.026	-0.179	0.991	0.100	
	(0.143)	(0.781)	(0.000)	(0.000)	
CGARCH (short memory process)	-2.312	3.091	0.790	2.363	
	(0.949)	(0.931)	(0.000)	(0.947)	
		Chile			
GARCH	0.169	0.806			
	(0.000)	(0.000)			
FIGARCH	0.339	0.599			0.528
	(0.000)	(0.000)			(0.000)
CGARCH (volatility)	0.138	0.360	0.983	0.120	
	(0.000)	(0.001)	(0.000)	(0.000)	
CGARCH (short memory process)	-0.424	0.908	0.535	0.595	
	(0.817)	(0.660)	(0.000)	(0.746)	

Table 7 (continued)

	β_r	β_σ	ρ	φ	d
		Mexico			
GARCH	0.084	0.909			
	(0.000)	(0.000)			
FIGARCH	0.239	0.629			0.501
	(0.000)	(0.000)			(0.000)
CGARCH (volatility)	0.064	0.891	0.997	0.034	
	(0.000)	(0.000)	(0.000)	(0.002)	
CGARCH (short memory process)	-0.932	1.740	0.823	0.992	
	(0.830)	(0.692)	(0.000)	(0.819)	
		Peru			
GARCH	0.255	0.729			
	(0.000)	(0.000)			
FIGARCH	-0.029	0.123			0.484
	(0.882)	(0.573)			(0.000)
CGARCH (volatility)	0.232	0.583	0.996	0.104	
	(0.000)	(0.000)	(0.000)	(0.000)	
CGARCH (short memory process)	-0.020	0.027	0.673	0.287	
	(0.116)	(0.947)	(0.000)	(0.000)	

p-values are reported in parentheses.

Table 8. Forecast Evaluations [$\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$]

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Argentina			
Basic RLS	0.753	4.728	12.475	37.205	209.381	860.698
	(0.038)	(0.019)	(0.004)	(0.000)	(0.000)	(0.000)
Threshold 1% RLS	0.758	4.739	12.456	36.995	208.492	861.245
	(0.002)	(0.019)	(0.004)	(0.009)	(0.000)	(0.000)
Mean Reversion RLS	0.750	4.668	12.204	35.878	198.130	804.210
	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)
Modified RLS	0.752	4.710	12.394	36.976	207.835	846.003
	(0.038)	(0.019)	(0.004)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.991	8.634	26.376	86.607	455.305	1630.945
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.942	7.403	21.462	67.010	332.064	1139.459
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
			Brazil			
Basic RLS	0.691	3.915	10.044	31.052	175.247	773.826
	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.298^{b})	(0.000)	(0.000)
Threshold 1% RLS	0.702	3.967	10.122	30.964	170.932	740.975
	(0.000)	(0.003)	(0.178^{b})	(0.298^{b})	(0.002)	(0.072)
Mean Reversion RLS	0.694	3.931	10.082	31.075	173.849	765.744
	(0.001)	(0.190^{b})	(0.336^{b})	(0.298^{b})	(0.000)	(0.000)
Modified RLS	0.702	3.960	10.813	30.901	170.627	740.144
	(0.000)	(0.007)	(0.000)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)
ARFIMA(0,d,0)	0.925	8.368	26.763	92.738	498.011	1792.570
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.879	7.215	22.140	74.181	383.240	1341.148
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table 8 (continued)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Chile			
Basic RLS	0.452	4.077	11.906	40.709	260.675	1002.936
	($1.000^{a,b}$)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Threshold 1% RLS	0.452	4.078	11.907	40.709	260.569	1002.177
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean Reversion RLS	0.470	3.868	10.993	36.616	230.168	901.681
	(0.000)	(0.218^{b})	(0.339^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.000)
Modified RLS	0.490	3.853	10.952	36.768	235.372	940.135
	(0.00)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.276^{b})	(0.018)	(0.000)
ARFIMA(0,d,0)	0.707	6.273	18.482	58.157	263.983	751.092
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.703	6.183	18.120	56.704	255.185	719.778
	(0.000)	(0.000)	(0.000)	(0.000)	(0.018)	($1.000^{a,b}$)
			Mexico			
Basic RLS	0.570	4.073	11.897	40.191	231.507	886.526
	($1.000^{a,b}$)	(0.000)	(0.000)	(0.000)	(0.000)	(0.052)
Threshold 1% RLS	0.593	4.368	12.831	42.601	232.507	850.812
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	($1.000^{a,b}$)
Mean Reversion RLS	0.602	3.653	10.301	35.287	220.786	928.209
	(0.000)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.217^{b})	(0.002)
Modified RLS	0.620	3.843	10.884	36.605	217.713	869.037
	(0.000)	(0.000)	(0.000)	(0.001)	($1.000^{a,b}$)	(0.263^{b})
ARFIMA(0,d,0)	0.788	7.066	22.669	77.332	392.473	1312.289
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.785	6.992	22.359	75.996	383.109	1272.077
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table 8 (continued)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Peru			
Basic RLS	0.509	5.510	16.675	54.607	308.784	1148.305
	(0.162^{b})	(0.000)	(0.000)	(0.003)	($1.000^{a,b}$)	($1.000^{a,b}$)
Threshold 1% RLS	0.505	5.454	16.511	54.525	320.168	1219.345
	(0.451^{b})	(0.001)	(0.000)	(0.000)	(0.003)	(0.000)
Mean Reversion RLS	0.504	5.247	15.997	53.400	322.688	1247.681
	($1.000^{a,b}$)	(1.000^{a})	(0.885^{b})	(0.091)	(0.003)	(0.000)
Modified RLS	0.509	5.285	15.983	52.805	320.776	1248.114
	(0.033)	(0.147^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.003)	(0.000)
ARFIMA(0,d,0)	0.921	9.602	30.121	97.585	464.321	1511.350
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.932	9.877	31.224	102.056	493.114	1630.218
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

Table 9. Forecast Evaluations $[\widehat{y}_{t+\tau|t} = E_t r^2_{t+\tau}]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Argentina			
Basic RLS	0.088	0.805	2.958	12.467	88.252	329.053
	(0.266^{b})	(0.136^{b})	(0.128^{b})	(0.015)	(0.206^{b})	(0.001)
Threshold 1% RLS	0.088	0.840	3.222	13.480	91.914	337.661
	(0.266^{b})	(0.119^{b})	(0.080)	(0.003)	(0.160^{b})	(0.000)
Mean Reversion RLS	0.088	0.813	3.082	13.673	95.404	346.340
	(0.266^{b})	(0.136^{b})	(0.128^{b})	(0.005)	(0.109^{b})	(0.000)
Modified RLS	0.088	0.825	3.137	13.855	95.825	345.895
	(0.266^{b})	(0.136^{b})	(0.115^{b})	(0.004)	(0.109^{b})	(0.000)
ARFIMA(0,d,0)	0.180	2.999	11.246	42.323	247.464	932.942
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.177	2.925	10.951	41.153	240.295	905.302
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
GARCH	0.084	0.708	2.649	10.564	94.209	647.874
	$(1.000^{a,b})$	$(1.000^{a,b})$	$(1.000^{a,b})$	$(1.000^{a,b})$	(0.109^{b})	(0.000)
FIGARCH	0.115	1.865	5.683	16.862	77.875	247.954
	(0.001)	(0.000)	(0.000)	(0.000)	$(1.000^{a,b})$	$(1.000^{a,b})$

Table 9 (continued)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Brazil			
Basic RLS	0.079	0.430	1.273	6.361	64.224	271.831
	(0.515^{b})	(0.446^{b})	(0.598^{b})	(0.248^{b})	(0.201^{b})	(0.117^{b})
Threshold 1% RLS	0.082	0.398	1.217	8.172	81.658	317.873
	(0.298^{b})	(0.446^{b})	(0.598^{b})	(0.001)	(0.000)	(0.002)
Mean Reversion RLS	0.079	0.426	1.248	6.294	63.855	270.734
	($1.000^{a,b}$)	(0.446^{b})	(0.598^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.120^{b})
Modified RLS	0.082	0.395	1.205	8.124	81.385	317.332
	(0.298^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.001)	(0.001)	(0.005)
ARFIMA(0,d,0)	0.139	9.602	7.012	26.442	146.844	510.622
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.125	9.877	4.153	15.019	76.163	233.284
	(0.000)	(0.000)	(0.000)	(0.000)	(0.150^{b})	($1.000^{a,b}$)
GARCH	0.083	0.526	1.936	9.829	75.456	311.991
	(0.114^{b})	(0.231^{b})	(0.019)	(0.000)	(0.084)	(0.003)
FIGARCH	0.151	2.310	6.544	20.061	95.785	300.663
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)

Table 9 (continued)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Chile			
Basic RLS	0.018	0.135	0.405	1.321	6.266	16.692
	(0.021)	(0.010)	(0.007)	(0.540^{b})	($1.000^{a,b}$)	(0.049)
Threshold 1% RLS	0.019	0.149	0.476	1.535	6.818	18.059
	(0.020)	(0.006)	(0.004)	(0.001)	(0.000)	(0.000)
Mean Reversion RLS	0.017	0.113	0.354	1.303	6.776	18.258
	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.000)	(0.000)
Modified RLS	0.017	0.113	0.355	1.304	6.769	18.238
	(0.219^{b})	(0.026)	(0.008)	(0.540^{b})	(0.012)	(0.000)
ARFIMA(0,d,0)	0.021	0.197	0.605	1.792	6.611	15.802
	(0.006)	(0.000)	(0.000)	(0.000)	(0.201^{b})	($1.000^{a,b}$)
ARFIMA(1,d,1)	0.021	0.197	0.605	1.794	6.626	15.874
	(0.006)	(0.000)	(0.000)	(0.000)	(0.197^{b})	(0.049)
GARCH	0.019	0.150	0.528	1.696	6.840	17.447
	(0.219^{b})	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
FIGARCH	0.020	0.266	0.917	2.733	11.387	33.908
	(0.020)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table 9 (continued)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Mexico			
Basic RLS	0.026	0.134	0.407	1.853	14.259	52.737
	(0.020)	(0.000)	(0.026)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.000)
Threshold 1% RLS	0.025	0.126	0.448	2.562	19.270	62.380
	(0.706^{b})	(0.003)	(0.000)	(0.001)	(0.000)	(0.000)
Mean Reversion RLS	0.025	0.108	0.332	1.922	17.097	59.465
	(0.706^{b})	(0.014)	(0.049)	(0.531^{b})	(0.000)	(0.000)
Modified RLS	0.025	0.106	0.328	1.921	17.170	59.634
	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.531^{b})	(0.000)	(0.000)
ARFIMA(0,d,0)	0.033	0.300	0.998	3.566	17.427	50.756
	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.033	0.303	1.013	3.625	17.796	52.262
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
GARCH	0.027	0.149	0.507	2.421	15.000	45.679
	(0.020)	(0.000)	(0.000)	(0.002)	(0.276^{b})	($1.000^{a,b}$)
FIGARCH	0.044	0.656	2.014	6.434	30.631	97.256
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table 9 (continued)

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Peru			
Basic RLS	0.083	0.957	3.019	11.800	67.769	190.331
	(0.688^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.014)	(0.000)
Threshold 1% RLS	0.079	1.096	3.609	16.118	90.852	237.225
	(0.983^{b})	(0.185^{b})	(0.031)	(0.012)	(0.000)	(0.000)
Mean Reversion RLS	0.078	0.993	3.285	14.720	82.740	222.070
	($1.000^{a,b}$)	(0.516^{b})	(0.142^{b})	(0.184^{b})	(0.000)	(0.000)
Modified RLS	0.078	1.003	3.370	14.702	81.312	218.141
	(0.983^{b})	(0.516^{b})	(0.035)	(0.044)	(0.004)	(0.000)
ARFIMA(0,d,0)	0.105	1.370	4.179	12.864	51.539	128.406
	(0.072)	(0.005)	(0.011)	(0.575^{b})	(0.017)	(0.000)
ARFIMA(1,d,1)	0.105	1.369	4.176	12.853	51.461	128.065
	(0.135^{b})	(0.005)	(0.012)	(0.608^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)
GARCH	0.078	1.134	3.386	12.256	58.040	165.165
	(0.983^{b})	(0.038)	(0.039)	(0.608^{b})	(0.017)	(0.000)
FIGARCH	0.086	1.665	5.900	18.118	78.197	228.297
	(0.619^{b})	(0.002)	(0.002)	(0.001)	(0.000)	(0.000)

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries and have been multiplied by 10^5; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

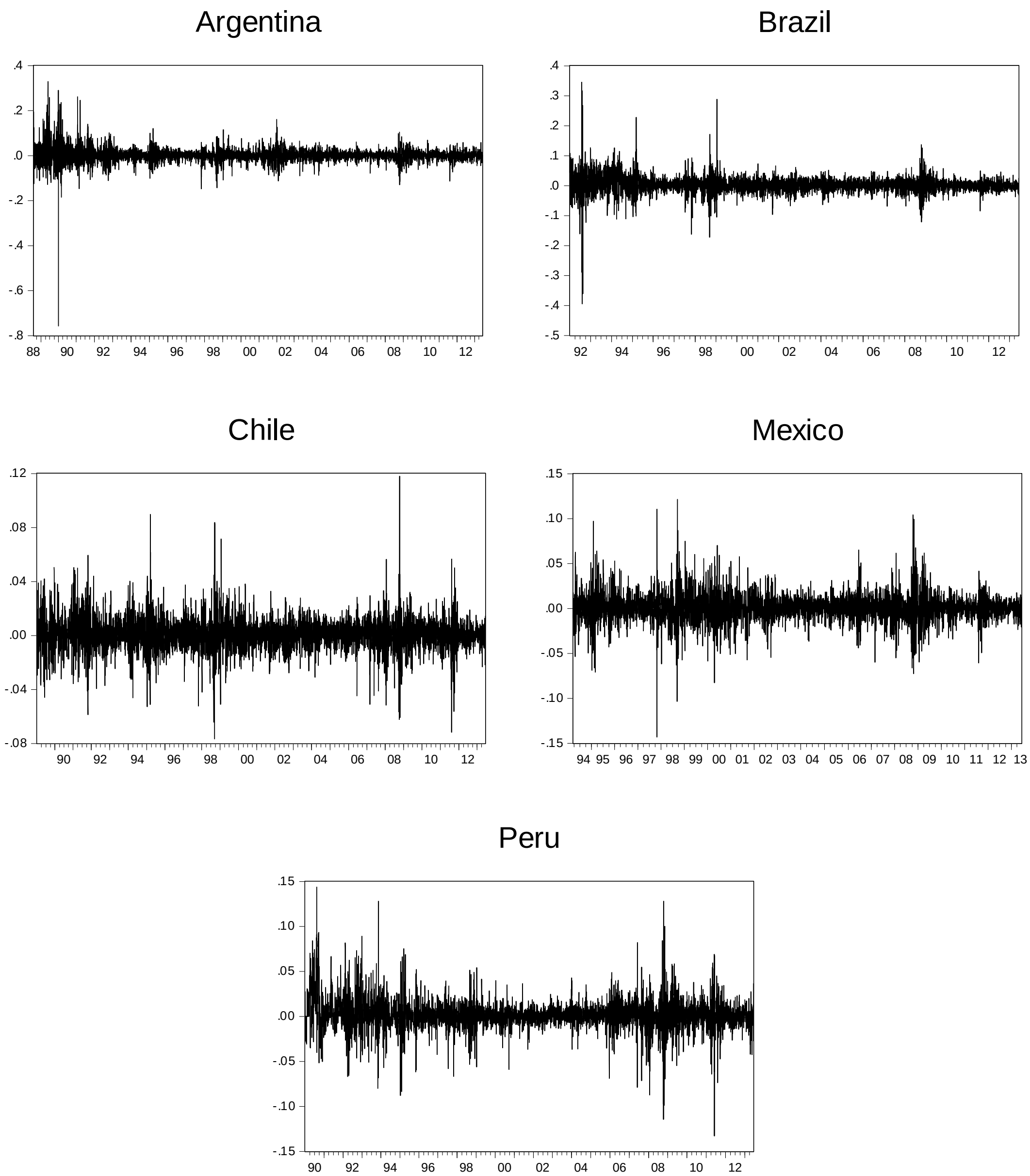

Figure 1. Daily Returns

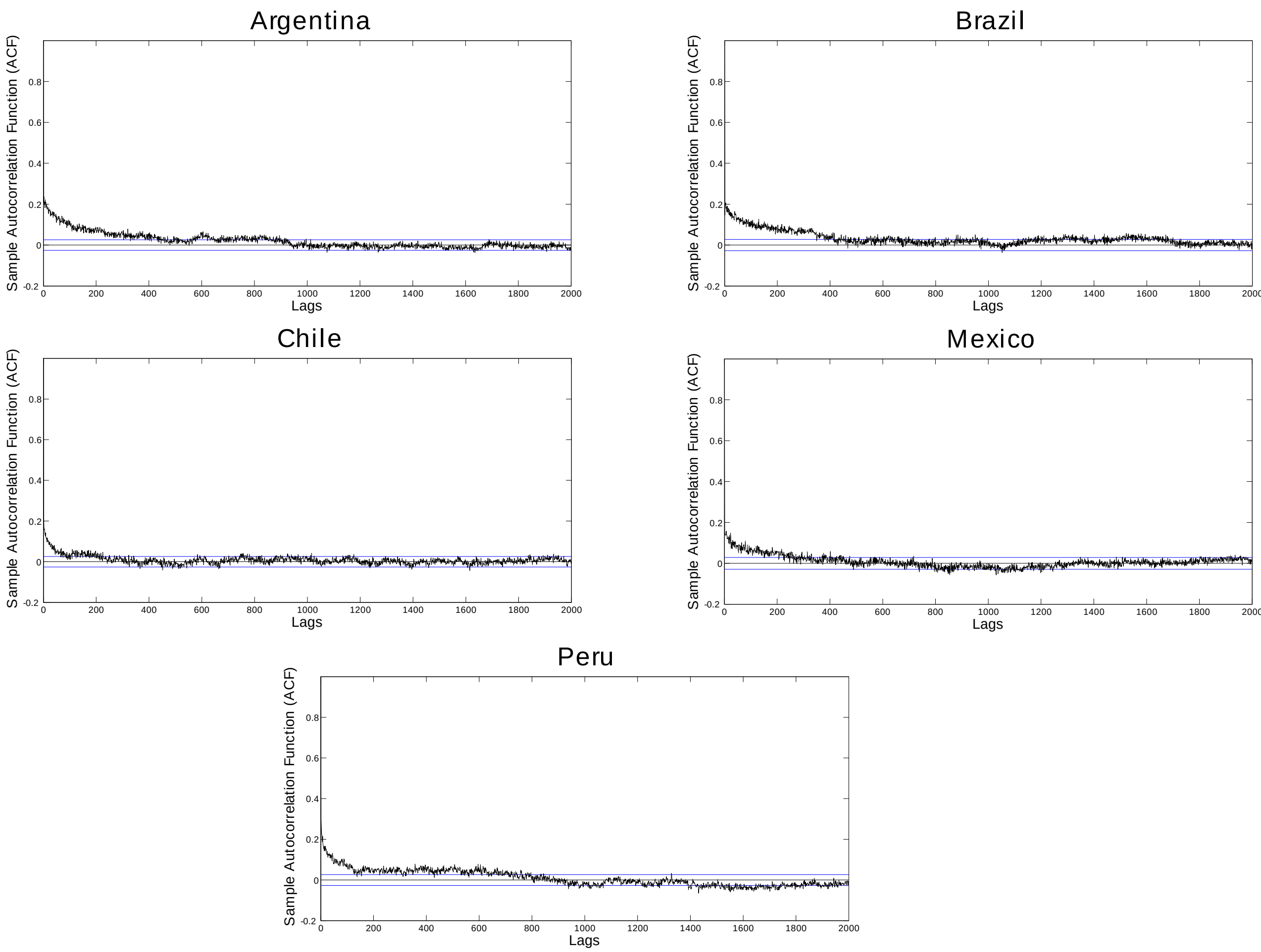

Figure 2. Sample Autocorelations of Volatility

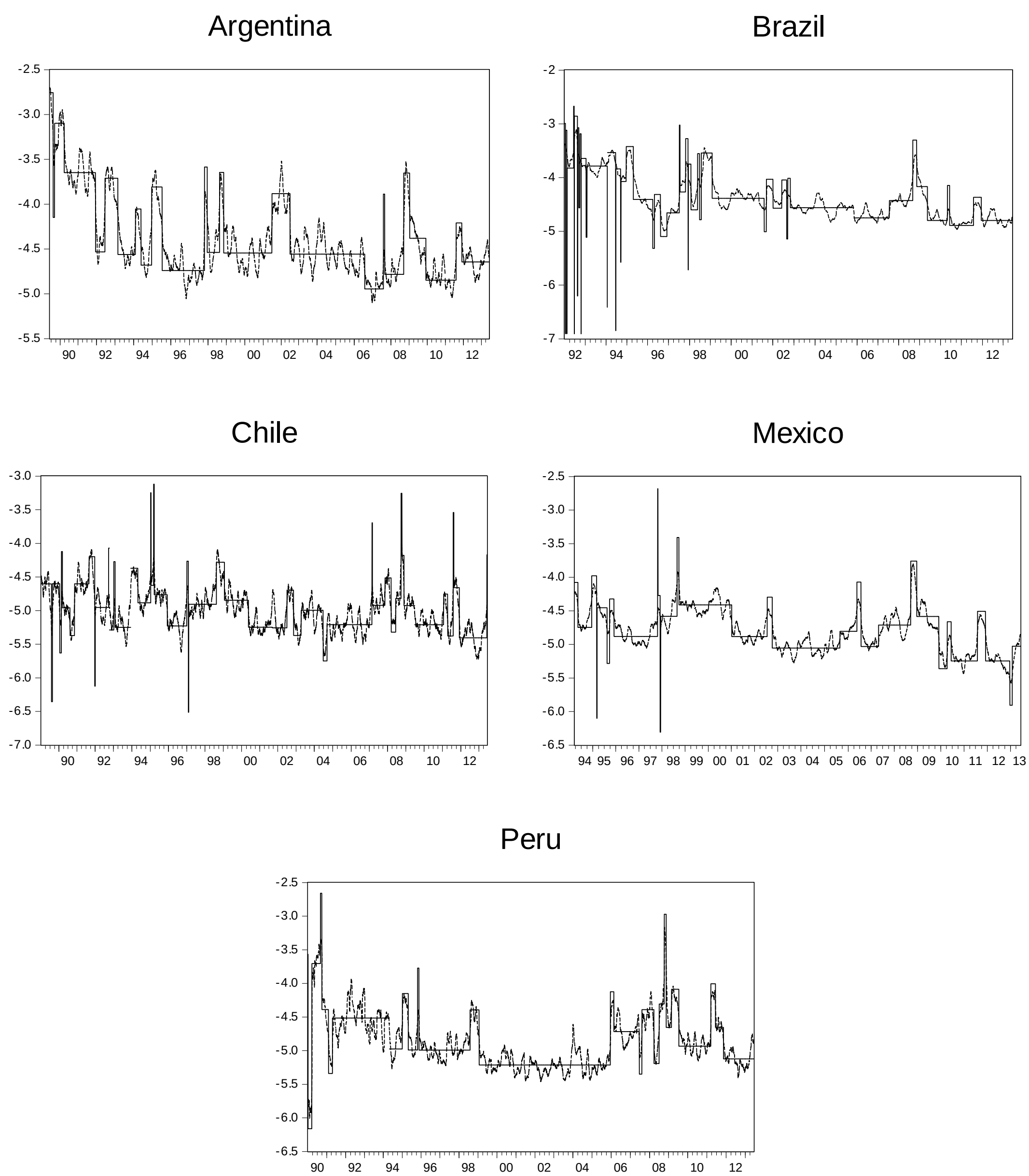

Figure 3. Basic RLS Model. Fitted Level Shift Component by Bai and Perron (2003): solid line and Smoothed Level Shift Component: dotted line

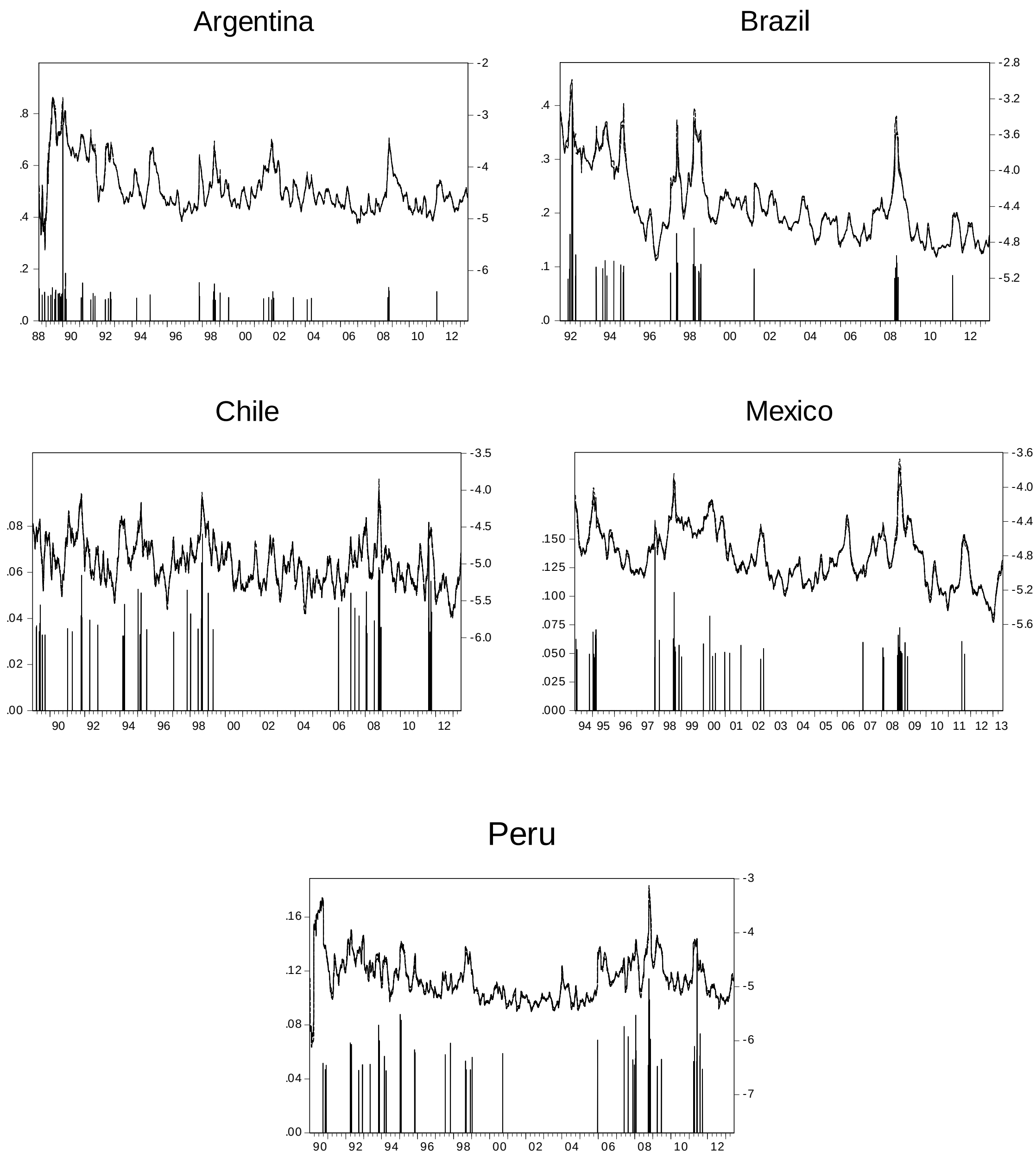

Figure 4. Smoothed Level Shift Component (rigth axis) and 1% Extreme Negative Past Returns (absolute values): left axes

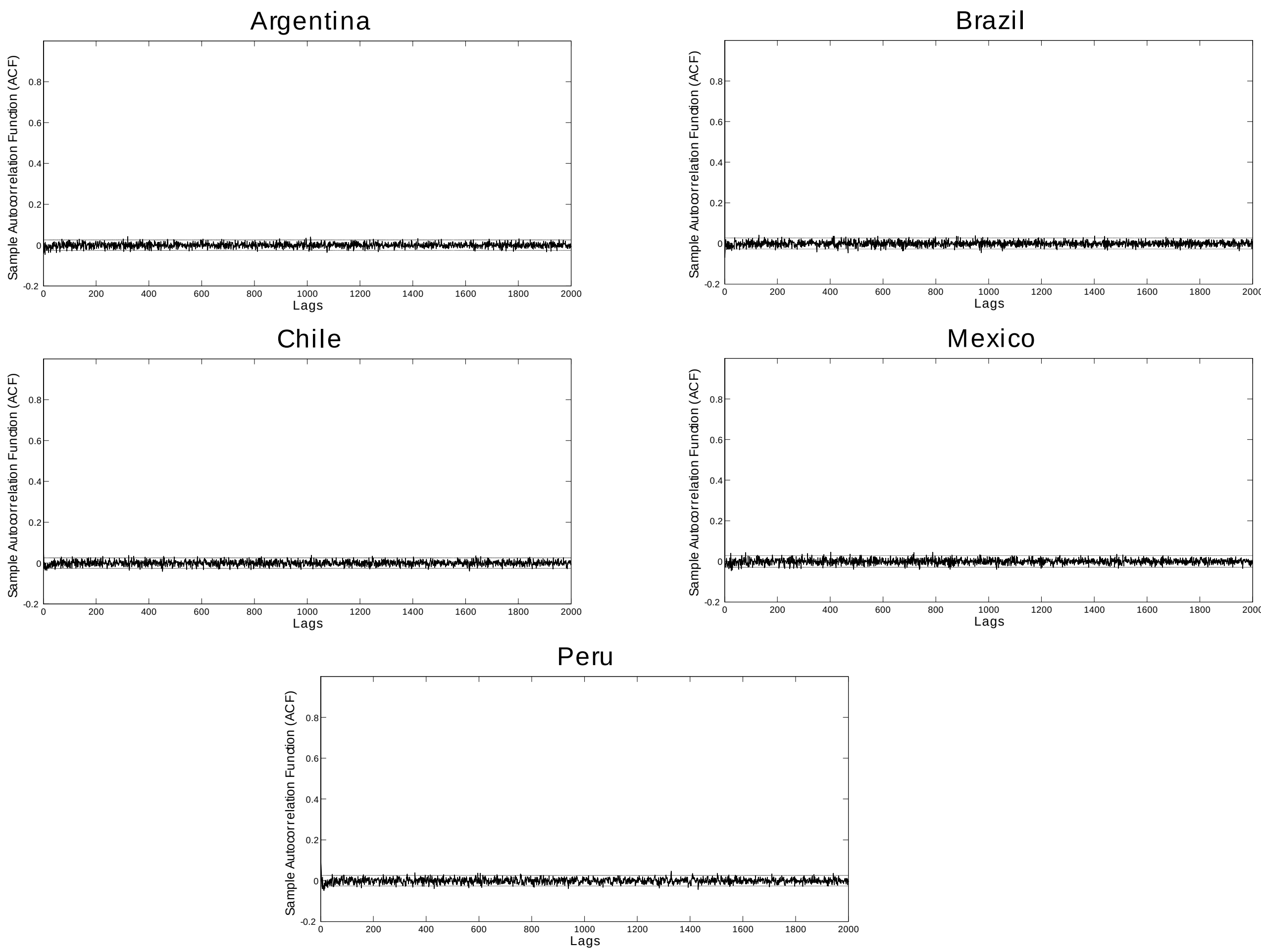

Figure 5. Sample Autocorrelations of Residuals

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