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FOR THE PERUVIAN STOCK AND FOREX MARKETS**

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Av. Universitaria 1801, Lima 32 – Perú.  
Teléfono: (51-1) 626-2000 anexos 4950 - 4951  
Fax: (51-1) 626-2874  
[econo@pucp.edu.pe](mailto:econo@pucp.edu.pe)  
[www.pucp.edu.pe/departamento/economia/](http://www.pucp.edu.pe/departamento/economia/)

Encargado de la Serie: Jorge Rojas Rojas  
Departamento de Economía – Pontificia Universidad Católica del Perú,  
[jorge.rojas@pucp.edu.pe](mailto:jorge.rojas@pucp.edu.pe)

Willy Alanya and Gabriel Rodríguez

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# Asymmetries in Volatility: An Empirical Study for the Peruvian Stock and Forex Markets

Willy Alanya  
The University of Warwick

Gabriel Rodríguez  
Pontificia Universidad Católica del Perú

## Abstract

Symmetric and asymmetric autoregressive conditional heteroskedasticity models and stochastic volatility models are applied to daily data of Peruvian stock and Forex markets returns for the period January 5, 1998 until December 30, 2011. Following the approach developed by Omori et al. (2007), Bayesian estimation methodology is used with different structures in the behavior of the disturbance terms. The results suggest the presence of asymmetric effects in both markets. In the stock market, we find that negative shocks generate higher volatility than positive shocks. In the Forex market, shocks related to episodes of depreciation create higher uncertainty in comparison with episodes of appreciation. Thus, the Central Reserve Bank faces relatively major difficulties in its intention of smoothing Forex volatility. The model with the best fit in both returns is the Asymmetric Stochastic Volatility with Normal errors. The stock market returns have greater periods of volatility; however, both markets react to shocks in the economy, as they display similar patterns and have a significant correlation for the sample period studied.

**JEL Classification:** C11, C12, C53, G12.

**KeyWords:** Asymmetries, GARCH, EGARCH, Stochastic Volatility, Stock Returns, Forex Returns, Bayesian Estimation.

## Resumen

Modelos de volatilidad estocástica y modelos de heterocedasticidad condicional autorregresiva simétricos y asimétricos son aplicados a datos diarios de los retornos bursátiles y cambiarios peruanos para el período desde el 5 de Enero de 1998 hasta el 30 de Diciembre de 2011. Siguiendo el enfoque desarrollado por Omori et al. (2007), se usa metodología Bayesiana con diferentes estructuras en el comportamiento de los términos de perturbación. Los resultados sugieren la presencia de efectos asimétricos en ambos mercados. En el mercado de valores, encontramos que los choques negativos generan una mayor volatilidad que los choques positivos. En el mercado cambiario, los choques relacionados con episodios de depreciación crean mayor incertidumbre en comparación con episodios de apreciación. Por lo tanto, en este caso, el Banco Central de Reserva del Perú enfrenta relativamente mayores dificultades en su intención de suavizar la volatilidad del tipo de cambio. El modelo con el mejor ajuste en ambos rendimientos es el modelo de volatilidad estocástica asimétrico con errores normales. Los rendimientos del mercado de valores tienen mayores períodos de volatilidad; sin embargo, los mercados reaccionan a las perturbaciones en la economía, ya que muestran patrones similares y tienen una correlación significativa para el período de la muestra estudiada.

**Clasificación JEL:** C11, C12, C53, G12.

**Palabras Claves:** Asimetrías, Modelos GARCH, EGARCH, Volatilidad Estocástica, Retornos Bursátiles, Retornos Cambiarios, Estimación Bayesiana.

# Asymmetries in Volatility: An Empirical Study for the Peruvian Stock and Forex Markets<sup>1</sup>

Willy Alanya  
The University of Warwick

Gabriel Rodríguez<sup>2</sup>  
Pontificia Universidad Católica del Perú

## 1 Introduction

Financial returns reflect the uncertain behavior that results from the investment decisions of economic agents, speculations, and the domestic and international environment. Financial series, especially stock and foreign exchange (Forex) market returns, are characterized by their high volatility which displays a clustering dynamic over time. Thus, there are periods in which they are very low (greater stability) and/or high (times of high uncertainty). Modeling financial series entails the incorporation of the dynamics of the volatility into the structure of a time series model. In the literature, this type of series was first modeled by Engle (1982) using an autoregressive conditional heteroskedasticity (ARCH) model applied to the volatility of inflation in the United Kingdom. Bollerslev (1986) notes that the model proposed by Engle (1982) does not capture the persistent dynamic of conditional volatility, even when a large number of lags are used, and develops a generalized autoregressive conditional heteroskedasticity (GARCH) model. On the other hand, the literature has found evidence that the returns of the financial series follow asymmetric patterns with their volatilities. For example, French et al. (1987) find that an unexpected shock in the returns is negatively related to the impact that this shock has on the volatility. Along these lines, Nelson (1991) proposes an exponential GARCH (EGARCH) model on the logs of the conditional variance, which consists of incorporating an asymmetry component into the returns and the variance. These so-called leverage effects occur because many financial series are more volatile in response to bad news, which affects the expectations of financial market returns more than would be the case with good news.

Moreover, in Glosten et al. (1993) the volatility asymmetries are studied based on the specifications of Nelson (1991), but incorporating seasonal effects. The authors show that when the data are of higher frequency, the estimated persistence of the estimated volatility is greater. Glosten et al. (1993) apply this model to the Center for Research in Security Prices (CRSP) index of the U.S. market and find that the effects on volatility depend on the type of shock; if it is positive, volatility reduces, while the opposite occurs in the case of a negative shock. Engle and Ng (1993) employ the asymmetric GARCH model proposed by Nelson (1991) to study the news impact curve; that is, how diverse shocks in the returns affect conditional volatility. Through statistical tests on the asymmetry parameter, the authors conclude that negative shocks generate greater volatility than positive shocks for the stock market of Japan<sup>3</sup>.

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<sup>2</sup>Address for Correspondence: Gabriel Rodríguez, Full Professor, Department of Economics, Pontificia Universidad Católica del Perú, 1801 Universitaria Avenue, Telephone: +511-626-2000 (4998), E-Mail Address: gabriel.rodriguez@pucp.edu.pe.

<sup>3</sup>The advance in the literature on various models or extensions of the GARCH and EGARCH model is extensive. Certain models have included long-memory characteristics, while others have introduced the possibility of a different order of powers in the variance. For a full and detailed review of the references, see Degiannakis and Xekalaki (2004),

On the other hand, in the literature on financial series, another group of models known as stochastic volatility (SV) has been developed. These models add greater flexibility to the volatility equation, as they factor in two terms of error: one for the returns equation, and another for the volatility equation, with which the fluctuating behavior of the volatility modeled is captured. These models were initially proposed in the literature, for the continuous case, by Hull and White (1987), who model the price of an European market option. In turn, the discrete version is developed in Jacquier et al. (1994), and Harvey et al. (1994). The SV models do not have an analytical representation for the maximum likelihood function as it involves latent variables (the volatilities) that do not render feasible the integration of variables of the models. For this reason, Kim et al. (1998) develop a Bayesian approach that overcomes this limitation, which is the most efficient compared with other algorithms, such as the method of moments proposed by Wiggins (1987) and the estimation of maximum likelihood of Harvey et al. (1994).

Kim et al. (1998) develop the SV model by implementing Bayesian Monte Carlo Markov Chain (MCMC) methods that improve the efficiency in the estimation of the parameters. The efficiency criteria consists of developing the lowest number of iterations necessary to establish the significant estimation of the parameters. Moreover, Kim et al. (1998) conduct a study of the autocorrelations (ACF) of the samples to determine their degree of randomness. In turn, they determine a better approximation to the term of error of the model that displays the characteristics of a  $\chi^2$  distribution. Moreover, in Kim et al. (1998) a range of statistical tests are applied to compare the fit of the SV model against the GARCH model. The tests are based on the criteria of the logarithm of the likelihood, simulation tests on estimated parameters, and a final test that factors in the information on the prior and posterior distributions of the logarithm of likelihood of the model.

For the Peruvian case, the main characteristics of the variables employed in this research are studied in Humala and Rodríguez (2013). The authors study the presence of non-Normal behavior in a statistical sense, such as asymmetries, elevated kurtosis, and clusters in the volatility, and conduct a study based on dynamic correlations on the moments of the series. In Alanya and Rodríguez (2014), an SV canonical model for the Peruvian stock and Forex markets is estimated; the periods of volatility of the stock and Forex markets in the Peruvian economy are identified; and the SV models with Normal errors are compared with the GARCH models by assuming Normal and t-Student distributions, respectively. The results indicate that the SV model exceeds the GARCH-N model, but not the specification of the GARCH-t, though it achieves a close fit. In the literature on the SV models, a similar model to EGARCH is developed that incorporates the asymmetries between the returns and the variance. For example, Yu (2005) studies a SV model and asymmetries that compare the results of the Bayesian algorithm using continuous and discrete approaches.

This study estimates the SV models proposed in Omori et al. (2007), which consists of the SV canonical model similar to the EGARCH specification; that is, it incorporates asymmetries between the returns and the volatility. Nonetheless, Omori et al. (2007) develop a methodology that improves the efficiency and speed of the parameter estimation, which consists of a combination of Normal distributions for the approximation of the term of error generated in the linearization of the SV model. Moreover, the model with and without asymmetries is extended to the case of t-Student distributions, for a log-Normal distribution and a combination of both<sup>4</sup>. These models are applied to the series of returns for the TOPIX index of Tokyo. As well as developing a new

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and Bedón and Rodríguez (2015), as well as the references cited in that paper.

<sup>4</sup>In this study we only employ the specifications of the asymmetric SV model for Normal and t-Student errors, denoted as ASV-N and ASV-t, respectively.

algorithm for the best estimation of the SV models, Omori et al. (2007) devise a new technique for the filtered estimations of the volatilities. The purpose of these methods is to assess the likelihood function of the SV models, and conduct forecasts on that basis. A first approximation is made in Gordon et al. (1993) who develop a particle filter based on random samples for the state vector in the state-space representation; the use of this algorithm is expanded for the non-linear case, and using errors that do not assume Normality.

Another filter is that of Pitt and Shephard (1999), known as the auxiliary particle filter. Moreover, Kim et al. (1998) take this filter as a basis and develop a variation related to the weighting employed when producing the samples for the filters. In Omori et al. (2007), the auxiliary particle filter is used on the state-space representation of the SV model that incorporates asymmetries between the returns and the variance.

This study models the volatility of the daily stock and Forex market returns for the period from January 5, 1998 to December 30, 2011, incorporating extensions to the canonical SV model estimated in Alanya and Rodríguez (2014). With this aim, various SV models are employed, as well as GARCH models, using the Bayesian methods developed in Omori et al. (2007) and Nakajima (2012).

The Bayesian MCMC algorithms are the most efficient methods in terms of convergence to the parameters; see Jaquier et al. (1994). Moreover, we contrast the fit to the data of the SV and GARCH models typically employed in the literature on financial series, using the marginal likelihood criterion of Chib (1995).

In this study, the following specifications are used for the SV and the autoregressive conditional heteroskedasticity models: (i) an SV model with Normal errors (canonical or SV-N model); (ii) an SV model with distributed errors following a  $t$ -Student distribution (SV-t); and (iii) an SV model that incorporates the existing relationship between the returns and the variance (asymmetry) in response to a shock in the volatility with Normal and  $t$ -Student errors, respectively (ASV-N and ASV-t). The GARCH models that are used have similar characteristics: (i) GARCH-N, GARCH-t; (ii) EGARCH-N and EGARCH-t. Likewise, conditional heteroskedasticity models are estimated by employing the same specifications through Bayesian algorithms. With respect to the estimated asymmetry parameters in the stock market, this has a negative sign in both specifications and under Normal and  $t$ -Student errors, which is characteristic for these markets, as a negative shock in this market generates more volatility than a positive shock. However, in the Forex market the parameter had a positive sign, which means that a depreciation shock (positive) in the exchange rate generates greater volatility than an appreciation shock (negative); thus, the Central Reserve Bank of Peru (BCRP) faces relatively major difficulties in its intention of smoothing Forex volatility. The model with the best fit in both returns is the ASV model with Normal errors. The stock market returns have greater periods of volatility; however, both markets react to shocks in the economy, as they display similar patterns and have a significant correlation for the sample period studied.

The paper is structured as follows. The next section briefly outlines the GARCH, EGARCH, SV and ASV models with brief remarks about the algorithm of estimation; Section 3 describes the data and provides an analysis of the results; Section 4 deals with conclusions.

## 2 The Models and The Algorithm of Estimation

This Section outlines the models used, as well as the respective algorithm. In order to save space, the GARCH (with N and  $t$ -Student errors) models are not presented as they are standard and

appear in most econometrics textbooks. In any case, the description may be found in Alanya and Rodríguez (2014).

## 2.1 The EGARCH Model

This model, proposed by Nelson (1991), incorporates into the GARCH model an asymmetry component between the mean and variance of the series analyzed. The EGARCH model is robust to negative values in the volatility due to its exponential form. Thus, the EGARCH (1,1) model takes into account the following specification with Normal errors:

$$\begin{aligned} y_t &= \sigma_t \epsilon_t, \\ \log \sigma_t^2 &= \omega + \beta \log \sigma_{t-1}^2 + \theta \left( \frac{y_{t-1}}{\sigma_{t-1}} \right) + \alpha \left( \left| \frac{y_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right), \\ \epsilon_t &\sim N(0, 1), \end{aligned} \tag{1}$$

where  $\beta$  is the parameter that represents the persistence of the logarithm of the conditional volatility,  $\alpha$  captures the magnitude of a shock in the volatility on the logarithm of the conditional variance, and  $\omega$  is the intercept or level of the model. The peculiarity of the EGARCH model in relation to the GARCH is the asymmetry parameter  $\theta$ . When this parameter takes negative values, it means that a negative shock on the volatility will generate a future increase in volatility. On the other hand, if the parameter is positive this shock generates a lower future volatility, with the existing asymmetry appreciating in response to a shock in the volatility.

The EGARCH model with t-Student (EGARCH-t) errors is specified:

$$\begin{aligned} y_t &= \sqrt{\lambda_t} \sigma_t \epsilon_t, \\ \log \sigma_t^2 &= \omega + \beta \log \sigma_{t-1}^2 + \theta \left( \frac{y_{t-1}}{\sigma_{t-1}} \right) + \alpha \left( \left| \frac{y_{t-1}}{\sigma_{t-1}} \right| - \zeta \right), \\ \lambda_t^{-1} &\sim \text{Gamma}(\nu/2, \nu/2), \\ \epsilon_t &\sim N(0, 1), \end{aligned} \tag{2}$$

where the component  $\sqrt{\lambda_t} \sigma_t$  characterizes a t-Student distribution with  $\nu$  degrees of freedom. Moreover, the term  $\zeta$  represents the expected value of a random variable  $z$  that in turn follows a t-Student distribution with  $\nu$  degrees of freedom.

## 2.2 The SV and ASV Models

The literature on SV models specifies volatility as a latent process. This allows the volatility to be represented in space-state form in which an inherent error or innovation is assumed. The innovations of the return and variance are assumed as *i.i.id.* Normal distributions. Accordingly, the discrete version of the canonical SV model is:



$$\begin{aligned}
y_t &= \sqrt{\lambda_t} \exp(h_t/2) \epsilon_t, \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \\
\beta &= \exp(\mu/2), \\
\lambda_t^{-1} &\sim \text{Gamma}(\nu/2, \nu/2), \\
\begin{bmatrix} \epsilon_t | \sigma \\ \eta_t | \sigma \end{bmatrix} &\sim i.i.d. N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & \sigma^2 \end{pmatrix} \right], \\
h_0 &\sim N \left[ \mu, \frac{\sigma^2}{1 - \phi^2} \right],
\end{aligned} \tag{3}$$

where  $h_t$  is the volatility of the return in time  $t$  and it is assumed that  $h_t$  follows a stationary process AR(1). Note that the specification in (3) collapses to an SV-N model when  $\lambda_t = 1$ . On the other hand, the parameter  $\sigma$  represents the volatility of the SV model, which is indicative of the fluctuations of cycles of volatility in the returns. The parameter  $\beta$  represents a factor of scale for the equation of variance, as it depends on the level term  $\mu$  which is understood as the long-term average process. Moreover, the SV model has an initial condition  $h_0$  with a distribution governed by its unconditional moments. In the case of a SV model with t-Student errors, the term  $\sqrt{\lambda_t} \epsilon_t$  of the equation for the returns will be distributed under a t-Student with  $\nu$  degrees of freedom.

The ASV model captures the asymmetries between the returns and the variance as in the EGARCH model of Nelson (1991). The discrete version of the model is presented in Harvey and Shephard (1996) where the main difference from the canonical model is the incorporation of the terms of errors of the returns and variance equations. The SV model with asymmetries is

$$\begin{aligned}
y_t &= \sqrt{\lambda_t} \exp(h_t/2) \epsilon_t, \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \\
\beta &= \exp(\mu/2), \\
\lambda_t^{-1} &\sim \text{Gamma}(\nu/2, \nu/2), \\
\begin{bmatrix} \epsilon_t | \sigma \\ \eta_t | \sigma \end{bmatrix} &\sim i.i.d. N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{pmatrix} \right], \\
h_0 &\sim \left[ \mu, \frac{\sigma^2}{1 - \phi^2} \right],
\end{aligned} \tag{4}$$

whereby the only parameter that differentiates this asymmetric model from the previous SV model is the presence of the parameter  $\rho$ ; that is, the relationship between the shocks that characterize the returns and the volatility. When this coefficient is less than zero, then a negative shock in the returns generates higher future volatility, and analogically a positive shock implies lower volatility. Likewise, in the case of the SV model without asymmetries, the term  $\sqrt{\lambda_t} \epsilon_t$  characterizes the returns under a t-Student distribution with  $\nu$  degrees of freedom, with which the distribution  $\lambda_t$  follows an Inverse Gamma distribution. Incorporation of the asymmetric effect and a t-Student error leads to methodological complications in the estimation of the canonical SV model analyzed in Kim et al. (1998).

### 2.3 Bayesian Estimation of EGARCH Models

The estimation of the GARCH-N and GARCH-t models under these algorithms for the Peruvian financial returns appear in Alanya and Rodríguez (2014). The intention of estimating these models using Bayesian statistics is to render comparable the GARCH and the SV models by making use of the marginal likelihood criteria of Chib (1995). Following the algorithm employed by Nakajima (2012), we denote the vector  $\vartheta = (\omega, \beta, \alpha, \theta)$  of the parameters of level  $\omega$ , of persistence  $\beta$ , to the moving averages component  $\alpha$  and the asymmetry parameter  $\theta$  of the EGARCH model. Moreover, in the case of the EGARCH-t model, the degrees of freedom parameter is represented by  $\nu$ . In this way, the Bayesian algorithm is summarized as follows: (i) initial values are given to the parameters  $(\vartheta, \nu)$ ; (ii) samples of  $\vartheta|\lambda, \nu, y$  are obtained; (iii) samples of  $\lambda|\vartheta, \nu, y$  are obtained; (iv) samples of  $\nu|\lambda$  are obtained; and (v) step (ii) is repeated.

In step (ii) the samples of the vector of parameters  $\vartheta|\lambda, \nu$  are obtained from the conditional distribution  $\pi(\vartheta|\lambda, \nu)$  using the Metropolis-Hastings algorithm, which is approximated using a Normal distribution. The proposed distribution for a candidate  $\vartheta^*$  is  $N(\mu_*, \Sigma_*)$  where:  $\mu_* = \vartheta^* + \Sigma_* \frac{\partial \log \pi(\vartheta|\lambda, \nu, y)}{\partial \vartheta} |_{\vartheta=\vartheta^*}$  and  $\Sigma_*^{-1} = -\frac{\partial^2 \log \pi(\vartheta|\lambda, \nu, y)}{\partial \vartheta \partial \vartheta'} |_{\vartheta=\vartheta^*}$ . Both moments of the distribution are obtained by approximating the conditional distribution  $\pi(\vartheta|\lambda, \nu, y)$  around the candidate  $\vartheta^*$  through a second-order Taylor series expansion.

In the EGARCH-t model, the equation that characterizes  $\log \pi(\vartheta|\lambda, \nu, y)$  is:

$$\begin{aligned} \log \pi(\vartheta|\lambda, \nu, y) &= \log \pi(\omega) + \log \pi(\beta) + \log \pi(\alpha) + \sum_{t=1}^n -\frac{1}{2} \log(\lambda_t \sigma_t^2) - \frac{y_t^2}{2\lambda_t \sigma_t^2}, \\ \sigma_t^2 &= \exp[\omega + \beta \log \sigma_{t-1}^2 + \theta \left( \frac{y_{t-1}}{\sigma_{t-1}} \right) + \alpha \left( \left| \frac{y_{t-1}}{\sigma_{t-1}} \right| - \zeta \right)]. \end{aligned}$$

The candidate  $\vartheta^*$  is accepted under a probability:  $\Pr(\vartheta_0, \vartheta^*) = \min\left\{ \frac{\pi(\vartheta^*|\lambda, \nu, y)q(\vartheta_0|\lambda, \nu, y)}{\pi(\vartheta_0|\lambda, \nu, y)q(\vartheta^*|\lambda, \nu, y)}, 1 \right\}$ , and  $\vartheta_0$  represents the last candidate accepted. In step (iii) the samples for the  $\lambda$  come from the following conditional distribution:  $\pi(\lambda_t|\vartheta, \nu, y) \propto \lambda_t^{-\frac{\nu}{2}-1} \exp\left\{ \frac{\nu}{2\lambda_t} \right\} \frac{1}{\sqrt{\lambda_t \sigma_t^2}} \exp\left\{ -\frac{y_t^2}{2\lambda_t \sigma_t^2} \right\}$ , and the Metropolis Hastings algorithm is also used, where the candidate samples come from an Inverse-Gamma distribution  $(\nu/2, \nu/2)$ . Finally, in step (iv) samples for (v) are generated using:  $\pi(\nu|\lambda) \propto \pi(\nu) \frac{(\frac{\nu}{2})^{\frac{n\nu}{2}}}{\Gamma(\frac{\nu}{2})^n} \prod_{t=1}^n (\lambda_t^{-\frac{\nu}{2}}) \left\{ \exp\left(-\frac{\nu}{2} \sum_{t=1}^n \lambda_t^{-1}\right) \right\}$ , and the estimation of this parameter is resolved using the Metropolis-Hastings algorithm once again.

### 2.4 Bayesian Estimation of the SV and ASV Models

In Omori et al. (2007) an efficient algorithm is developed for the estimation of the SV models in the presence of asymmetries and assuming various types of error in the returns. This algorithm is based on the approach of Kim et al. (1998), which approximates the error by transforming the distribution of the SV model with a combination of Normal distributions, but also generates re-weightings on the results to minimize the previously obtained residuals. The SV model, using the notation of Omori et al. (2007), can be represented as follows:

$$\begin{bmatrix} y_t^* \\ h_{t+1} \end{bmatrix} = \begin{bmatrix} h_t \\ \mu + \phi(h_t - \mu) \end{bmatrix} + \begin{bmatrix} \epsilon_t^* \\ \eta_t \end{bmatrix},$$

such that,  $y_t^* = \log y_t^2 = h + \epsilon_t^*$  and  $\epsilon_t^* = \log(\epsilon_t^2)$ , because of which  $y_t = d_t \exp(y_t^*/2)$ , with  $d_t$  being a composition of dummy variables that capture the asymmetric affect between returns and variance errors:  $d_t = \mathbf{1}(\epsilon_t \geq 0) - \mathbf{1}(\epsilon_t < 0)$ .

Following Omori et al. (2007), for the approximation of the term  $\epsilon_t^*$ , ten (10) Normal distributions are employed, unlike the seven approximations suggested in Kim et al. (1998). The authors show that this mixed distribution fits better in relative terms to an  $\chi^2$  distribution when, for the term of error  $\epsilon_t^*$ , the logarithm functions  $\log(\epsilon_t^2)$  and square root  $\sqrt{\epsilon_t^2}$  are factored in. We denote the index that refers to one of the Normal distributions that comprise the mixed distribution as  $s_n$ . Thus, the algorithm for the estimation of the ASV-t model is summarized as follows: (i) we give the initial values to  $s_n, h_n, \mu, \vartheta$  where  $\vartheta = (\phi, \rho, \sigma)$ ; (ii) we generate samples for  $s_n | d_n, h_n, \mu, \vartheta, y_n^*$ ; (iii) we generate samples for  $\vartheta | d_n, s_n, y_n^*$ ; (iv) we generate samples for  $h_n, \mu | d_n, s_n, \vartheta, y_n^*$ ; (v) we obtain samples for  $\lambda | h_n, \mu, d_n, s_n, \vartheta, y_n^*$ ; and (vi) we obtain samples for  $\nu | \lambda$ . In step (ii) we get the following conditional distribution:

$$\begin{aligned} \pi(s_t = j | y_n^*, d_n, h_n, \mu, \vartheta) & \propto \pi(s_t = j | \epsilon_t^*, \eta_t, d_t, \mu, \vartheta) \\ & \propto \Pr(s_t = j) v_j^{-1} \exp \left\{ -\frac{(\epsilon_t^* - m_j)^2}{2v_j} - \frac{[\eta_t - d_t \rho \exp(m_j/2) \{a_j + b_j(\epsilon_t^* - m_j)\}]^2}{\sigma^2(1 - \rho^2)} \right\}, \end{aligned}$$

where  $\Pr(s_t = j)$  follows a discrete distribution, in which samples are generated under the inverse distribution method. For step (iii) the samples generated for  $\vartheta$  follow the distribution:  $g(\vartheta | y_n^*, d_n, s_n) \propto g(y_n^* | d_n, s_n, \vartheta) \pi(\vartheta)$ .

Firstly, the distribution for  $g(y_n^* | d_n, s_n, \vartheta)$  is obtained on applying the Kalman filter to the following equations:

$$\begin{aligned} \begin{bmatrix} y_t^* \\ h_{t+1} \\ \tilde{\mu}_{t+1} \end{bmatrix} & = \begin{bmatrix} h_t \\ \tilde{\mu}_t + \phi(h_t - \tilde{\mu}_t) \\ \tilde{\mu}_t \end{bmatrix} + \begin{bmatrix} \epsilon_t^* \\ \eta_t \\ 0 \end{bmatrix}, \\ \begin{bmatrix} h_1 \\ \tilde{\mu}_1 \end{bmatrix} & \sim N \left[ \begin{bmatrix} h_0 \\ \mu_0 \end{bmatrix}, \begin{bmatrix} \frac{\sigma^2}{(1-\phi^2)} + \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 \end{bmatrix} \right]. \end{aligned}$$

This state-space representation of the SV model allows estimation using the Kalman filter, where  $\tilde{\mu}_1 = \tilde{\mu}_2 = \dots = \tilde{\mu}_n = \mu$ . Subsequently, the joint distribution  $g(\vartheta | y_n^*, d_n, s_n)$  is approximated using the Metropolis-Hastings algorithm, namely, the estimated parameters denoted as  $\hat{\vartheta}$ . Finally,  $\gamma^*$  candidates are generated in accordance with a truncated normal distribution  $TN_R(\hat{\vartheta}, \Sigma_*)$ , with:  $\Sigma_*^{-1} = -\frac{\partial^2 \log g(y_n^* | d_n, s_n, \vartheta) \pi(\vartheta)}{\partial \vartheta \partial \vartheta^2} \Big|_{\vartheta = \hat{\vartheta}}$ , and the truncated area is represented by  $R = \{\gamma : |\phi| < 1, \sigma^2 > 0, |\rho| < 1\}$ . This candidate is accepted or rejected based on a probability by

following the Metropolis Hastings algorithm. In step (iv) the sample of the latent volatilities in moment  $n$  and the level parameter  $h_n, \mu | d_n, s_n, \vartheta, y_n^*$  is carried out using the filter developed by Koopman et al. (1996), with which latent parameters can be estimated using the smoothed Gaussian simulator. To sample the parameter associated with the t-Student distribution  $\lambda$  in step (v), the following conditional distribution is employed:

$$\pi(\lambda | h_n, \mu, d_n, s_n, \vartheta, y_n^*) \propto \lambda_t^{-\frac{\nu}{2}-1} \exp \left\{ -\frac{\nu}{2\lambda_t} - \frac{(\log \lambda_t - \mu_{\lambda_t})^2}{2\sigma_{\lambda_t}^2} \right\},$$

where  $\mu_{\lambda_t}$  and  $\sigma_{\lambda_t}^2$  are obtained from the results of the state-space representation determined in the previous step and whose analytical representation is set out in Nakajima and Omori (2009). Then, through the Metropolis-Hastings algorithm, the candidates derived from the prior distribution of the  $\lambda_t^{-1}$  parameter are obtained. Finally, the conditional distribution for the degrees of freedom  $\nu$  from the previous step of the algorithm is given by:

$$\pi(\nu | \lambda) \propto \pi(\nu) \frac{(\frac{\nu}{2})^{\frac{n\nu}{2}}}{\Gamma(\frac{\nu}{2})^n} \prod_{t=1}^n (\lambda_t^{-\frac{\nu}{2}}) \left\{ \exp(-\frac{\nu}{2} \sum_{t=1}^n \lambda_t^{-1}) \right\}.$$

As with the algorithm of the EGARCH models, the samples of this parameter are resolved through an acceptance and rejection procedure for the candidates and the use of the Metropolis-Hastings algorithm.

## 2.5 Filtered Estimation of Stochastic Volatility

The particle filter proposed by Omori et al. (2007) is employed to estimate the filtered volatilities of the model. The objective of this filter is to approximate the likelihood function of the SV model using this sequential algorithm. Thus, through these approximations the estimation of the logarithm of likelihood of the models is obtained.

The auxiliary particle filter for the ASV-N model under the state-space representation of the SV model is characterized by the following equations:

$$f(y_t | h_t) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{(h_t - y_t^2 \exp(-h_t))^2}{2} \right]$$

which is the density or measurement equation, while:

$$\begin{aligned} f(h_{t+1} | y_t, h_t, \mu, \theta) &= \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma}} \exp \left[ -\frac{(h_{t+1} - \mu_{t+1})^2}{2(1-\rho^2)\sigma} \right], \\ \mu_{t+1} &= \mu + \phi(h_t - \mu) + \sigma\rho \exp(-h_t/2)y_t. \end{aligned}$$

is the equation of evolution or transition. The steps are as follows: (i) for time  $t = 0$  of the algorithm, the volatilities  $h_{t+1}^i, h_t^i$  are generated for  $i = 1, 2, \dots, I$  based on the unconditional distribution; (ii)  $J$  samples are generated for the volatilities  $h_t^i$  and  $h_{t+1}^i$  through the function of importance  $g(h_{t+1}, h_t | y_{t+1}, \mu, \vartheta) : g(h_{t+1}, h_t^i | y_{t+1}, \mu, \vartheta) \propto f(h_{t+1} | y_t, h_t^i, \mu, \vartheta) g(h_t^i | y_{t+1}, \mu, \vartheta)$ , where  $g(h_t^i | y_{t+1}, \mu, \vartheta) = \frac{f(y_{t+1} | \mu_{t+1}^i) \hat{f}(h_t^i | y_t, \mu, \vartheta)}{\sum_{j=1}^I f(y_{t+1} | \mu_{t+1}^j) \hat{f}(h_t^j | y_t, \mu, \vartheta)}$  and  $f(y_{t+1} | \mu_{t+1}^j)$  are obtained by evaluating this distribution in the measurement equation of the state-space representation. Moreover,  $\hat{f}(h_t^i | y_t, \mu, \theta)$

is a discrete uniform distribution that approximates  $f(h_t^i|y_t, \mu, \vartheta)$  from which samples are generated for  $f(h_{t+1}|y_t, h_t, \mu, \vartheta)$ . Thus, for each  $i$ ,  $J$  components  $w_j$  are generated in accordance with  $w_j = \frac{f(y_{t+1}|h_{t+1}^j)\hat{f}(h_t^i|y_t, \mu, \vartheta)}{g(h_t^i|y_{t+1}, \mu, \vartheta)}$ , the following results are kept:  $\bar{w}_{t+1} = \frac{1}{IJ} \sum_{j=1}^J \sum_{i=1}^I w_{i,j}$ ; (iii) a re-sample is generated using as probabilities the results stored in  $w_j$  for each  $i$ , with which the final samples  $I$  are obtained; (iv) step (ii) is resumed, and the algorithm is applied to the following period. Under this algorithm, Omori et al. (2007) show that if  $I, J \rightarrow \infty$ :  $\sum_{t=1}^n \log(\bar{w}_t) \rightarrow \sum_{t=1}^n \log f(y_t|y_1, \dots, y_{t-1}, \mu, \vartheta)$ , where  $\sum_{t=1}^n \log(\bar{w}_t)$  is a consistent estimator of the logarithm of likelihood, which is the main input for calculating the marginal likelihood of Chib (1995).

### 3 Empirical Results

In this Section the results of the parameter estimations for the models EGARCH-N, EGARCH-t, SV-N, SV-t, ASV-N and ASV-t are presented. Moreover, to assess the fit of the model to the data in both methodologies, the marginal likelihood criteria of Chib (1995) is employed<sup>5</sup>. In the estimation of the parameters of all models, 15000 iterations are employed, the first 10000 having been discarded.

The data used in this study are the stock market returns calculated as the first differences in the General Index of the Lima Stock Exchange (IGBVL) and the Forex rate returns obtained from the differences in the nominal purchase exchange rate published by Peru's Superintendency of Banking, Insurance, and Pension Fund Administrators (SBS). Thus, the returns are calculated using the following formula:  $y_t = \ln(P_t) - \ln(P_{t-1})$ , where  $P_t$  is the respective market index or exchange rate, respectively. The returns are daily and cover the period from January 5, 1998 to December 30, 2011 for both series. In 2011, the Peruvian stock market posted a fall in its index of around 17%, while market capitalization dropped by 24.4 % from 2010, explained by greater uncertainty in the global markets and the general election process that year. However, the trading volume totalled 7,817 million Soles and more transactions were carried out, worth 365,202. The stock market returns are shown in Figure 1a. Moreover, Figure 1b shows the Forex returns, and clustering can be appreciated right across the sample, especially in the period of the 2008 international financial crisis.

The main statistics on both financial returns and volatilities are shown in Table 1. The first moment of the returns approximates the value of zero, implying that there are as many observations with positive results as there are negatives, which is an indicator of high volatility and erratic behavior in the series. Moreover, the standard deviation of the stock market returns is 0.015 greater than the standard deviation of the Forex returns 0.002, from which it is inferred that there was greater fluctuation in the stock market returns. Moreover, the kurtosis shows expected results for financial returns, which are characterized for having data very far from the mean. As for the stock market returns, the kurtosis is 13.188 while for the foreign exchange returns, it is 15.635. It should be stated that despite the returns having a mean close to zero, for the estimations of all models in this study, both series are corrected for the mean, and our study is centered on explaining the dynamic of the second moment of the returns.

Moreover, when the models are estimated using Bayesian algorithms, the Geweke and inefficiency indices are determined. The inefficiency of the estimated parameters is calculated as a

<sup>5</sup>In the case of the GARCH-N and GARCH-t models, only the results of this test are presented. However, the results of the estimation of the algorithm of Gilks and Wild (1992) are reported in Alanya and Rodríguez (2014).

function of the ACF of the samples generated:  $1 + 2 \sum_{s=1}^{\infty} \rho_s$ , where  $\rho_s$  represents the autocorrelation of the samples generated in the lag  $s$ .

### 3.1 The EGARCH Models

In order to save space the GARCH (Normal and Student-t) are not presented here, but can be found in Alanya and Rodríguez (2014). However, they are considered in the selection of the models. To carry out the Bayesian algorithms, the prior distributions for the EGARCH models of Nakajima et al. (2012) are used. The parameters follow the following prior distributions:  $\omega \sim N(0, 1)$ ,  $\beta \sim \text{Beta}(8, 1)$ ,  $\alpha \sim N(0, 1)$ ,  $\theta \sim N(0, 1)$  and  $\nu \sim \text{Gamma}(16, 0.8)$ . The results of the parameter estimations of the EGARCH-N and EGARCH-t models for the stock market returns are shown in Table 2. The persistence  $\beta$  in the conditional volatility for both models is high, which indicates that a shock in the estimated conditional variance will have a transitory but long-lasting effect. The estimated levels of persistence are 0.947 and 0.956, slightly higher in the case of the EGARCH-t model. The estimated values suggest that in response to a conditional volatility shock, we will get an effect of 14 and 17 days, respectively. Moreover, the parameter  $\alpha$ , which reflects the symmetric effect or magnitude of the shock on the volatility, is 0.403 and 0.310 for the EGARCH-N and EGARCH-t models, respectively. This symmetric effect is greater in the EGARCH-N model because for the EGARCH-t model, part of this symmetric effect is explained by the non-Normal error.

The estimated EGARCH models for the stock market returns show that there is an asymmetric relationship between the returns and the variance. Indeed, the parameter  $\theta$  for the EGARCH-N model is  $-0.037$  and  $-0.021$  for the EGARCH-t model. This implies that when a negative shock or an event that concerns the stock market occurs, the volatility generated is greater than when there is a positive shock in the economy. Such asymmetric behavior in stock market returns has been observed in the literature for various series. However, the asymmetric effects are not high<sup>6</sup>.

On the other hand, the estimated level parameter  $\omega$  in both models is  $-0.45$  on average. With respect to the convergence criteria, the Geweke and inefficiency indices show us that the algorithm employed generates Markov chains rapidly, so a significant estimation is arrived at. These results can be seen in the paths of the iterations and their autocorrelations in Figures 2a and 2b, which establish the randomness in the iterations realized and the posterior densities. The parameter that characterizes the t-Student distribution in the EGARCH-t model,  $\nu$ , has a mean of 7.136, which implies that factoring this type of distribution into the module captures the information on atypical observations of the model, in the framework of the conditional heteroskedasticity models. Note that the autocorrelation function (ACF) of the parameter  $\nu$  decays slowly.

In the case of the Forex returns, the results of the estimation and the convergence criteria are found in Table 2, and display high persistence  $\beta$  in response to a shock in the volatility, taking a value of 0.96 in both models, which is equivalent to an half life of 18 days. As regards the parameter  $\alpha$  that captures the GARCH effect, the results give an estimate of 0.399 in the model with Normal errors and 0.336 in the model that assumes a t-Student, as this distribution affects the expected value of a shock in the volatility  $\zeta$ . In turn, the asymmetric effect, captured in the parameter  $\theta$ , has a mean in the posterior density of 0.064 for the case of EGARCH-N, and of 0.045 for the EGARCH-t model. These results suggest that a positive shock on the foreign exchange returns in this market generates slightly greater volatility than is the case for a negative shock. In consequence, exchange rate depreciation is more sensitive to volatility than appreciation. This

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<sup>6</sup>Similar results are found in Lengua et al. (2015) using a more complicated approach.

means that though the Central Reserve Bank of Peru (BCRP) controls foreign exchange volatility with its floating exchange rate and interventionist regime, the issuing entity faces greater difficulties in its objective of smoothing the volatility when there is a currency depreciation, as compared to when appreciations occur.

With respect to estimations of the level parameter  $\omega$ , this has a mean of  $-0.386$  for the EGARCH-N model and  $-0.542$  for the EGARCH-t model. Moreover, the degrees of freedom parameter of this model is estimated at 4.510, so the EGARCH-t model captures, to a large extent, the new distribution with fat tails in the data. With respect to inefficiency levels, these are low in most of the parameters and the posterior densities are well-formed around the mean of the distribution. The paths of the iterations, the correlations, and the densities of the parameters of the EGARCH-N and EGARCH-t models are shown in Figures 3a and 3b. The ACF of parameter  $\nu$  shows better behavior.

### 3.2 The SV and ASV Models

Table 3 shows the results of the estimation of the four SV models for the stock and Forex returns, for which the posterior distributions established in Nakajima (2012) are used. These distributions are:  $\frac{\theta+1}{2} \sim Beta(20, 1.5)$ ,  $\sigma^2 \sim Inverse - Gamma(2.5, 0.025)$ ,  $\mu \sim N(-10, 1)$ ,  $\frac{\rho+1}{2} \sim Beta(1, 1)$ ,  $\nu \sim Inverse - Gamma(16, 0.8)$ . The persistence of the stochastic volatility  $\phi$  is high for all models, fluctuating around 0.96 in all cases, with the value estimated for asymmetric models being relatively low. Thus, a shock in the stochastic volatility, considering that our data are daily and regardless of the asymmetry, will have a half life of 19 days for the models that assume Normality, and 22 days for the models with errors distributed with a t-Student.

The parameter that measures the variability of the stochastic volatility  $\sigma$  fluctuates around 0.29 in the models with Normal distribution, and 0.26 for models with t-Student distribution, in the case of the stock market returns. Conversely, for the case of the Forex returns, this parameter is estimated at 0.31 for the SV-N and ASV-N models, and at 0.28 for the SV-t and ASV-t models, which suggests that the stochastic volatility has longer periods of uncertainty for the stock market returns, and this is consistent with the interventionist exchange rate policy of the BCRP.

Similarly, Table 3 shows the results of the estimation of the parameter that measures the asymmetry  $\rho$  between the returns and the variance in the ASV-N and ASVt models, with estimated values of  $-0.081$  and  $-0.091$ , respectively. These results are consistent with the estimates of the EGARCH-N and EGARCH-t models. This means that a negative shock in the equation of the returns, because of the negative covariance between the shocks in the returns and the volatility equations, generates greater volatility than when there is a positive shock in the returns. This fact is recurrent in the stock markets, in which great uncertainty is generated when there is a fall in the respective indices on account of the shocks faced by the local and world economy. However Table 3 shows that for the stock market the parameter of asymmetry has a confidence interval including zero or positive values. It implies the possibility that asymmetric effects are inexistent in this market. The results for the ASV-N and aSV-t models indicate the same conclusion. In comparison with the EGARCH models, the existence of asymmetric effects is weakly. The result is interesting because Lengua Lafosse et al. (2015) find similar results for the Latin American stock markets using a more sophisticated model belonging to the ASV family models.

For the SV-t and ASV-t models, the estimated degrees of freedom are 24.345 and 23.428, respectively. This implies that incorporating a t-Student distribution into the SV model and ASV

into the stock market data does not provide more information on the behavior of the series, as a high number of degrees of freedom converge on a model with Normal distribution. Thus, with the exception of the parameter of the degrees of freedom<sup>7</sup>, the estimates display low levels of inference and the iterations prove to be random, as shown in Figures 4a, 4b, 5a, and 5b.

For the Forex returns, the results of the Bayesian estimation of the SV and ASV models are shown in Table 3, where the standard MCMC deviation, the confidence interval, and the inefficiency criteria can also be appreciated. Thus, the estimated persistence  $\phi$  of the stochastic volatility is high for all models. For the SV-N and ASV-t specifications, a shock in the mean equation will have an effect of up to 26 days. For the SVt model, this effect will last approximately 31 days, while in the ASV-t model the effect is extended over 24 periods. Moreover, the parameter that measures the asymmetry  $\rho$  for the foreign exchange returns is 0.210 for the ASV-N model and 0.223 in the ASV-t model, implying that negative shocks in the stochastic volatility generate lower volatility than the positive shocks. These results coincide with the predictions of the EGARCH models estimated earlier, though their estimates are greater in magnitude. Thus, exchange rate depreciation affects the Forex stochastic volatility more than appreciation. However, the results show that the interventions of the issuing entity are more sensitive to a lower supply of dollars and/or increases in demand for foreign currency.

The estimated values of the degrees of freedom of the t-Student distribution  $\nu$  are high, with a posterior mean of 27.050 and 29.527 in the SVt and ASVt models, which suggests that under the stochastic volatilities approach, a model assuming Normality is sufficient to effectively capture the volatility cycles of the Forex returns. On the other hand, in Figures 6a, 6b, 7a, and 7b the paths of the 5000 iterations performed for each of the four volatility models are shown; in each of the cases except for degrees of freedom, the randomness of the iterations and the posterior densities estimated around the mean are appreciated, which suggest an efficient estimation of the parameters.

### 3.3 Chib's (1995) Marginal Likelihood

The marginal likelihood of Chib (1995) takes into account the prior information on the distributions of the parameters, the posterior densities, or the final distributions of the parameters, and incorporates both results into the logarithm of likelihood of each of the models. Thus, the formal definition of the marginal likelihood is  $VM = \log f(y|M, \theta^*) + \log f(\theta^*) - \log f(\theta^*|M, y)$ , where the first term of the equation is the logarithm of the maximum likelihood of the model  $M$ , the term  $\log f(\theta^*)$  is the logarithm of the prior distributions assessed at the means of the posterior distributions  $\theta^*$  and the final term  $\log f(\theta^*|M, y)$  is the logarithm of the posterior distributions also assessed at the mean, which is obtained by evaluating the density through a Gaussian kernel.

The results of this test for the stock market and the Forex returns are shown in Table 4. In the case of the stock market returns, the SV models have a better fit than the GARCH models for each of the specifications adopted. The SV model that incorporates asymmetries possesses the greatest marginal likelihood. On the other hand, the EGARCH model has the best fit among the conditional heteroskedasticity models. Moreover, utilizing an EGARCH-t results in a better fit to the data, which is expressed in the lower estimated values of the degrees of freedom and in greater marginal likelihood. In the case of the SV models, incorporating t-Student distribution does not provide a better fit among the models of this type, which is reflected in the high degrees of freedom estimated.

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<sup>7</sup>However, the results are robust when the parameters are estimated with a larger number of iterations.



With respect to the series of Forex returns considering the models of conditional heteroskedasticity, the EGARCH-t model has the best fit, as it significantly estimates the degrees of freedom and the asymmetric response to shocks in the conditional volatility. In the case of the SV models, the specification with the best results is the ASV-N model. In light of the estimations that report a high number of degrees of freedom, a model that assumes Normality in these returns is the most useful.

Thus, for the stock and the Forex markets returns, the ASV model has a better fit among the models analyzed. In particular, for the stock rate series, all stochastic volatility models fit better to the data than the EGARCH models. In the case of the series of Forex, the EGARCH-t model has a better fit than the SV, ASV-t and SV-t models; however, these specifications considerably exceed the other GARCH models. In general, according to the marginal likelihood, the SV performs well.

### 3.4 Explaining the Stochastic Volatilities

Figures 8a and 8b show the smoothed volatilities for all models of stock and Forex market returns, respectively. The smoothed volatilities are obtained using a mean of the realizations of the implicit volatilities within the estimated samples:  $\sum_{t=1}^H h_t/H$ , with  $H$  being the number of iterations performed. It can be noted that the volatilities follow the same pattern in all the SV models. In particular, the ASV model accentuates peaks in the volatilities in the sample studied for both returns.

On the other hand, it can be appreciated that the stochastic volatility of the Peruvian stock market and Forex returns was affected by the international crises of 1997 and 1999, reflected in high volatility, due to the economic problems in Asia and Russia. Likewise, the financial crisis in the United States has had serious repercussions on the behavior of the volatility of the stock and Forex markets. Moreover, it can also be appreciated that in 2010 there were no major shocks in these markets. However, in 2011 the crisis was accentuated in the countries of the European Community and North America, causing uncertainty in the markets of emerging economies such as Peru.

Moreover, Figures 9a and 9b provide a comparison between filtered<sup>8</sup> and smoothed volatility for the ASV model, which results in a better fit to the data for both returns. It can be seen that the filtered procedure that estimates volatility period-by-period evidences certain peaks in the estimated volatility, while the smoothed volatility, on taking the information from all the samples generated, displays smoothed behavior.

On comparing the evolution of the volatilities, it can be observed that the volatilities, when affected by the same shocks on the economy, show a similar dynamic in time. Moreover, the coefficient of correlation between the stochastic volatilities of the returns is 0.45, which indicates similar propagation of the uncertainty in both markets. As is shown in our estimations of the ASV models, the parameter that measures the mean of the volatility in the long term  $\beta$  for the stock market returns is 0.0103, and for the foreign exchange returns is 0.0017; that is, the stock market is more volatile on average than the foreign exchange market, which is because the volatility in the Forex market is controlled at certain degree by the BCRP, whereas the stock market more directly reflects the uncertainty brought about by multiple shocks to the economy.

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<sup>8</sup>The filtered estimations of the volatilities are carried out using  $J = 10$  and  $I = 2500$  in the auxiliary particle filter of Omori et al. (2007).

## 4 Conclusions

Following the study of the Peruvian financial returns based on the canonical SV approach of Alanya and Rodríguez (2014), this study extends the canonical SV model to new specifications that take into account a t-Student error and asymmetries between the returns and the variance. Likewise, conditional heteroskedasticity models are estimated by employing the same specifications through Bayesian algorithms.

With respect to the estimated asymmetry parameters in the stock market, this has a negative sign in both specifications and under Normal and t-Student errors, which is characteristic of these markets, because a negative shock in this market generates more volatility than a positive shock. The results in Table 3 shows that for the stock market the parameter of asymmetry has a confidence interval including zero or positive values. It implies the possibility that asymmetric effects are inexistent in this market. The results for the ASV-N and ASV-t models indicate the same conclusion. In comparison with the EGARCH models, the existence of asymmetric effects is weakly. The result is interesting because Lengua Lafosse et al. (2015) find similar results for the Latin American stock markets using a more sophisticated model belonging to the ASV family models.

In the Forex market the parameter has a positive sign, which means that a depreciation shock (positive) in the exchange rate generates greater volatility than an appreciation shock (negative); thus, the BCRP faces relatively major problems in its intention of smoothing the Forex exchange volatility.

On comparing the evolution of the volatilities, it can be observed that the volatilities, when affected by the same shocks on the economy, show a similar dynamic in time. Moreover, the coefficient of correlation between the stochastic volatilities of the returns is 0.45, which indicates similar propagation of the uncertainty in both markets. As is shown in our estimations of the ASV models, the parameter that measures the mean of the volatility in the long term  $\beta$  for the stock market returns is 0.0103, and for the foreign exchange returns is 0.0017; that is, the stock market is more volatile on average than the foreign exchange market, which is because the volatility in the Forex market is controlled at certain degree by the BCRP, whereas the stock market more directly reflects the uncertainty brought about by multiple shocks to the economy.

The estimations show little inefficiency in the estimation of the parameters measured under the Geweke index, except for the case of the SV models that factor in t-Student errors, given that the estimated degrees of freedom are high, with which these SV models do not necessarily produce a better fit than models that assume Normal errors in the returns employed using the marginal likelihood statistic. The model with the best fit in both returns is the ASV model with Normal errors. The stock market returns have greater periods of volatility; however, both markets react to shocks in the economy, as they display similar patterns and have a significant correlation for the sample period studied. This suggests, as a topic for the agenda, that a multivariate model of asymmetric stochastic volatility would be capable of identifying these common volatility factors and better explaining the volatilities displayed in both Peruvian financial series.

## References

- [1] Alanya, W., and G. Rodríguez (2014), “Stochastic Volatility in Peruvian Stock Market and Exchange Rate Returns: A Bayesian Approximation,” Working Paper **392**, Department of

Economics, Pontificia Universidad Católica del Perú.

- [2] Bedón, P., and G. Rodríguez (2015), “Univariate Autoregressive Conditional Heteroskedasticity Models: An Application to the Peruvian Stock Market Returns,” Working Paper **400**, Department of Economics, Pontificia Universidad Católica del Perú.
- [3] Bollerslev, T. (1986), “Generalized Autoregressive Conditional Heteroskedasticity,” *Journal of Econometrics* **31**, 307-327.
- [4] Chib, S. (1995), “Marginal likelihood from the Gibbs output,” *Journal of the American Statistical Association* **90**, 1313-1321.
- [5] Degiannakis, S. and E. Xekalaki (2004), “Autoregressive Conditional Heteroskedasticity (ARCH) Models: A Review,” *Quality Technology and Quantitative Management* **1**, 271-324.
- [6] Engle, R. (1982), “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation”, *Econometrica* **50**, 987-1008.
- [7] Engle, R., and V. Ng (1993), “Measuring and testing the impact of news on volatility”, *Journal of Finance* **43**, 1749-1777.
- [8] French, K. R., G. W. Schwert, and R. E. Stambaugh (1987), “Expected Stock Returns and Volatility,” *Journal of Financial Economics* **19**, 3-29.
- [9] Gilks, W. R. and P. Wild (1992), “Adaptive REjection Sampling for Gibbs Sampling,” *Applied Statistics* **41(2)**, 337-348.
- [10] Glosten, L., R. Jagannathan, and D. Runkle (1993), “On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks,” *Journal of Finance, American Finance Association*, **48(5)**, 1779-1801.
- [11] Gordon, N. J., D. J. Salmond, and A. F. M. Smith (1993), “A novel approach to non-linear and non-Gaussian Bayesian state estimation,” *IEE-Proceedings F* **140**, 107-133.
- [12] Harvey, A. C., E. Ruiz, and N. Shephard (1994), “Multivariate stochastic models,” *Review of Financial Studies* **61**, 247-264.
- [13] Harvey, A. C., and N. Shephard (1996), “Estimation of an asymmetric model of asset prices,” *Journal of Business & Economic Statistics* **14**, 429-434.
- [14] Hull, J. C., and A. White (1987), “The Pricing of Options on Assets with Stochastic Volatilities,” *Journal of Finance* **42**, 281-300.
- [15] Humala, A., and G. Rodríguez (2013), “Some stylized facts of returns in the foreign exchange and stock markets in Peru,” *Studies in Economics and Finance* **30(2)**, 139-158.
- [16] Jacquier, E., N. G. Polson, and P. E. Rossi (1994), “Bayesian Analysis of Stochastic Volatility Models,” *Journal of Business & Economic Statistics* **12**, 371-389.
- [17] Lengua Lafosse, P., C. Bayes and G. Rodríguez (2015), “A Stochastic Volatility Model with GH Skew Student’s t-Distribution: Application to Latin-American Stock Returns,” Working Paper **405**, Department of Economics, Pontificia Universidad Católica del Perú.

- [18] Kim, S., N. Shephard, and S. Chib (1998), “Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models,” *The Review of Economic Studies* **65**, 361-393.
- [19] Koopman, S. J., N. Shephard, and J. A. Doornik (1996), “SSFPack 1.1: Filtering, Smoothing and Simulation Algorithms for State-Space Models in Ox,” Unpublished paper and software available at <http://www.nuff.ac.uk/users/shepards/ox>.
- [20] Nakajima, J., and Y. Omori (2009), “Leverage, heavy-tails and correlated jumps in stochastic volatility models,” *Computational Statistics and Data Analysis* **53**, 2335-2353.
- [21] Nakajima, J. (2012), “Bayesian analysis of generalized autoregressive conditional heteroskedasticity and stochastic volatility: modeling leverage, jumps and heavy-tails for financial time series,” *Japanese Economic Review* **63**, 81-103.
- [22] Nelson, D. B. (1991), “Conditional Heteroskedasticity in Asset Returns: A New Approach”, *Econometrica* **59**, 347-370.
- [23] Omori, Y., S. Chib , N. Shephard, and J. Nakajima (2007), “Stochastic Volatility with Leverage: Fast Likelihood Inference,” *Journal of Econometrics* **140**, 425-449.
- [24] Pitt, M., and N. Shephard (1999), “Filtering via Simulation: Auxiliary Particle Filters,” *Journal of the American Statistical Association* **94**, 590-599.
- [25] Wiggins, J. B. (1987), “Option Values under Stochastic Volatilities,” *Journal of Financial Economics* **19**, 351-372.
- [26] Yu, J. (2005), “On leverage in a stochastic volatility model,” *Journal of Econometrics* **127**, 165-178.

Table 1. Descriptive Statistics for Returns and Volatility

Values	Stock		Forex	
	Returns	Volatility	Returns	Volatility
	(1998:01–2011:12)		(1998:01–2011:12)	
Mean	0.001	-10.454	0.000	-14.930
Median	0.001	-10.127	0.000	-14.124
Maximum	0.128	-4.028	0.022	-7.540
Minimum	-0.133	-27.253	-0.023	-23.748
Standard Deviation	0.015	2.538	0.002	3.700
Skewness	-0.441	-1.088	0.281	-1.324
Kurtosis	13.188	5.555	15.635	4.198
Observations	3374		3377	

Table 2. Estimation of EGARCH, EGARCHt Models

Parameters	Mean	SD	Confidence Interval	Geweke	Inefficiency
		MCMC	at 95%	Index	
Stock Market					
EGARCH-N					
$\omega$	-0.458	0.051	[-0.551, -0.363]	0.508	6.06
$\alpha$	0.403	0.025	[0.353, 0.453]	0.294	4.74
$\beta$	0.947	0.006	[0.936, 0.957]	0.532	6.04
$\theta$	-0.037	0.011	[-0.059, -0.016]	0.905	3.13
EGARCH-t					
$\omega$	-0.454	0.077	[-0.645, -0.322]	0.810	18.21
$\alpha$	0.310	0.027	[0.258, 0.356]	0.540	9.38
$\beta$	0.956	0.008	[0.936, 0.970]	0.835	18.03
$\theta$	-0.021	0.011	[-0.044, -0.001]	0.538	6.18
$\nu$	7.136	0.793	[5.816, 8.880]	0.378	33.23
Forex Market					
EGARCH-N					
$\omega$	-0.386	0.046	[-0.468, -0.297]	0.189	17.84
$\alpha$	0.399	0.020	[0.358, 0.439]	0.502	6.24
$\beta$	0.967	0.004	[0.960, 0.974]	0.179	17.14
$\theta$	0.064	0.010	[0.044, 0.085]	0.622	4.13
EGARCH-t					
$\omega$	-0.542	0.083	[-0.710, -0.390]	0.410	10.68
$\alpha$	0.336	0.026	[0.286, 0.388]	0.302	10.07
$\beta$	0.965	0.006	[0.953, 0.975]	0.436	10.01
$\theta$	0.045	0.011	[0.024, 0.067]	0.968	4.06
$\nu$	4.510	0.387	[3.848, 5.333]	0.364	28.07

Table 3. Estimation of SV, SVt, ASV, ASVt Models

Parameters	Mean	Standard Deviation	Confidence Interval	Geweke	Inefficiency
		MCMC	at 95%	Index	
Stock Market					
SV-N					
$\phi$	0.9646	0.0068	[0.9502, 0.9765]	0.295	4.78
$\sigma$	0.2881	0.0229	[0.2473, 0.3369]	0.717	9.57
$\beta$	0.0103	0.0008	[0.0088, 0.0118]	0.418	1.06
SV-t					
$\phi$	0.9694	0.0064	[0.9559, 0.9810]	0.968	11.94
$\sigma$	0.2638	0.0230	[0.2203, 0.3099]	0.672	17.96
$\beta$	0.0099	0.0008	[0.0085, 0.0115]	0.339	0.64
$\nu$	24.3452	3.7498	[16.9650, 31.6409]	0.162	94.91
ASV-N					
$\phi$	0.9641	0.0065	[0.9508, 0.9761]	0.317	1.46
$\sigma$	0.2889	0.0213	[0.2486, 0.3306]	0.126	1.64
$\beta$	0.0103	0.0007	[0.0089, 0.0118]	0.000	1.72
$\rho$	-0.0809	0.0456	[-0.1675, 0.0089]	0.110	3.63
ASV-t					
$\phi$	0.9690	0.0063	[0.9555, 0.9805]	0.001	11.93
$\sigma$	0.2630	0.0220	[0.2228, 0.3088]	0.000	30.30
$\beta$	0.0099	0.0008	[0.0085, 0.0116]	0.000	3.83
$\rho$	-0.0906	0.0484	[-0.1819, 0.0074]	0.276	4.56
$\nu$	23.4283	4.5933	[16.1060, 33.3119]	0.000	153.77

Table 3 (continues). Estimation of SV, SVt, ASV, ASVt Models

Parameters	Mean	Standard Deviation	Confidence Interval	Geweke	Inefficiency
		MCMC	at 95%	Index	
Forex Market					
SV-N					
$\phi$	0.9744	0.0053	[0.9633, 0.9843]	0.103	3.28
$\sigma$	0.3082	0.0219	[0.2673, 0.3516]	0.851	5.48
$\beta$	0.0016	0.0002	[0.0013, 0.0020]	0.064	0.95
SV-t					
$\phi$	0.9779	0.0050	[0.9675, 0.9875]	0.533	7.27
$\sigma$	0.2817	0.0214	[0.2408, 0.3238]	0.328	18.14
$\beta$	0.0016	0.0002	[0.0013, 0.0020]	0.000	1.97
$\nu$	27.0504	4.9889	[19.3613, 39.8727]	0.008	106.67
ASV-N					
$\phi$	0.9709	0.0052	[0.9602, 0.9807]	0.512	7.69
$\sigma$	0.3057	0.0212	[0.2648, 0.3481]	0.427	14.86
$\beta$	0.0017	0.0002	[0.0014, 0.0021]	0.042	0.62
$\rho$	0.2102	0.0456	[0.1193, 0.2976]	0.169	4.39
ASV-t					
$\phi$	0.9739	0.0051	[0.9635, 0.9835]	0.487	5.98
$\sigma$	0.2829	0.0216	[0.2414, 0.3252]	0.265	17.33
$\beta$	0.0017	0.0002	[0.0014, 0.0021]	0.019	2.25
$\rho$	0.2268	0.0483	[0.1319, 0.3217]	0.006	5.29
$\nu$	29.5265	5.5579	[19.5011, 40.9226]	0.576	170.62



Table 4. Marginal Likelihood Estimation for GARCH, EGARCH, SV and ASV Models

Model	Marginal Likelihood
Stock Market	
GARCH-N	10252.536
GARCH-t	10338.708
EGARCH-N	10268.751
EGARCH-t	10345.472
SV-N	10355.871
SV-t	10349.855
ASV-N	10358.478
ASV-t	10352.638
Forex Market	
GARCH-N	15707.927
GARCH-t	16570.616
EGARCH-N	16788.658
EGARCH-t	17008.202
SV-N	17001.796
SV-t	16996.787
ASV-N	17009.383
ASV-t	17004.490

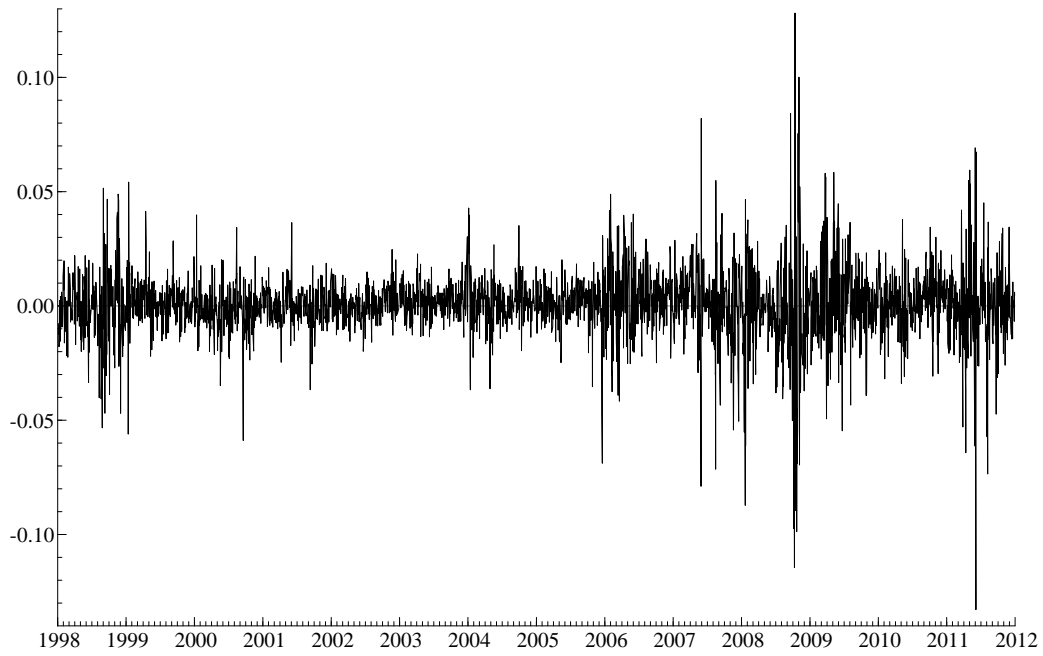


Figure 1a. Stock Market Returns

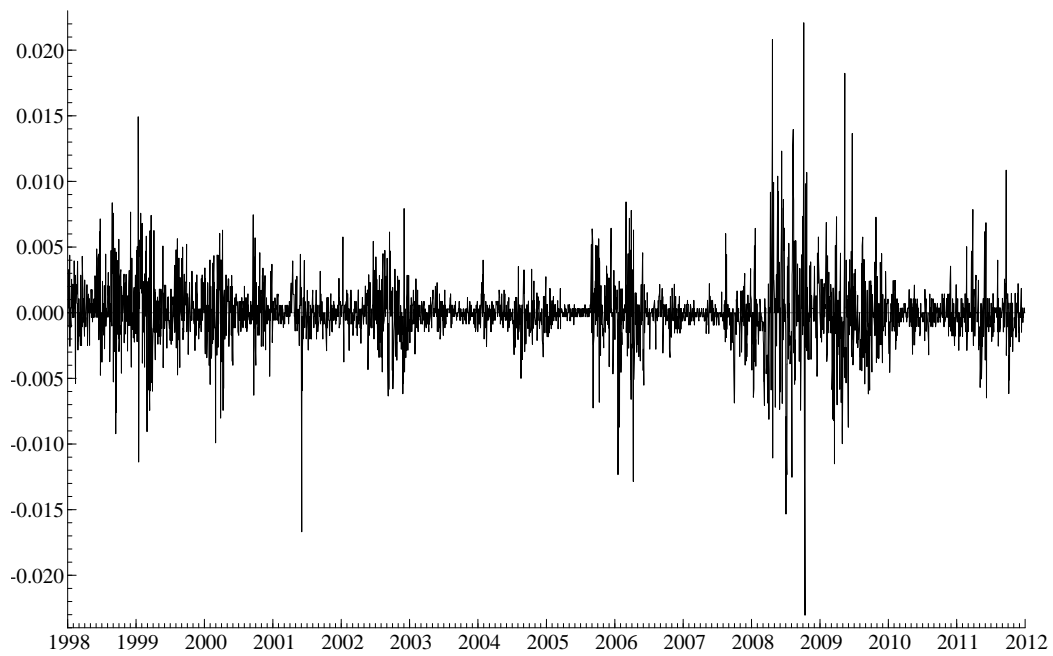


Figure 1b. Forex Returns

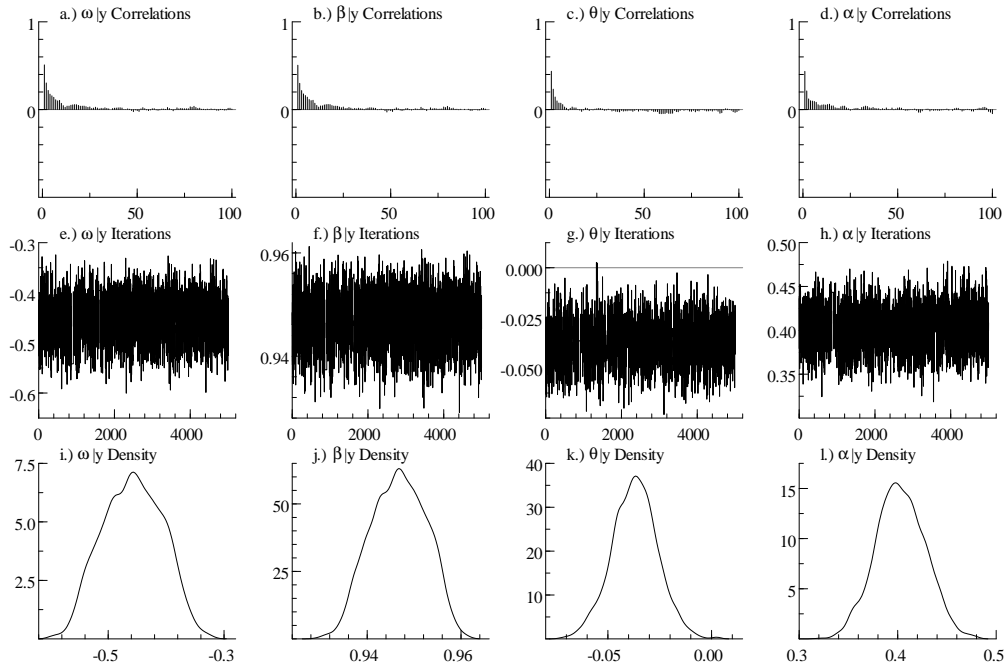


Figure 2a. Correlations, Iterations and Posterior Distributions for EGARCH-N Model in Sock Returns

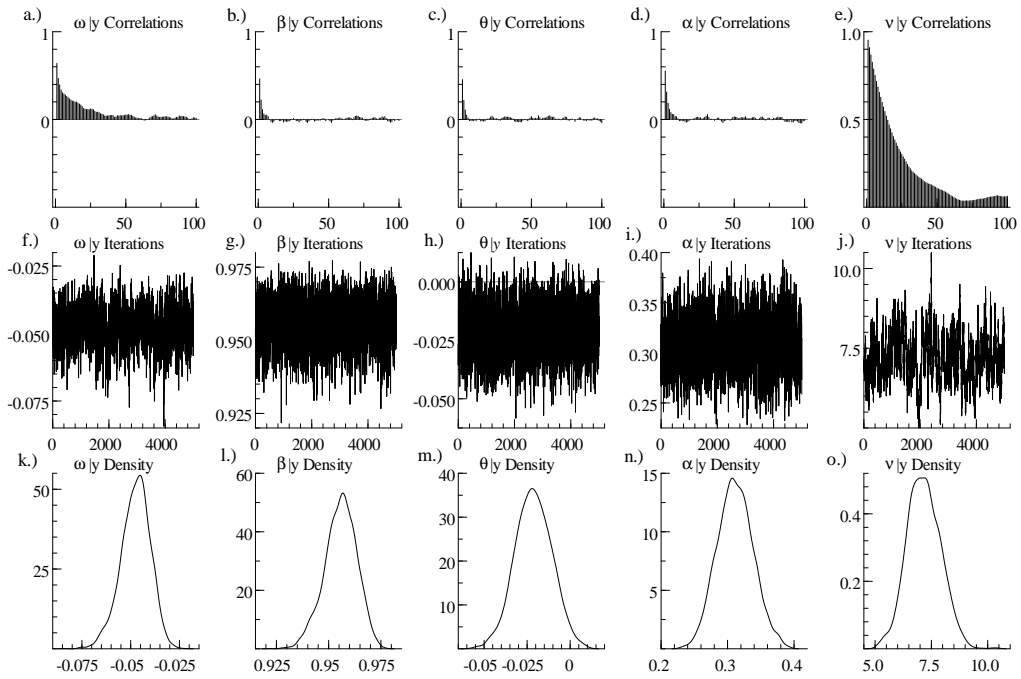


Figure 2b. Correlations, Iterations and Posterior Distributions for EGARCH-t Model in Stock Returns

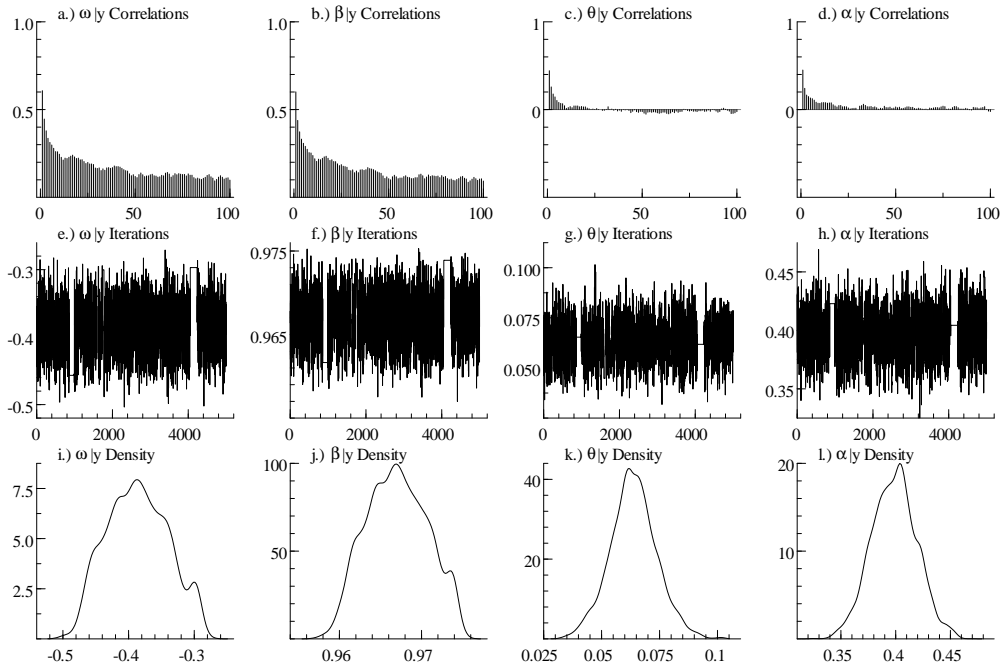


Figure 3a. Correlations, Iterations and Posterior Distributions for EGARCH-N Model in Forex Returns

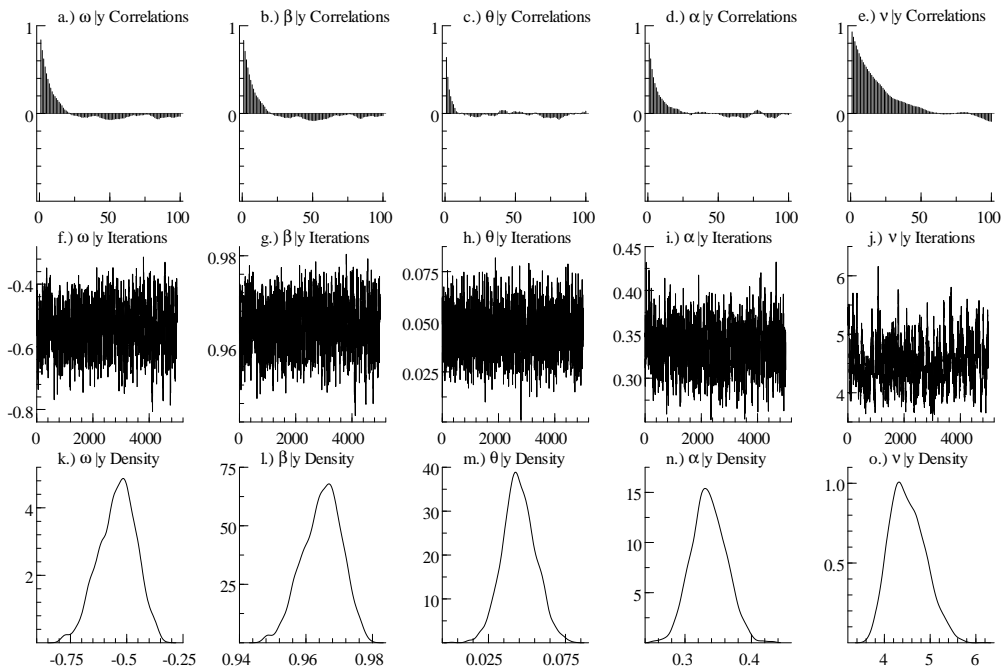


Figure 3b. Correlations, Iterations and Posterior Distributions for EGARCH-t Model in Forex Returns

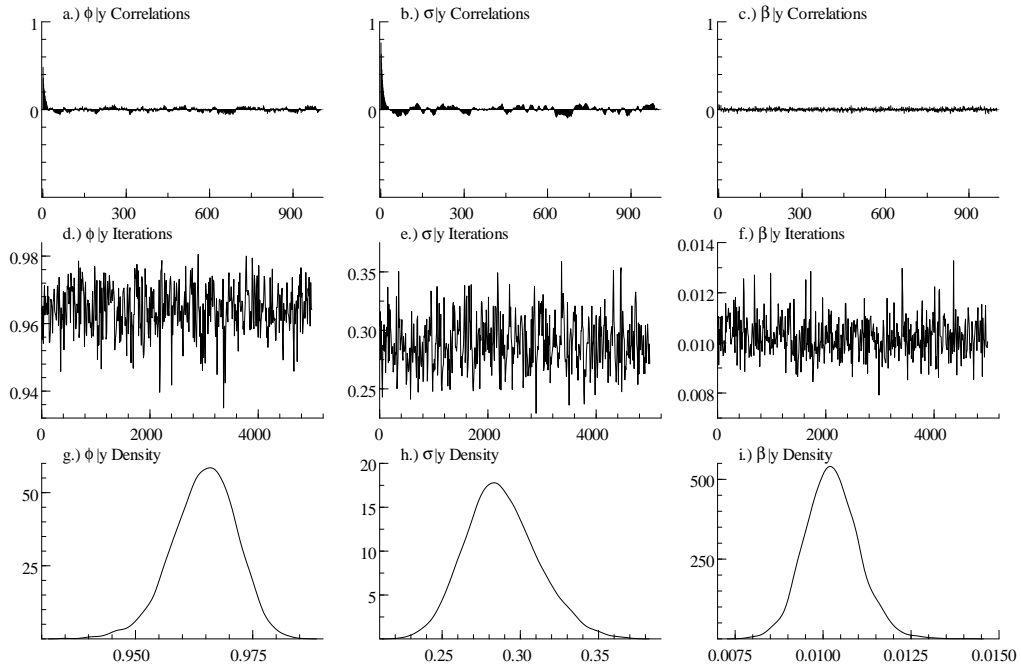


Figure 4a. Correlations, Iterations and Posterior Distributions for SV-N Model in Stock Returns

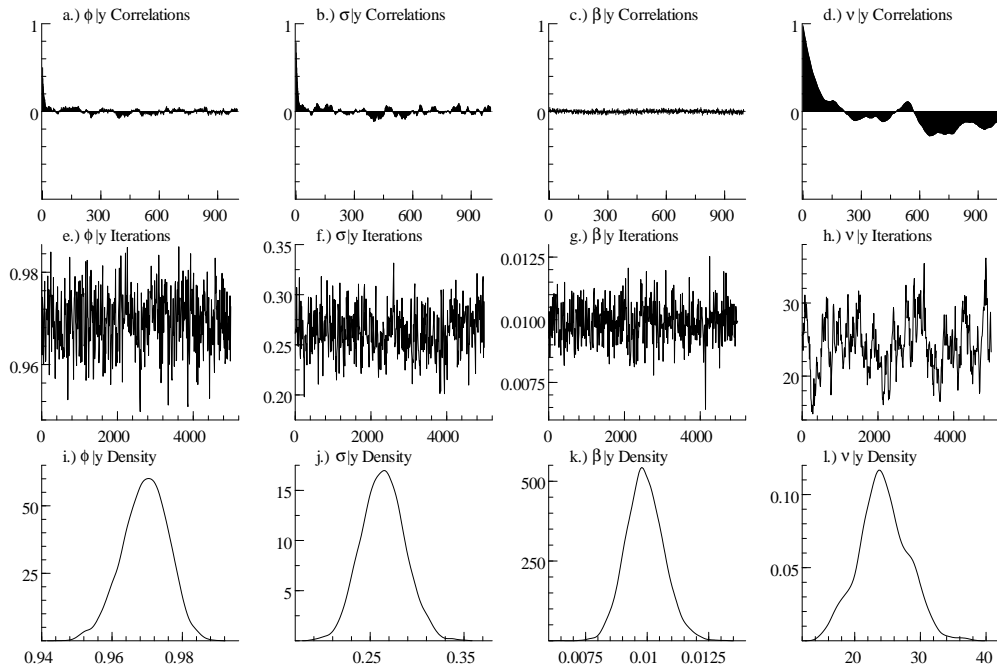


Figure 4b. Correlations, Iterations and Posterior Distributions for SV-t Model in Stock Returns

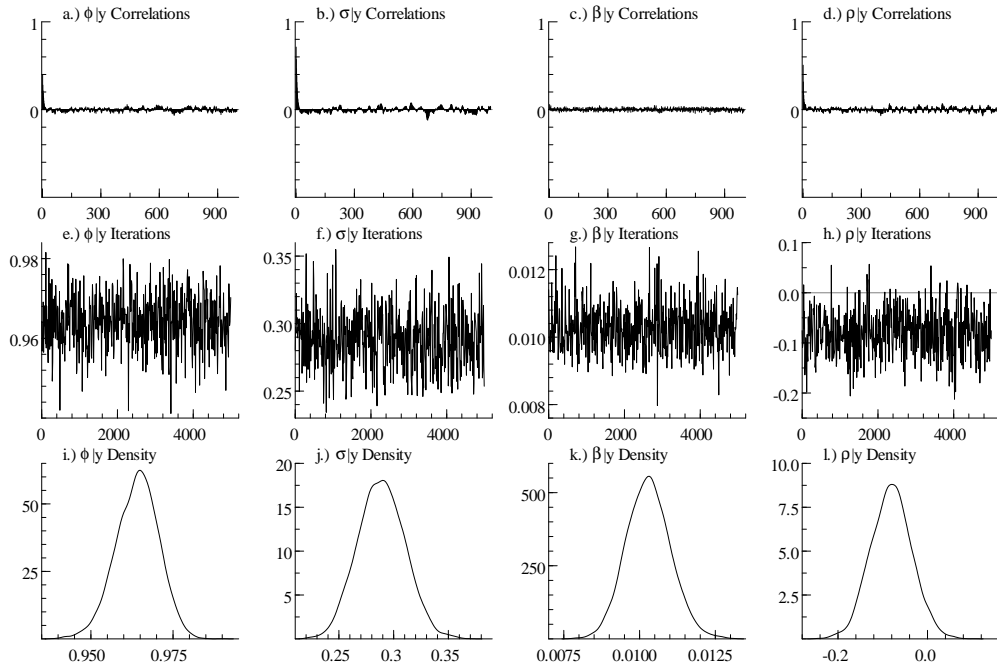


Figure 5a. Correlations, Iterations and Posterior Distributions for ASV-N Model in Stock Returns

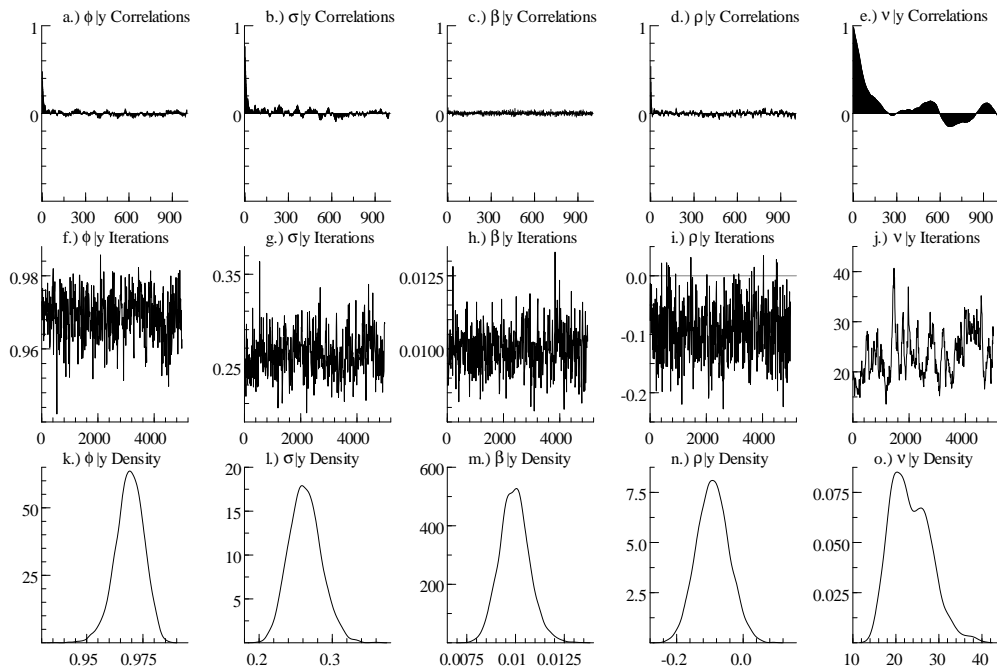


Figure 5b. Correlations, Iterations and Posterior Distributions for ASV-t Model in Stock Returns

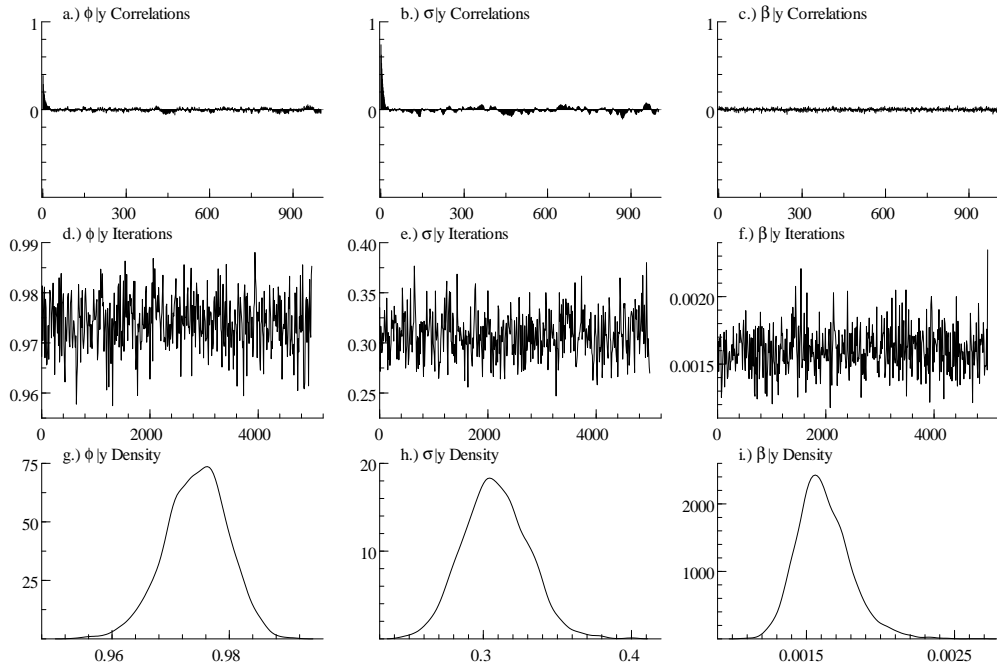


Figure 6a. Correlations, Iterations and Posterior Distributions for SV-N Model in Forex Returns

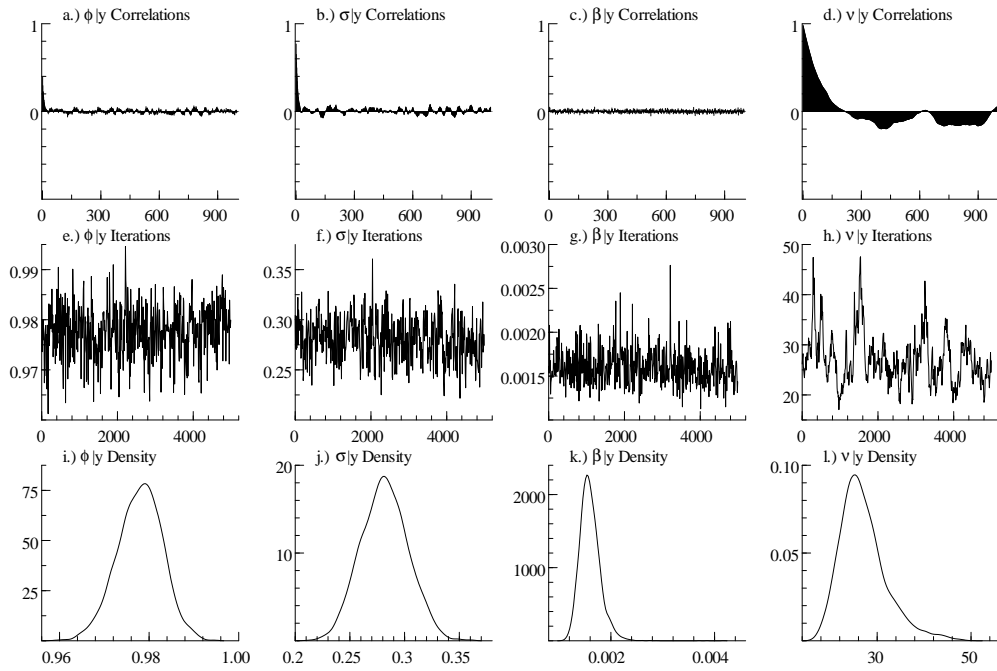


Figure 6b. Correlations, Iterations and Posterior Distributions for SV-t Model in Forex Returns

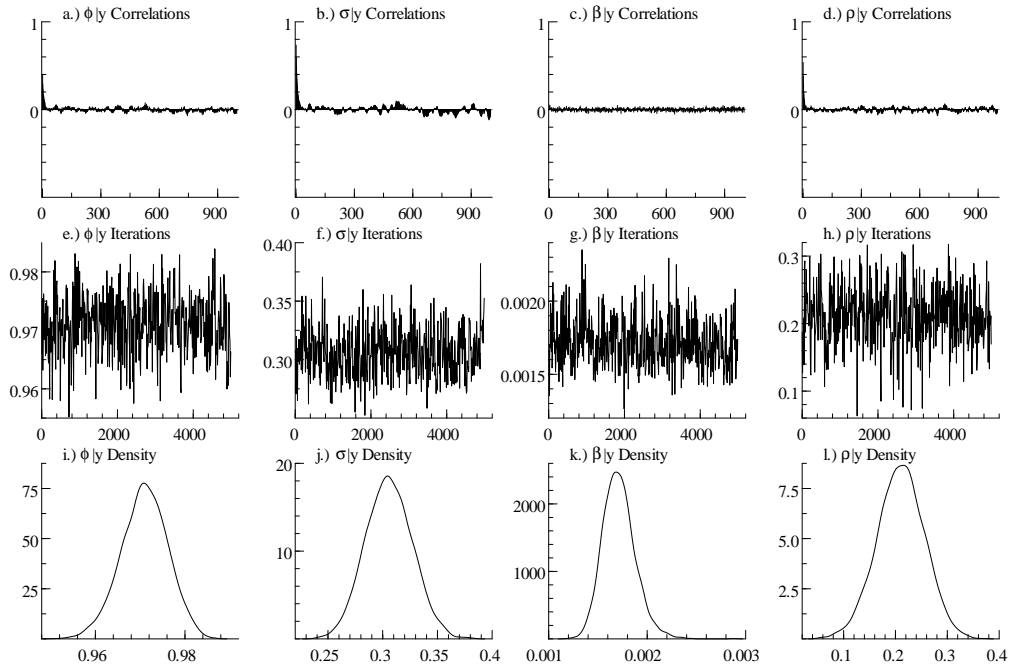


Figure 7a. Correlations, Iterations and Posterior Distributions for ASV-N Model in Forex Returns

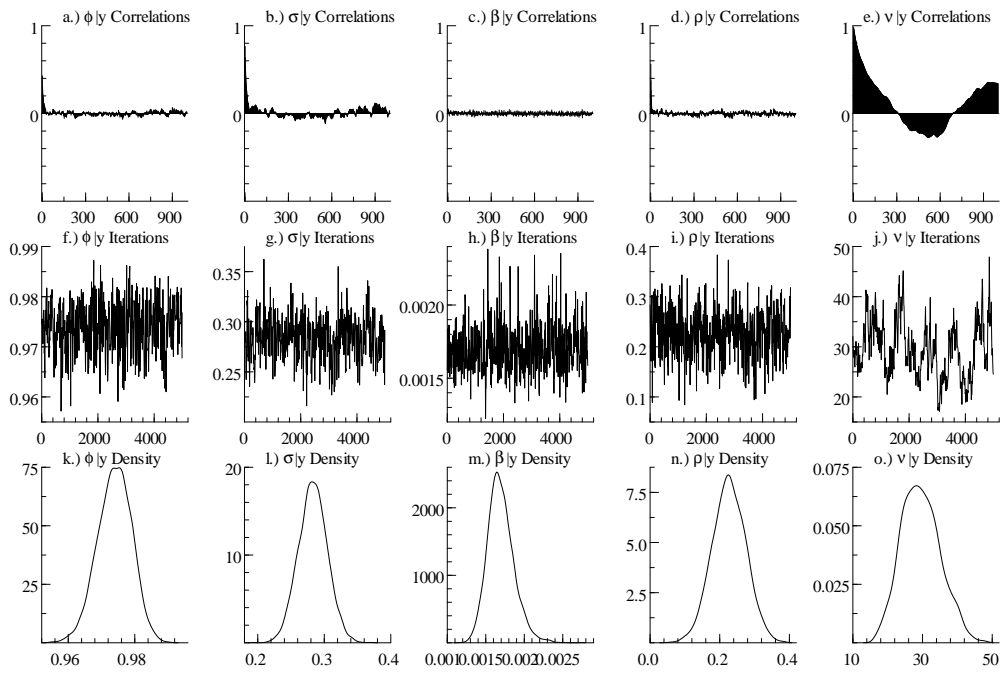


Figure 7b. Correlations, Iterations and Posterior Distributions for ASV-t Model in Forex Returns



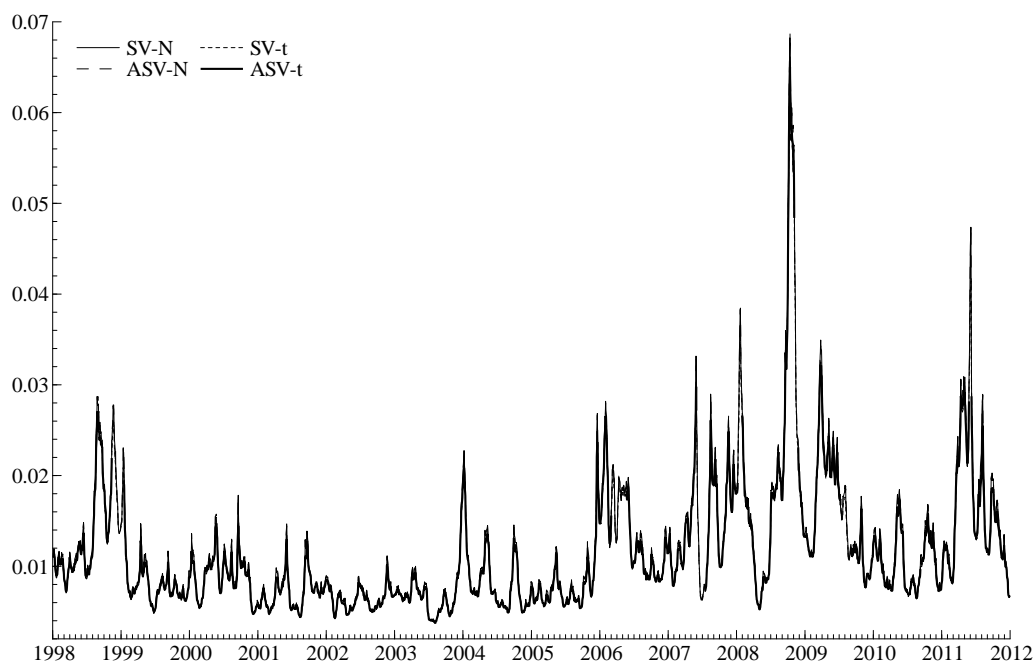


Figure 8a. Smoothed Volatility for SV, SVt, ASV, ASVt Models in Stock Returns

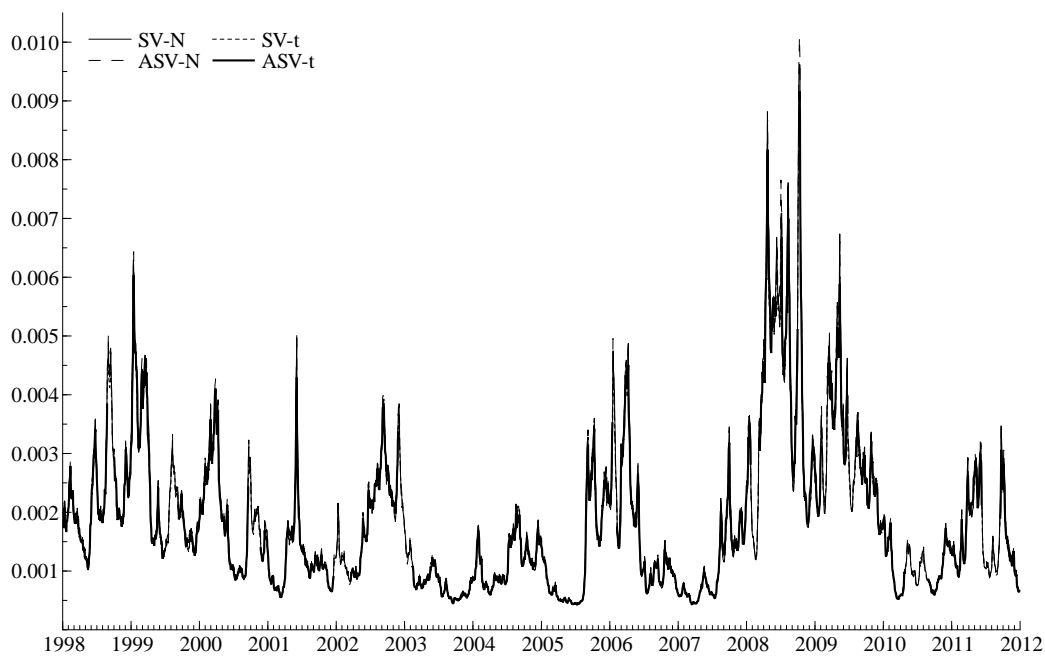


Figure 8b. Smoothed Volatility for SV, SVt, ASV, ASVt Models in Forex Returns

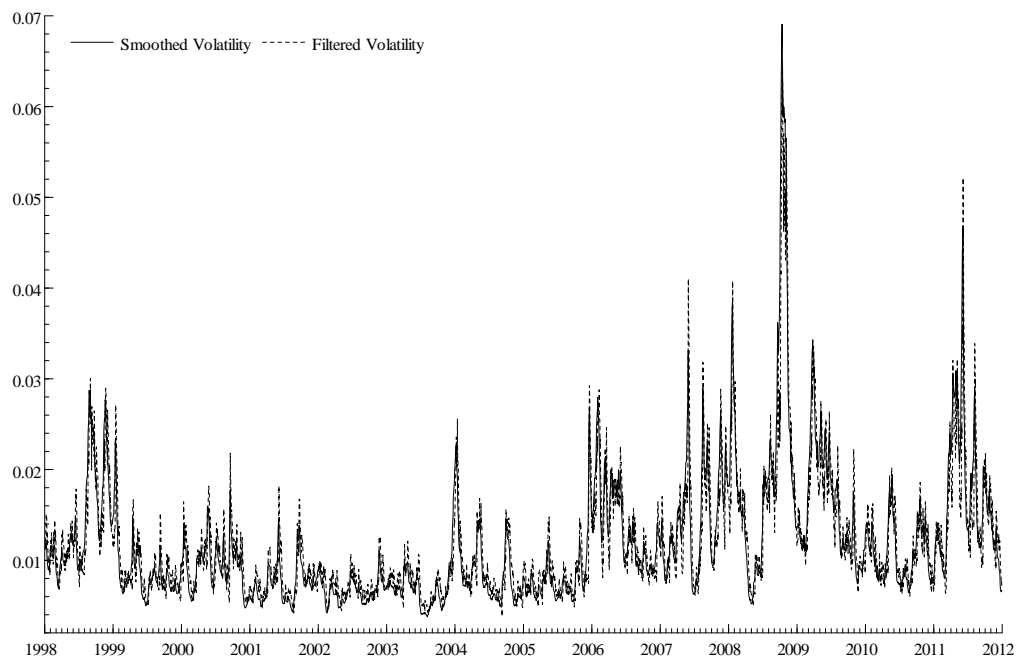


Figure 9a. Smoothed and Filtered Volatility for the ASV Model in Stock Returns

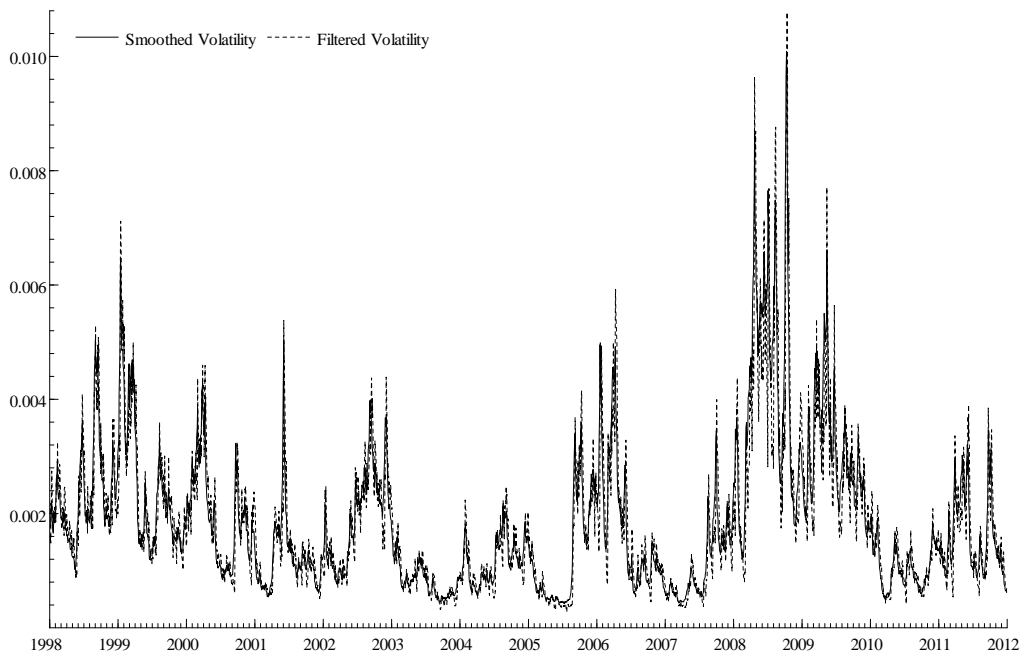


Figure 9b. Smoothed and Filtered Volatility for the ASV Model in Forex Returns

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