DOCUMENTO DE TRABAJO N° 415

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DOCUMENTO DE TRABAJO 415
http://files.pucp.edu.pe/departamento/economia/DDD415.pdf

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Lima, Departamento de Economía, 2016
(Documento de Trabajo 415)

PALABRAS CLAVE: Random Level Shifts, Long Memory, Forex Returns Volatility, Latin American Forex Markets, Time Varying Probability, Mean Reversion, Forecasting.

Hecho el Depósito Legal en la Biblioteca Nacional del Perú Nº 2016-03785.
ISSN 2079-8466 (Impresa)
ISSN 2079-8474 (En línea)

Impreso en Kolores Industria Gráfica E.I.R.L.
Jr. La Chasca 119, Int. 264, Lima 36, Perú.
Tiraje: 100 ejemplares

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Abstract

Following Xu and Perron (2014), this paper uses daily data for six Forex Latin American markets. Four models of the family of the Random Level Shift (RLS) model are estimated: a basic model where probabilities of level shift are driven by a Bernouilli variable but probability is constant; a model where varying probabilities are allowed and introduced via past extreme returns; a model with mean reversion mechanism; and a model incorporating these two features. Our results prove three striking features: first, the four RLS models fit well the data, with almost all the estimates highly significant; second, the long memory property disappears completely from the ACF, including the GARCH effects; and third, the forecasting performance is much better for the RLS models against an overall of four competitor models: GARCH, FIGARCH and two ARFIMA models.

JEL Classification: C22, C52, G12.

Keywords: Random Level Shifts, Long memory, Forex Return Volatility, Latin American Forex Markets, Time Varying Probability, Mean Reversion, Forecasting.

Resumen

Siguiendo el trabajo de Xu y Perron (2014), este documento utiliza datos diarios de volatilidades de retornos cambiarios para seis mercados de América Latina. Cuatro modelos del tipo Random Level Shifts (RLS) son estimados: un modelo básico donde las probabilidades de cambios de nivel son gobernadas por una variable del tipo Bernouilli pero dicha probabilidad es constante; un modelo donde las probabilidades son cambiantes en el tiempo y dependen de los retornos bursátiles extremos negativos del periodo anterior; un modelo con reversión a la media; y un modelo que incorpora los dos aspectos mencionados anteriormente. Los resultados sugieren tres importantes aspectos: el primero es que los cuatro modelos RLS ajustan bien los datos con prácticamente todos los estimados altamente significativos; segundo, la característica de larga memoria desaparece completamente de la ACF, incluyendo los efectos GARCH; y, tercero, la performance de los cuatro modelos en términos de predicción es buena contra diferentes modelos rivales como los modelos GARCH, FIGARCH, y dos modelos ARFIMA.

Clasificación JEL: C22, C52 G12.

Palabras Claves: Cambio de Nivel Aleatorios, Larga Memoria, Volatilidad de Retornos Cambiarios, Mercados Cambiarios en América Latina, Probabilidad Variante en el Tiempo, reversión a la Media, Predicción.

An Empirical Application of a Random Level Shifts Model with Time-Varying Probability and Mean Reversion to the Volatility of Latin-American Forex Markets Returns[1]

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1 Introduction

A sizeable branch of the econometric literature on time series argues that financial asset return volatilities exhibit long-term dependence. In formal terms, the definition of the long memory property is consistent with the notion that a time series has an autocorrelation function (ACF) that slowly decays in its lags; or equivalently, if its spectral density function has an infinite value at the frequency of zero. Another branch of the literature has proposed that long-memory behavior is spurious and due to the presence of rare level shifts. This idea extends that exposed by Perron (1989) who showed that structural change and unit roots are easily confused: when a stationary process is contaminated by structural changes, the estimate of the sum of its autoregressive coefficients is biased toward 1 and tests of the null hypothesis of a unit root are biased toward non-rejection. This phenomenon has been shown to apply to the long-memory context as well. That is, when a stationary short-memory process is contaminated by structural change in levels, the estimate of the long-memory parameter is biased away from 0 and the autocovariance function (and the ACF) of the process exhibits a slow rate of decay. Relevant references on this issue include Diebold and Inoue (2001), Engle and Smith (1999), Gourieroux and Jasiak (2001), Granger and Ding (1996), Granger and Hyung (2004), Lobato and Savin (1998), Mikosch and Stărică (2004a,b), Parke (1999) and Teverovsky and Taqqu (1997).

Recently, Lu and Perron (2010) directly estimate a structural model where the series of interest is the sum of a short-memory process and a jump or level shift component. This model is named the random level shift (RLS) model. In its basic specification, the probability of level shifts are considered constant. This model has been recently extended by Xu and Perron (2014) in order to allow for time varying probabilities for the level shifts and the introduction of a mean reversion mechanism. According to the RLS models (any of them), if the level shifts are taken into account, the presence of long memory disappears implying that the presence of long memory in standard models is spurious. Similar evidence applies to the presence of GARCH effects.

The presence of genuine long memory means that volatility has high persistence and shocks to this variable have lasting effects. In the RLS models, only the shocks that have permanent effects are the level shifts and the rest is a component of short memory. On the other hand, the level changes are important in themselves because they are associated with domestic or foreign financial crises or even to domestic issues affecting financial markets (such as electoral processes as in the case of Latin American countries).

[1] This paper is drawn from the Thesis of José Carlos Gonzáles Tanaka, Department of Economics, Pontificia Universidad Católica del Perú. We thank useful comments from Paul Castillo (Central Reserve Bank of Peru), Jiawen Xu (Shangai University of Finance and Economics), Zhongjun Qu and Pierre Perron (Boston University), Jorge Rojas (PUCP) and Patricia Lengua Lafosse (PUCP). Any remaining errors are our responsibility.

Our perspective is that there are enough episodes of turbulence in Forex markets in Latin America to support the use of the RLS models. In addition, empirical evidence for Latin American Forex markets is very scarce. In that sense, the contribution of this paper is fundamentally empirical. Our results may be summarized as follows: (i) the four RLS models fit well; (ii) the presence of level shifts is sporadic or rare but still significant; after taking this into account for these observations, no evidence of long memory is appreciated in the ACFs; (iii) there is no fractional integration evidence; and (iv) good performance of the RLS models in terms of forecasting for short, medium and long horizons.

The paper is structured as follows. Section 2 presents a brief revision of the literature. Section 3 presents the basic RLS model and describes the two extensions proposed by Xu and Perron (2014). In order to gain fluency and continuity in the text, a brief description of the estimation algorithm is relegated to the Appendix. Section 4 deals with the data and the results of the estimation of the different models. Moreover, a comparison with the ARFIMA(p,d,q), GARCH and Components GARCH (CGARCH) models is presented. Section 5 shows the prediction results, while Section 6 discusses the main conclusions.

2 Brief Literature Revision

The literature provides us with several possible formalizations for this definition; see McLeod and Hipel (1978), Taylor (1986), Dacarogna et al. (1993), Ding et al. (1993), Beran (1994), Robinson (1994), and Baillie (1996), among others. Following notations and definitions in Perron and Qu (2010), let $\{x\}_{t=1}^{T}$ be a time series that is stationary with spectral density function $f_x(\omega)$ at frequency ω, so x_t has long memory if $f_x(\omega) = g(\omega)\omega^{-2d}$, for $\omega \to 0$, where $g(\omega)$ is a smooth variation function in a vicinity of the origin, which indicates that for all real numbers t, it is proved that $g(t\omega)/g(\omega) \to 1$ for $\omega \to 0$. When $d > 0$, the function of the spectral density increases for frequencies increasingly close to the origin. The infinite rate that is divergent depends on the parameter value d. Besides, let $\gamma_x(\tau)$ be the ACF of x_t, so x_t has long memory if $\gamma_x(\tau) = c(\tau)\tau^{2d-1}$, for $\tau \to \infty$, where $c(\tau)$ is a smooth variation function. When $0 < d < 1/2$, the ACF decays at a slow rate that will depend on the parameter value of d.

Granger and Joyeux (1980) and Hosking (1981) introduce the ARFIMA(p,d,q) model as a parametric way of capturing long memory dynamics. There is also literature on semiparametric estimators of the fractional parameter d where the most used estimators are the proposed by Geweke and Porter-Hudak (1983) using the log-periodogram; see also Robinson (1995a), and the the local Whittle estimator of Kunsch (1987) and Robinson (1995b); see also Andersen et al. (2003). Another way to capture the long-memory behavior is by mixing it with GARCH effects, as in the Fractional Integrated GARCH (FIGARCH) model proposed by Baillie et al. (1996). In this model, the conditional variance of the process is assumed to have a slow hyperbolic rate of decay due to the influence of lagged squared innovations. The main characteristic of all these models is the assumption of long memory. Furthermore, Bollerslev and Mikkelsen (1996) propose the FIEGARCH model. In both of these two models, the fractional parameter is significant and, with the latter model, asymmetries are found in the series. In addition, Ding et al. (1993) conclude that the ACF of the absolute value of the returns is greater than the ACF of the returns, especially when $d = 1$. Besides, they propose the Asymmetric Power ARCH (APARCH) model in which they allow the series to be affected by asymmetric impacts in the variable.

Lobato and Savin (1998) apply a semiparametric test, which proves to be robust when there is weak dependence, to detect the presence of long-range dependence in the daily returns and squared returns of the S&P500 market. Since the null hypothesis is a short memory process, for the case of the Stock returns the null is not rejected, while for the squared returns and the absolute value of the returns the null is rejected. However, the authors state that the results may be spurious due to the non-stationarity of the series in the squared returns: when they partitioned the sample in two, with January of 1973 as a breakpoint, no evidence is found of structural break causing long memory. Teverovsky and Taqqu (1997), present a method that could distinguish between the effects of long memory and level shift. Gourieroux and Jasiak (2001) study the relationship between the presence of long memory and infrequent breaks by estimating the correlogram rather than the fractional parameter. They find that non-linear time series with sporadic breaks could have long memory. On the other hand, Diebold and Inoue (2001) find that the long memory property and the structural change phenomenon are related through the following models: the Markov-Switching model of Hamilton (1989) and the simple mixture permanent stochastic breaks model of Engle and Smith (1999). The authors' analysis shows that stochastic regime shifts are readily confused with long memory, even asymptotically, once it is assured that the structural break probabilities are small. Through Monte Carlo simulations, they argue that the confusion is not only a theoretical issue, but a reality in empirical economic and financial applications.

Other authors like Granger and Hyung (2004) have found evidence that the fractionally integrated models and the slow decay in the ACF are caused by infrequent breaks. Analytically, they show that structural breaks cause bias in the fractional parameter estimated through the method of Geweke and Porter-Hudak (1983), and that the ACF exhibits slow decay. To prove their analysis empirically, they compare the fractional integration and structural break models to analyze the absolute value of the daily S&P500 Stock returns from 1928 to 2002. They reach the conclusion that the long-memory presence could be highly dependent on the breaks that occur in the sample. Further analysis and evidence is found in Mikosch and Stărică (2004a, 2004b). See also Stărică and Granger (2005).

Following the above path, Perron and Qu (2010) propose a model and methodology to discern between level shifts and the long-memory property using the ACF, the estimates of the fractional parameter d, and the periodogram. These authors establish a simple mixture model that integrates a short-memory process with a random level shifts component affected by a variable of occurrence related to a Bernoulli Process. They apply the so-called RLS model to the log-squared returns of four major indices (AMEX, Dow Jones, NASDAQ and S&P500), concluding that their model best describes the volatility behavior. Meanwhile, Lu and Perron (2010), and Li and Perron (2013) apply the RLS model to the stock market and Forex returns, respectively. It is interesting to note that after accounting for level shifts, no evidence of long memory remains present and so too are GARCH effects eliminated. Xu and Perron (2014) extend the basic RLS model in two regards: (i) by introducing a time-varying probability, and (ii) a mean reversion mechanism. A final model is a mixture of these two models. By applying these different models to the above-mentioned data, the results reinforce the above-mentioned results.

In the case of Latin American financial markets, the recent models have been applied to different contexts. For example, Herrera Aramburú and Rodríguez (2014) opt for a testing approach to verify whether Peruvian financial markets present long memory. A similar approach is used by Pardo Figueroa and Rodríguez (2014). With respect to modelling, Ojeda Cunya and Rodríguez (2016) have applied the basic RLS to the Peruvian financial markets while Rodríguez and Tramontana (2015) have applied these tools to the Latin-American stock markets. The extended model proposed by Xu and Perron has been used by Rodríguez (2016) to analyze the stock markets in Latin America. The results obtained are similar to the original proposals of Lu and Perron (2010), and Xu and Perron (2014); that is, the artificial presence of long-memory behavior due to the (sporadic or rare) presence of level shifts.

3 Methodology

This Section presents the Basic RLS model that considers a constant probability of level shifts. Then, the two extensions to this model are presented. For non-specialized readers, the brief technical details related to the method and algorithm of estimation are relegated to the Appendix.

3.1 The Basic RLS Model

Following Lu and Perron (2010), we use a simple mixture model, which is a combination of a short-memory process and a level shift component that depends on a Bernouilli distribution. Following same notation, the basic RLS is specified as follows:

$$\begin{aligned} y_t &= a + \tau_t + c_t, \\ \tau_t &= \tau_{t-1} + \delta_t, \\ \delta_t &= \pi_t \eta_t, \end{aligned} \tag{1}$$

where a is a constant, τ_t is the level-shift component, c_t is the short-memory component, and π_t is a Bernouilli variable, which takes the value of 1 with probability α and the value of 0 with probability $(1-\alpha)$. In this way, following the third expression in (1), when π_t assumes the value of 1, a random level shift η_t occurs with a distribution $\eta_t \sim i.i.d.\ N(0, \sigma_\eta^2)$. Note that the process δ_t can be described as $\delta_t = \pi_t \eta_{1t} + (1 - \pi_t)\eta_{2t}$, with $\eta_{it} \sim i.i.d. N(0, \sigma_{\eta_i}^2)$ for $i = 1, 2$ and $\sigma_{\eta_1}^2 = \sigma_\eta^2$, $\sigma_{\eta_2}^2 = 0$. The short-memory process (in its general form) is defined by the process $c_t = C(L)e_t$, with $e_t \sim i.i.d.$ $N(0, \sigma_e^2)$ and $E|e_t|^r < \infty$ for values $r > 2$, where $C(L) = \sum_{i=0}^{\infty} c_i L^i$, $\sum_{i=0}^{\infty} i|c_i| < \infty$ and $C(1) \neq 0$. Moreover, it is assumed that π_t, η_t and c_t are mutually independent. Based on the results of Lu and Perron (2010) and Li and Perron (2013), even when it would be useful to consider the component e_t as a noise variable, in this paper we model this component as an AR(1) process, that is, $c_t = \phi c_{t-1} + e_t$.[2]

3.2 Extensions to the Basic RLS Model

As pointed out in Xu and Perron (2014), level shifts usually occur in clusters in certain periods of time related to financial crisis. This phenomenon of clustering indicates that level shifts are not *i.i.d.*, but that the probability of these shifts varies in accordance with economic, political, and social conditions in the country.

Following on from the notation used in Xu and Perron (2014), the probability of level shift is defined as $p_t = f(p, x_{t-1})$, where p is a constant and x_{t-1} are the covariables that help to better predict the probability of level shifts. According to the study by Martens et al. (2004), there is a strong relationship between current volatility and past returns, also known as the leverage effect. This effect is modeled through the *news impact curve* proposed by Engle and Ng (1993) as follows:

[2] Note that this model can be extended to model the short-memory component as an ARMA(p,q) process. However, the estimates show no statistical significance beyond an AR(1) process.

$\log(\sigma_t^2) = \beta_0 + \beta_1 \mathbf{1}(r_{t-1} < 0) + \beta_2 |r_{t-1}| \mathbf{1}(r_{t-1} < 0)$, where σ_t^2 represents the volatility and $\mathbf{1}(A)$ is the indicator function that takes the value of one when the event A occurs. Given that our objective in this part of the study is not to model the volatility but the probability of level shifts, the variable x_{t-1} is not represented by past returns (r_{t-1}). Instead, extreme past returns that are below a threshold κ will be used. Therefore, we employ the returns that belong to 1%, 2.5% and 5% of the distribution of the returns ($\kappa = 1.0\%, 2.5\%, 5.0\%$). Thus, the probability of level shifts is given by:

$$f(p, x_{t-1}) = \left\{ \begin{array}{ll} \Phi(p + \gamma_1 \mathbf{1}\{x_{t-1} < 0\} + \gamma_2 \mathbf{1}\{x_{t-1} < 0\} |x_{t-1}|) & \text{for } |x_{t-1}| > \kappa \\ \Phi(p) \quad \text{other cases,} & \end{array} \right\}, \tag{2}$$

where $\Phi(.)$ is a function of Normal accumulated distribution, with which I ensure that $f(p, x_{t-1})$ is between 0 and 1.

The second extension of Basic RLS models is that level shifts occur around a mean; that is, each time a level shift occurs and the volatility of the series increases, a similar change occurs in the opposite direction, which makes the mean of the volatility remains at a given value. This process of mean reversion is modeled as follows: $\eta_{1t} = \beta(\tau_{t|t-1} - \overline{\tau}_t) + \widetilde{\eta}_{1t}$, where $\widetilde{\eta}_{1t}$ is distributed Normally with mean 0 and variance σ_η^2, $\tau_{t|t-1}$ is the estimated level shift component at time t, and $\overline{\tau}_t$ is the mean of all level-shift components estimated from the start of the sample to time t. The process of mean reversion occurs when $\beta < 0$ and this parameter represents the velocity at which the volatility returns to its mean. The final model combines the two stated characteristics, giving us four models to estimate.

4 Empirical Results

In this Section we briefly describe the data. We also analyze the results in terms of the presence of both long-memory behavior and GARCH effects. We also conduct a forecasting comparison exercise.

4.1 The Data

We use daily data for six Latin American Forex markets. The returns are calculated as $r_t = \ln(P_t) - \ln(P_{t-1})$ where P_t is the value of the exchange rate of the respective Latin America country against the US dollar. Following recent literature (see Lu and Perron (2010), Li and Perron (2010), and Xu and Perron (2010), among others), we model log-absolute returns[3]. When returns are zero or close to it, the log-absolute transformation implies extreme negative values. Using the estimation method described above, these outliers would be attributed to the level shifts component and would thus bias the probability of shifts upward. To avoid this drawback, we bound absolute returns away from zero by adding a small constant, i.e., we use $y_t = \log(|r_t| + 0.001)$, a technique introduced to the

[3] Using this measure has two advantages: (i) it does not suffer from a non-negativity constraint as do, for example, absolute or squared returns. In fact, it is a similar argument as that used in the EGARCH(1,1) model proposed by Nelson (1991). The dependent variable is $\log(\sigma_t^2)$ in order to avoid the problems of negativity when the dependent variable is σ_t^2 as in the standard GARCH models and other relatives models; (ii) there is no loss related to using square returns in identifying level shifts since log-absolute returns are a monotonic transformation. It is true that log-absolute returns are quite noisy, but this is not problematic since the algorithm used is robust to the presence of noise.

stochastic volatility literature by Fuller (1996). The results are robust to alternative specifications; for example, using another value for this so-called offset parameter, deleting zero observations, or replacing them with a small value.

With respect to the construction of the volatility series, several points should be noted. We use daily returns as opposed to realized volatility series constructed from intra-daily high frequency data, which has recently become popular. Even though it is true that realized volatility series are a less noisy measure of volatility, their use would be problematic in the current context for the following reasons: (i) these series are typically available only for a short span, whereas the use of a long span is imperative in making reliable estimates of the probability of occurrence of level shifts, given that level shifts are relatively rare; (ii) such series are available only for specific assets, as opposed to market indices. In our framework, the intent of the level shift model is to have a framework which allows for special events affecting overall Forex markets. Using data on a specific asset would confound such market-wide events with idiosyncratic ones associated with the particular asset used; (iii) we wish to re-evaluate the adequacy of GARCH models applied to daily returns when taking into account the possibility of level shifts. Hence, it is important to have estimates of these level shifts for squared daily returns which are equivalent to those obtained using log-absolute returns.

The data sample is as follows: Argentina (02/01/2002-02/01/2014; 2958 observations), Brazil (01/04/1999-02/07/2014; 3785 observations), Chile (01/04/1993-02/07/2014; 5282 observations), Colombia (08/20/1992-02/07/2014; 5259 observations), Mexico (01/02/1992-02/07/2014; 5636 observations) and Peru (01/03/1997-07/02/2014; 4251 observations).

Table 1 sets out summary statistics of the volatility series and shows their unconditional distribution characteristics. The six Forex return volatility series have similar characteristics: mean, standard deviation, and extreme values. All of them show a positive skewness; that is, a right-tailed distribution, while Argentina has a markedly higher value than the other series. The kurtosis of Brazil, Chile and Colombia are less than 3. However for the other countries, the results are greater.

Figure 1 illustrates the evolution and behavior of the Forex returns, and we observe that Argentina and Mexico have less turbulence than the other countries. A common characteristic among all countries is the great variation of the returns in the financial crisis of 2008. Peru, in contrast to the others, does not post big extreme values (negative or positive) as well as the other countries. Figure 2 shows the well-known fact about the ACF of financial volatility series: long memory behavior. For all countries there is little accommodation of the confidence bands, which means all values of the autocorrelations are significant. For more details about stylized facts in the Peruvian Forex market, see Humala and Rodríguez (2013).

4.2 Results of the Estimations

Four models are estimated: The Basic RLS, the Threshold $\kappa\%$ RLS, The Mean Reversion RLS, and the Modified RLS. The results of these models are presented in Table 2, 3, 4, and 5, respectively.

Table 2 presents the results from the estimation of the Basic RLS model. All parameters are significant even at a level of significance of 1%. Argentina and Mexico clearly show a great dispersion from the mean in the level shift, σ_η, unlike the other countries. The σ_e estimate is very similar across all countries with the exception of Argentina and Peru, which have lower estimates. The estimates of the AR(1) coefficient is significant only for Colombia, Mexico and Peru. The estimates of the jump probability is close to 1.5% for all countries except for Mexico. Given this number and the sample size, we have the number of breaks for each country's volatility: 44, 62, 45, 99, 17 and 71 for Argentina, Brazil, Chile, Colombia, Mexico, and Peru, respectively. For the last country, our results are consistent with those found in Ojeda Cunya and Rodríguez (2016) even though we have more observations. For the other countries, the estimates of the level shifts are in accordance with those found by Rodríguez and Tramontana Tocto (2015).

In Table 3, we present the estimation results when a time-varying probability mechanism is incorporated into the RLS model. For each series, we consider three different threshold levels ($\kappa\% = 1\%, 2.5\%, 5\%$) in order to evaluate the robustness of our results. The estimate of σ_η clearly shows great similarities for all the countries with respect to the Basic RLS; however, our second model presents relatively smaller values.

When it comes to the estimates of p, the value estimated from the varying probability can be converted to a constant probability that could resemble the value of the Basic RLS α. Accordingly, our α that results from this constant for Argentina is 0.015, 0.017 and 0.015 for the 5, 2.5, and 1% thresholds, respectively. For Brazil, 0.017 for all thresholds. For Peru, 0.016, 0.019 and 0.018 for 5, 2.5, and 1% thresholds, respectively. For Chile, 0.008, 0.008, and 0.009 for 5, 2.5, and 1% thresholds, respectively. For Colombia, p gives a probability of 0.022, 0.029, and 0.029 for 5, 2.5 and 1% thresholds, respectively. For Mexico, p gives 0.002, 0.002 and 0.003 for 5, 2.5, and 1% thresholds, respectively. As shown, the estimate of the probability (p) gives results that are very similar to the Basic RLS that is set out in Table 2.

The estimates of γ_1 and γ_2, which correspond to the components $\mathbf{1}\{x_{t-1} < 0\}$ and $\mathbf{1}\{x_{t-1} < 0\}|x_{t-1}|$ (respectively) in the specification of the time-varying probability, are positive in all cases, which is in keeping with our specification. At $\kappa = 5\%$, γ_1 (γ_2) prove significant for all countries except for Argentina (Argentina, Brazil, and Peru). Since γ_1 and γ_2 are not significant at this threshold, Argentina is the only country that presents clear evidence of no varying-jump probability. At a threshold of 2.5%, γ_1 (γ_2) shows significance for all countries except for Colombia (Argentina). At a threshold of 1%, which will be of interest for the fourth model, the results present γ_1 as significant for all the countries except for Argentina, Colombia and Peru. As to γ_2, all the estimates are significant.

In Brazil, γ_1 is similar for $\kappa = 2.5\%, 1.0\%$ thresholds (although at $\kappa = 1\%$ it is relatively greater), and the same is true for γ_2. Nevertheless, it is not the case at $\kappa = 5\%$, but the estimates here are not significant. In Chile, γ_1 is greater at 2.5% and 1% than at 5%. For γ_2, the results are similar at thresholds of 5% and 2.5%; at 1%, it is greater. In Colombia, at 1% and 2.5%, γ_2 results greater than at 1%. γ_1 is significant just at 5%. In Mexico, for γ_1 at 5%, the estimate is less than at the other thresholds. For γ_2 at 2.5%, the estimate is less than at the other thresholds. Finally, in Peru, for γ_1, at 5% and 2.5% the estimates vary greatly; for γ_1, at 2.5% the estimate is greater than at 1%. At 5%, the result is much bigger, however, not significant.

In Table 4, we show the results in cases where only a mean reversion mechanism is incorporated in the RLS model. In this case, all the estimates of β are significantly negative. This clearly indicates that the mean-reverting process is present in the volatility series. As to σ_η, it is highly significant for Argentina, Chile, Mexico and Peru. The value is less for Argentina and Mexico with respect to the Basic RLS; however, for the other countries, the estimate is even less.

Table 5 presents the estimates of the Modified RLS combining both the time varying jump probability and the mean reversion mechanism using a threshold value of $\kappa = 1\%$. First of all, the estimates β are again significantly negative, which tells us this variable is present for the level-shift components. Besides, this variable in both Tables 4 and 5 has similar results, which confirms the robustness of our findings. With respect to the estimates of γ_1 and γ_2, these are positive and significant in all cases, except for γ_1 in Argentina. For σ_η and σ_e, the estimates for each country resemble the estimates of Table 4; this clearly tell us that the mean reverting process has a high participation in the model, in contrast to Tables 2 and 3.

Figure 4 shows the ACF of the short-memory components only for the Basic RLS model. It is calculated as the residuals between the volatility and the level-shift component. Because we are only using the results from the Basic RLS models, the level-shift component has been calculated using the method of Bai and Perron (2003)[4]. The message of the Figure 4 is that any evidence of long memory behavior disappears completely. The short memory presents no evidence of this behavior once the level shifts are taken into account by extracting them from the volatility series.

This new evidence provides the same conclusions as those reached in Lu and Perron (2010), Li and Perron (2013), and Xu and Perron (2014), but using Forex data from emerging economies. The message is that long memory is artificially present in these financial markets. However, if we model and extract the level shifts, this behavior is discarded.

4.3 Effect of Level Shifts on Long Memory and ARFIMA Models

In order to confirm our results more clearly, we proceed to estimate two models: the ARFIMA(0,d,0) and the ARFIMA(1,d,1). We estimate these models to our volatility variable and to the short-memory component obtained through the four RLS models. The results were very similar, so for the sake of saving space, we only show the estimation with the short-memory component of the Basic RLS model where the short memory component is extracted using the approach of Bai and Perron (2003).

The results are presented in Table 6. For the ARFIMA(0,d,0), the estimates of the parameter d fluctuate between 0.197 and 0.291 and are significant in all countries. Nonetheless, when we assess the short-memory component, which reflects the extraction of the level shifts, we can see that the value of d becomes negative in all cases, except for Mexico, where it is nonetheless small. These results show that the time series no longer present long-range dependence. In the case of the ARFIMA(1,d,1) model and in the volatility series, the estimates of the parameter d again shows a value that signals long-memory behavior. On the other hand, in the short-memory component, antipersistence is clearly stated with a large negative value of d. By way of conclusion, it can be stated that the long-memory behavior present in the volatility of the Forex time series is artificially introduced by the presence of rare level shifts. After accounting for these, no evidence of long memory is found any longer.

4.4 Effect of Level Shifts in GARCH, FIGARCH and CCGARCH

Given the above conclusion, it would be interesting to analyze the effect of level shifts within the presence of conditional heteroskedasticity. There is some consensus that stock and Forex returns exhibit conditional heteroskedasticity. For that reason, the GARCH(1,1) model introduced by Bollerslev (1986) has been extensively used to model these returns and volatility. Although Lamoureux and Lastrapes (1990) proposed the conclusion that structural changes in the level of variance can magnify the evidence of conditional heteroskedasticity, it was not until Lu and Perron

[4] We also have estimates of the short-memory component extracted from the other three RLS models. The smoothed estimates are very similar to the estimate obtained using the Basic RLS model. In order to save space, we exclude these Figures. All these materials are available upon request.

(2010) that an assessment was presented to prove whether these regime changes can completely eliminate all traces of conditional heteroskedasticity. In order to compare the estimates with the CGARCH model, we include the dummy variables associated with the level shifts detected using the method of Bai and Perron (2003). We then apply five models: a GARCH, a FIGARCH, a CGARCH, a CGARCH with short-memory estimated using the method of Bai and Perron (2003), and a CGARCH with the short memory calculated using a smoothed level-shift component ($\hat{\tau}_t$).

For the demeaned return process $\widetilde{r}_t$, the GARCH (1,1) model is:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \\ \sigma_t^2 &= \mu + \beta_r \widetilde{r}_{t-1}^2 + \beta_\sigma \sigma_{t-1}^2, \end{aligned} \tag{3}$$

where ϵ_t is *i.i.d.* Student-t distributed with mean 0 and variance 1.

The second model is a FIGARCH:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \\ (1-L)^d \sigma_t^2 &= \mu + \beta_r \widetilde{r}_{t-1}^2 + \beta_\sigma \sigma_{t-1}^2. \end{aligned} \tag{4}$$

The third model is a Component GARCH (CGARCH) with the following specification:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \\ n_t &= \mu + \rho(n_{t-1} - \mu) + \psi(\widetilde{r}_{t-1}^2 + \sigma_{t-1}^2), \\ (\sigma_t^2 - n_t) &= \beta_r(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_\sigma(\sigma_{t-1}^2 - n_{t-1}). \end{aligned} \tag{5}$$

Also, we employ a CGARCH model with the level-shift component extracted using the method of Bai and Perron (2003). We incorporate these features with the following specification:

$$\begin{aligned} \widetilde{r}_t &= \sigma_t \epsilon_t, \\ (\sigma_t^2 - n_t) &= \beta_r(\widetilde{r}_{t-1}^2 - n_{t-1}) + \beta_\sigma(\sigma_{t-1}^2 - n_{t-1}), \\ n_t &= \mu + \rho(n_{t-1} - \mu) + \psi(\widetilde{r}_{t-1}^2 + \sigma_{t-1}^2) + A_t, \end{aligned} \tag{6}$$

where $A_t = \sum_{i=2}^{m+1} D_{i,t}\gamma_i$ with $D_{i,t} = 1$ if t is in regime i, that is, $t \in \{T_{i-1}+1, \ldots, T_i\}$, and 0**.** Otherwise, with T_i ($i = 1, \ldots, m$) being the break dates obtained using the method of Bai and Perron (2003) with the change in long-run mean (again $T_0 = 0$ and $T_{m+1} = T$, the number of breaks is obtained from the point estimate of α). The coefficients γ_i, which index the magnitude of the shifts, are parameters that are going to be estimated with the others, while the number of breaks is obtained from the point estimate of α. We also include estimations using a CGARCH where the level-shift component has been extracted using a smoothing procedure. In this case, we replace $A_t = \hat{\tau}_t$.

The results are shown in Table 7. In the cases of the GARCH (1,1) models, the two parameters (β_σ and β_r) are highly significant and together sum close to 1. The second parameter β_σ is higher implying a strong persistence in the variance of the Forex returns. In the case of the estimates of the FIGARCH model, the estimates of the fractional parameter d is greater than 0.5 in all countries. According to this fact, long memory is present in the behavior of the volatility of the Forex returns. The CGARCH model is also estimated without the level shifts according to equation (5). The results gives similar information as before. In this case, the estimates of the parameter ρ results very close to 1 implying strong persistence in the volatilities.

The results obtained with the two last specifications of the CGARCH model are quite different. In the case of the CGARCH using the level-shift component estimated using the method of Bai and Perron (2003), we find across all countries that the estimates of the parameter β_σ are not significant. The estimate of β_r is not significant in the case of Peru alone; however, the coefficient itself presents little value. Besides, and interestingly, the estimates of ρ are now well below one, with an average value of 0.55 for all countries. In the last experiment, a CGARCH model is estimated with the level shift component estimated using a smoothing kernel. The results are quite similar, which testifies to their robustness.

Additional evidence is obtained from the estimates of the half-life of the shocks. The half-life for the GARCH models shows an infinite persistence because the sum between β_σ and β_r is greater than one or very close to one. For the first CGARCH (without level shifts), the average for the six countries shows a number of around 220 days, evidencing the long-memory behavior. However, when we incorporate the level shifts using the method of Bai and Perron (2003), the half-life of the shocks is around 1.34 days on average for countries. The half-life for the last CGARCH (with the smoothed level-shift component) has a similar value: 1.01 days. These last two numbers stand as clear support of the hypothesis that long-memory is confused with structural breaks or rare level shifts.

4.5 Forecasting Performance

We use the following forecasting horizons: $\tau = 1, 5, 10, 20, 50, 100$. The mean square forecast error (MSFE) criterion proposed by Hansen and Lunde (2006) and Patton (2011), is defined by: $MSFE_{\tau,i} = \frac{1}{T_{out}} \sum_{t=1}^{T_{out}} (\bar{\sigma}^2_{t,\tau} - \bar{y}_{t+\tau,i|t})^2$ where T_{out} is the number of forecasts (or number of observations left to forecast), $\bar{\sigma}^2_{t,\tau} = \sum_{s=1}^{\tau} y_{t+s}$, and $\bar{y}_{t+\tau,i|t} = \sum_{s=1}^{\tau} y_{t+s,i|t}$ with i indexing the model. The relative performance of models i and j at time t is defined as: $d_{ij,t} = (\bar{\sigma}^2_{t,\tau} - \bar{y}_{t+\tau,i|t})^2 - (\bar{\sigma}^2_{t,\tau} - \bar{y}_{t+\tau,j|t})^2$. The different model forecasting performances are evaluated and compared using the 10% model confidence set (MCS) of Hansen et al. (2011). The MCS offers better model evaluation by showing the best model in cases where the data are quite informative. However, where this is not the case, it shows various models as the best ones.

We have carried out the forecasting procedure as follows: since we use countries with few total observations, we keep only 500 and 1028 observations for Argentina and Brazil, respectively; we choose these numbers based on the beginning of a particular year for each country. We keep the last 2022, 1982, 2066 and 2041 observations for Chile, Colombia, Mexico and Peru respectively. Therefore, the start date for the forecasts are: 01/17/2012 (Argentina), 01/04/2010 (Brazil), 01/03/2006 (Chile and Colombia) and 01/02/2006 (Mexico and Peru). The reasons for using this period is that it contains a range of very different calm and turbulent episodes, including the last financial crisis of 2008 and the later period of quantitative easing applied in the United States. We estimate the models just once without the last observations chosen for each country. The forecasts are then made conditional on the parameter estimates obtained.

For the comparison with other models, we proposed two different volatilities and four different competitor models. With respect to the volatilities, the first one is that used throughout the paper, which is the logarithm of absolute value of the returns. The predictions are obtained using the equation of $\hat{y}_{t+\tau|t}$. As in the studies in Ojeda Cunya and Rodríguez (2016), Rodríguez (2016), and Rodríguez and Tramontana Tocto (2015), for this specification, the comparisons are made using the four RLS models, the ARFIMA(0,d,0) and the ARFIMA(1,d,1) models.

The second volatility series used is that of squared returns. In this case, we incorporate the GARCH and FIGARCH as another two competitor models. In this case, because our RLS models were estimated with the first volatility type, we proceed to make some transformations to obtain the squared returns. All the details involved in these transformations can be found in Lu and Perron (2010).

The results shown in Table 8 correspond to the logarithm of the absolute value of the returns as a measure of volatility. We must clarify that when we cite one model as the best one, this is true in statistical terms, since the MCS is a random subset that shows the best models with a certain level of confidence (see Hansen et al., 2011). We utilize six countries and six horizons, which gives us 36 cases to consider for each model. As regards the Basic RLS, this model belongs to the 10% MCS in 14 out of the 36; for the Threshold $\kappa = 1\%$, it belongs in 13 cases; for the Mean Reversion RLS, we have 26 cases; and for the Modified RLS there are 18 out of 36 cases where the model belongs to the 10% MCS. The Mean Reversion RLS model performs the best, based on the number of times the model belongs to the MCS. Taken together, the two ARFIMA models belong to the 10% MCS in 1 out of 36 cases. The conclusion is clear. The RLS models not only fit the data well, but also allow for a good forecasting performance in the majority of countries and for all steps.

The results with the squared return volatility series are presented in Table 9. The Basic, 1% Threshold, Mean Reversion, and Modified RLS models belong to the 10% MCS in 19, 16, 29 and 24 cases out of 36. When it comes to the GARCH and FIGARCH together, they add up to 14 cases out of 36, while the ARFIMAs belong to the MCS in 6 out 36 cases (adding both together). As we found in the previous Table, even when the results of the competitor models are added together, the RLS models performs the best in the forecasting analysis across most cases. Figures 4 and 5 illustrate the results found in Table 9. Figure 4 suggests that the FIGARCH model is never able to dominate any member of the family of the RLS models, which is very interesting because it means that long memory is not an ingredient needed to dominate other models, such as RLS models. On the other hand, Figure 5 illustrates the results shown in Table 9 with respect to the GARCH model's performance. There are some periods for which the GARCH model is not dominated by a member of the RLS family of models. However, we argue that in most cases, a member of the family of the RLS models surpasses the performance of the GARCH models.

5 Conclusions

A sizeable branch of the literature on financial econometrics has proposed that long-memory behavior is spurious and due to the presence of rare level shifts. Lu and Perron (2010) and Li and Perron (2013) apply the RLS model to the stock market and Forex returns, respectively. The interesting issue is that after taking into account the level shifts, no evidence of long memory is present and even GARCH effects are eliminated. Xu and Perron (2014) extend the Basic RLS model in two aspects: (i) by introducing a time-varying probability; and (ii) a mean reversion mechanism. A final model is a mixture of these two mentioned models. Applying these different models to the above mentioned data, the results reinforce the results before mentioned.

In the case of Latin-American financial markets, the recent models have been applied to different contexts. For example, Herrera Aramburú and Rodríguez (2016) opt for a testing approach to verify whether Peruvian financial markets present long memory. A similar approach is used by Pardo Figueroa and Rodríguez (2014). As regards modelling, Ojeda Cunya and Rodríguez (2016) apply the Basic RLS to the Peruvian financial markets while Rodríguez and Tramontana (2015) applied it to the Latin American stock markets. Meanwhile, the extended model proposed by Xu and Perron has been used by Rodríguez (2016) to analyze the stock markets in Latin America. The results obtained in this regard are similar to the original proposals of Lu and Perron (2010) and Xu and Perron (2014); that is, showing an artificial presence of long-memory behavior due to the (sporadic or rare) presence of level shifts.

The objective of this paper is to estimate the four different RLS models suggested by the mentioned literature using Latin American Forex markets volatility. After estimation, we compare the forecasting performance of these four models with other very well know models; namely, the ARFIMA, GARCH and FIGARCH models. Our results may be summarized as follows: (i) the four RLS models fit well; (ii) there the presence of level shifts is sporadic or rare but still important. After taking into account these observations, long memory is not appreciated in the ACFs; that is, there is no fractional integration evidence; (iii) the RLS models perform well in terms of forecasting for short, medium and long horizons compared to competitive models as the ARFIMA (p,d,q), GARCH (1,1) and FIGARCH.

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Technical Appendix

The first-differences of the model (1), with the aim of eliminating the autoregressive process of the level shift component, depends solely on the Bernouilli process: $\Delta y_t = \tau_t - \tau_{t-1} + c_t - c_{t-1} = c_t - c_{t-1} + \delta_t$, and moving to the state-space form, the mean and transition equations are obtained, respectively: $\Delta y_t = c_t - c_{t-1} + \delta_t$, $c_t = \phi c_{t-1} + e_t$. In matrix form $\Delta y_t = HX_t + \delta_t$ and $X_t = FX_{t-1} + U_t$ are obtained, where $X_t = [c_t, c_{t-1}]$, $F = \begin{bmatrix} \phi & 0 \\ 1 & 0 \end{bmatrix}$, $H = [1, -1]'$. In this case, the first row of the matrix F shows the coefficient ϕ of the autoregressive part of the short-memory component. Moreover, U is a Normally distributed vector of dimension 2 with mean 0 and variance: $Q = \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & 0 \end{bmatrix}$. In comparison with the standard state-space model, the important difference in the current model is that the distribution of δ_t is a mixture of two Normal distributions with variance σ_η^2 and 0, occurring with probabilities α and $1 - \alpha$, respectively[5].

The model described above is a special version of the models included in Wada and Perron (2006) and Perron and Wada (2009). In this case, there are only shocks that affect the level of the series, and the restriction is imposed that the variance of one of the components of the mixture of distributions is zero. The basic input for the estimation is the increase in the states through the realizations of the mixture at time t so that the Kalman filter can be used to construct the likelihood function, conditional to the realizations of the states. The latent states are eliminated from the final expression of the likelihood by summing over all the possible realizations of the states. In consequence, despite its fundamental differences, the model takes a structure that is similar to that of the Markov-Switching model of Hamilton (1989, 1994). Let $Y_t = (\Delta y_1, ..., \Delta y_t)$ be the vector of observations available at time t and denote the vector of parameters by $\theta = [\sigma_\eta^2, \alpha, \sigma_e^2, \phi]$. Adopting the notation used in Hamilton (1994), $\mathbf{1}(.)$ represents a vector of ones of dimension (4×1), the symbol $\odot$ denotes element-by-element multiplication, $\widehat{\xi}^{ij}_{t|t-1} = vec(\widetilde{\xi}_{t|t-1})$ with the (i, j)th element of $\widetilde{\xi}_{t|t-1}$ being $\Pr(s_{t-1} = i, s_t = j|Y_{t-1}; \theta)$ and $\omega_t = vec(\widetilde{\omega}_t)$ with the (i, j)th element of $\widetilde{\omega}_t$ being $f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}; \theta)$ for $i, j \in \{1, 2\}$. Thus, I have $s_t = 1$ when $\pi_t = 1$, that is, a level shift occurs. Using the same notation as Lu and Perron (2010), the logarithm of the likelihood function is $\ln(L) = \sum_{t=1}^{T} \ln f(\Delta y_t|Y_{t-1}; \theta)$, where

$$\begin{aligned} f(\Delta y_t|Y_{t-1}, \theta) &= \sum_{i=1}^{2}\sum_{j=1}^{2} f(\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1}, \theta) \Pr(s_{t-1} = i, s_t = j|Y_{t-1}, \theta) \\ &\equiv \mathbf{1}'(\widehat{\xi}_{t|t-1} \odot \omega_t). \end{aligned}$$

By applying rules of conditional probabilities, Bayes's rule and the independence of s_t with respect to past realizations, I obtain $\widetilde{\xi}^{ki}_{t|t-1} = \Pr(s_{t-2} = k, s_{t-1} = i|Y_{t-1}; \theta)$. The evolution of $\widehat{\xi}_{t|t-1}$

[5] In comparison with the Markov-Switching model of Hamilton (1989), this model does not limit the magnitude of the level shifts, so any number of regimes is possible. Moreover, the probability 0 or 1 does not depend on past events, unlike the Markov model.

can be expressed as:

$$\begin{bmatrix} \tilde{\xi}^{11}_{t+1|t} \\ \tilde{\xi}^{21}_{t+1|t} \\ \tilde{\xi}^{12}_{t+1|t} \\ \tilde{\xi}^{22}_{t+1|t} \end{bmatrix} = \begin{bmatrix} \alpha & \alpha & 0 & 0 \\ 0 & 0 & \alpha & \alpha \\ 1-\alpha & 1-\alpha & 0 & 0 \\ 0 & 0 & 1-\alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} \tilde{\xi}^{11}_{t|t} \\ \tilde{\xi}^{21}_{t|t} \\ \tilde{\xi}^{12}_{t|t} \\ \tilde{\xi}^{22}_{t|t} \end{bmatrix}, \tag{A.1}$$

which is equal to $\widehat{\xi}_{t+1|t} = \Pi\widehat{\xi}_{t|t}$ with $\widehat{\xi}_{t|t} = \frac{(\widehat{\xi}_{t|t-1}\odot\omega_t)}{\mathbf{1}'(\widehat{\xi}_{t|t-1}\odot\omega_t)}$. Note that thus far the model includes the probabilities of level shift (α) as constant. Thus, once the specific estimate of α is obtained, a possible approach is the use of a smoothed estimate of the level shift component $\widehat{\tau}_t$. However, in the present context of abrupt structural shifts, the conventional smoothers may perform poorly. In place of this, I use the method proposed by Bai and Perron (1998, 2003) to obtain the dates on which the level shifts occur, as well as the means (averages) within each segment. Indeed, I use the estimation of α to obtain an estimate of the number of level shifts, and the method of Bai and Perron (1998, 2003) to obtain estimates of the break dates that globally minimize the following sum of squared residuals: $\sum_{i=1}^{m+1}\sum_{t=T_{i-1}+1}^{T_i}[y_t-\mu_i]^2$, where m is the number of breaks, T_i $(i=1,2,...;m)$ are the break dates $T_0=0$, and $T_{m+1}=T$ and μ_i $(i=1,2,...,m+1)$ are the means (averages) inside each regime, which can be estimated once the date breaks have been estimated or known . This method is efficient and can handle a large number of observations; see Bai and Perron (2003) for further details[6].

In consequence, the conditional likelihood function for Δy_t corresponds to the following Normal density:

$$\widetilde{\omega}^{ij}_t = f(\Delta y_t|s_{t-1}=i, s_t=j, Y_{t-1},\theta) = \frac{1}{\sqrt{2\pi}}|f^{ij}_t|^{-1/2}\exp(-\frac{v^{ij\prime}_t(f^{ij}_t)^{-1/2}v^{ij}_t}{2}),$$

where v^{ij}_t is the prediction error and f^{ij}_t is its variance, and these terms are defined as:

$$\begin{aligned} v^{ij}_t &= \Delta y_t - \Delta y^i_{t|t-1} = \Delta y_t - E[\Delta y_t|s_t=i, Y_{t-1};\theta], \\ f^{ij}_t &= E(v^{ij}_t v^{ij\prime}_t). \end{aligned}$$

The best predictions for the state variable and its respective conditional variance in $s_{t-1}=i$ are $X^i_{t|t-1}=FX^i_{t-1|t-1}$, and $P^i_{t|t-1}=FP^i_{t-1|t-1}F'+Q$, respectively. Furthermore, the mean equation is $\Delta y_t = HX_t+\delta_t$, where the error δ_t has a mean 0 and a variance that can take values $R_1=\sigma^2_\eta$ with probability α or $R_2=0$ with probability $(1-\alpha)$. Thus, the prediction error is $v^{ij}_t=\Delta y_t - HX^i_{t|t-1}$ and its variance is $f^{ij}_t = HP^i_{t|t-1}H'+R_j$. In this way, given that $s_t=j$ and $s_{t-1}=i$ and using

[6]Note that because the model permits consecutive level shifts, we set (in the empirical application of the Basic RLS model) the minimum length of a segment at only one observation.

updating formulas:

$$
\begin{aligned}
X^i_{t|t} &= X^i_{t|t-1} + P^i_{t|t-1}H'(HP^i_{t|t-1}H' + R_j)^{-1}(\Delta y_t - HX^i_{t|t-1}),\\
P^{ij}_{t|t-1} &= P^i_{t|t-1} - P^i_{t|t-1}H'(HP^i_{t|t-1}H' + R_j)^{-1}HP^i_{t|t-1},
\end{aligned}
$$

are obtained. In order to reduce the dimensionality problem in the estimation, Lu and Perron (2010) use the recollapsing procedure proposed by Harrison and Stevens (1976). In so doing, $\widetilde{\omega}^{ij}_t$ is unaffected by the history of the states before time $t-1$. The, I have four possible states corresponding to $S_t = 1$ when $(s_t = 1, s_{t-1} = 1)$, $S_t = 2$ when $(s_t = 1, s_{t-1} = 2)$, $S_t = 3$ when $(s_t = 2, s_{t-1} = 2)$ and $S_t = 4$ when $(s_t = 2, s_{t-1} = 2)$ and the matrix Π is defined as (A.1). Taking the definitions of $\widetilde{\omega}_t$, $\widehat{\xi}_{t|t}$, $\widehat{\xi}_{t+1|t}$, the set of conditional probabilities and the one-period forward predictions, the same structure as a version of the Markov model of Hamilton (1989, 1994) is obtained. However, the EM algorithm cannot be used. This is because the mean and the variance in the conditional density function are non-linear functions of the parameters θ and of past realizations $\{\Delta y_{t-j}; j \geq 1\}$. Likewise, the conditional probability of being in a determined regime $\widehat{\xi}_{t|t}$ is inseparable from the conditional densities $\widetilde{\omega}_t$. For further details, see Lu and Perron (2010), Li and Perron (2013), and Wada and Perron (2006).

The estimation method is based on the work of Xu and Perron (2014), which is an extension of the basic RLS model by Lu and Perron (2010) and Li and Perron (2013). The first difference compared with the basic model is that the vector of parameters is different: $\theta = [\sigma^2_\eta, p, \sigma^2_e, \phi, \gamma_1, \gamma_2, \beta]$[7]. The second important difference is that, given the probability of level shifts is now varying, the equation (A.1) is replaced by:

$$
\begin{bmatrix} \widetilde{\xi}^{11}_{t+1|t} \\ \widetilde{\xi}^{21}_{t+1|t} \\ \widetilde{\xi}^{12}_{t+1|t} \\ \widetilde{\xi}^{22}_{t+1|t} \end{bmatrix} = \begin{bmatrix} p_{t+1} & p_{t+1} & 0 & 0 \\ 0 & 0 & p_{t+1} & p_{t+1} \\ (1-p_{t+1}) & (1-p_{t+1}) & 0 & 0 \\ 0 & 0 & (1-p_{t+1}) & (1-p_{t+1}) \end{bmatrix} \begin{bmatrix} \widetilde{\xi}^{11}_{t|t} \\ \widetilde{\xi}^{21}_{t|t} \\ \widetilde{\xi}^{12}_{t|t} \\ \widetilde{\xi}^{22}_{t|t} \end{bmatrix}. \tag{A.2}
$$

Therefore, the conditional likelihood function for Δy_t follows the Normal density:

$$
\widetilde{\omega}^{ij}_t = f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}} |f^{ij}_t|^{-1/2} \exp(-\frac{v^{ij\prime}_t (f^{ij}_t)^{-1/2} v^{ij}_t}{2}),
$$

where v^{ij}_t is the prediction error and f^{ij}_t is its variance and is defined as: $v^{ij}_t = \Delta y_t - \Delta y^{ij}_{t|t-1} = \Delta y_t - E[\Delta y_t | s_t = i, s_{t-1} = j, Y_{t-1}, \theta]$ and $f^{ij}_t = E(v^{ij}_t v^{ij\prime}_t)$. Note that $\Delta y^{ij}_{t|t-1}$ depends only on the information contained in $t-1$. The predictions for the variable of state and its respective conditional variance to $s_{t-1} = i$ are: $X^i_{t|t-1} = FX^i_{t-1|t-1}$ and $P^i_{t|t-1} = FP^i_{t-1|t-1}F' + Q$. The mean equation is $\Delta y_t = HX_t + \delta_t$, where the error δ_t has zero mean and a variance that can take values

[7] This vector of parameters corresponds to the model that contains the two extensions, that is, the Modified RLS model. In the case of the Threshold κ% RLS model (only varying probabilities), the vector of parameters is $\theta = [\sigma^2_\eta, p, \sigma^2_e, \phi, \gamma_1, \gamma_2]$, while in the case of the Mean Reversion RLS model, the set of parameters is $\theta = [\sigma^2_\eta, p, \sigma^2_e, \phi, \beta]$.

$R_1 = \sigma_\eta^2$ or values $R_2 = 0$, so the prediction error is $v_t^{ij} = \Delta y_t - HX_{t|t-1}^i$ and is associated with a variance $f_t^{ij} = HP_{t|t-1}^i H' + R_j$. Then, given $s_t = j$ and $s_{t-1} = i$ and using the updating formula I have:

$$
\begin{aligned}
X_{t|t}^{ij} &= X_{t|t-1}^i + P_{t|t-1}^i H'(HP_{t|t-1}^i H' + R_j)^{-1}(\Delta y_t - HX_{t|t-1}^i), \\
P_{t|t}^{ij} &= P_{t|t-1}^i - P_{t|t-1}^i H'(HP_{t|t-1}^i H' + R_j)^{-1} HP_{t|t-1}^i.
\end{aligned}
$$

As in Perron and Wada (2009), I reduce the estimation problem by using the recollapsing process proposed by Harrison and Stevens (1976):

$$
\begin{aligned}
X_{t|t}^i &= \frac{\sum_{i=1}^2 \Pr(s_{t-1} = i, s_t = j|Y_t, \theta) X_{t|t}^{ij}}{\Pr(s_t = j|Y_t, \theta)} = \frac{\sum_{i=1}^2 \widetilde{\xi}_{t|t}^{ij} X_{t|t}^{ij}}{\sum_{i=1}^2 \widetilde{\xi}_{t|t}^{ij}}, \\
P_{t|t}^i &= \frac{\sum_{i=1}^2 \Pr(s_{t-1} = i, s_t = j|Y_t, \theta)[P_{t|t}^{ij} + (X_{t|t}^i - X_{t|t}^{ij})(X_{t|t}^i - X_{t|t}^{ij})']}{\Pr(s_t = j|Y_t, \theta)} \\
&= \frac{\sum_{i=1}^2 \widetilde{\xi}_{t|t}^{ij}[P_{t|t}^{ij} + (X_{t|t}^j - X_{t|t}^{ij})(X_{t|t}^j - X_{t|t}^{ij})']}{\sum_{i=1}^2 \widetilde{\xi}_{t|t}^{ij}}.
\end{aligned}
$$

For the Mean Reversion RLS model, certain modifications are necessary. The prediction error v_t^{ij} of the previous expressions is no longer Normally distributed with mean 0 and variance that depends on the value of the state, but is modeled as: $y_t = a + c_t + \tau_t$,

$$
\Delta y_t = \tau_t - \tau_{t-1} + c_t - c_{t-1}, \tau_t - \tau_{t-1} = \pi_t[\beta(\tau_{t|t-1} - \overline{\tau}_t) + \widetilde{\eta}_{1t}] + (1 - \pi_t)\eta_{2t}.
$$

Moreover,

$$
\begin{aligned}
\widetilde{\omega}_t^{ij} &= f(\Delta y_t | s_{t-1} = i, s_t = j, Y_{t-1}, \theta) = \frac{1}{\sqrt{2\pi}} |f_t^{ij}|^{-1/2} \exp(-\frac{\widetilde{v}_t^{ij\prime}(f_t^{ij})^{-1/2} \widetilde{v}_t^{ij}}{2}), \\
\widetilde{v}_t^{ij} &= \left\{ \begin{array}{c} v_t^{11} - \beta(\tau_{t|t-1}^{11} - \overline{\tau}_t^{11}) \\ v_t^{12} \\ v_t^{21} - \beta(\tau_{t|t-1}^{21} - \overline{\tau}_t^{21}) \\ v_t^{22} \end{array} \right\},
\end{aligned}
$$

and $f_t^{ij} = E(\widetilde{v}_t^{ij} \widetilde{v}_t^{ij\prime}) = HP_{t|t-1}^i H' + R_j$. Further details appear in Xu and Perron (2014).

Table 1. Descriptive Statistics of the Volatility Series

Volatility	Mean	SD	Maximum	Minimum	Skewness	Kurtosis
Argentina	-6.026	0.739	-1.503	-6.908	1.473	5.838
Brazil	-5.145	0.857	-2.259	-6.908	0.065	2.569
Chile	-5.616	0.736	-3.044	-6.908	0.245	2.440
Colombia	-5.668	0.767	-2.564	-6.908	0.400	2.637
Mexico	-5.576	0.817	-1.679	-6.908	0.440	3.148
Peru	-6.148	0.597	-3.738	-6.908	0.940	3.524

Table 2. Estimates of the Basic RLS Model

	σ_η	α	σ_e	ϕ	Likelihood
Argentina	1.309^a	0.015^a	0.496^a		2413.592
	(0.198)	(0.003)	(0.008)		
Brazil	0.535^a	0.016^b	0.745^a		4414.095
	(0.120)	(0.007)	(0.009)		
Chile	0.477^a	0.009^b	0.636^a		5272.241
	(0.128)	(0.004)	(0.007)		
Colombia	0.435^a	0.0188^b	0.647^a	0.066^a	5358.872
	(0.110)	(0.009)	(0.007)	(0.016)	
Mexico	1.072^a	0.003^a	0.670^a	0.071^a	5895.033
	(0.072)	(0.001)	(0.007)	(0.015)	
Peru	0.513^a	0.017^a	0.490^a	0.104^a	3229.701
	(0.084)	(0.005)	(0.006)	(0.020)	

Standard errors are in parentheses; estimates with a, b, c are significant at the 1%, 5%, 10% levels respectively.

Table 3. Estimates of the RLS Model with Time Varying Probabilities

Threshold	σ_η	p	σ_e	γ_1	γ_2	Likelihood
			Argentina			
5%	1.120^a	-2.175^a	0.477^a	0.519	142.698	2379.683
	(0.150)	(0.184)	(0.008)	(0.374)	(10869.088)	
2.5%	1.244^a	-2.115^a	0.477^a	2.381^a	0.166	2386.231
	(0.140)	(0.167)	(0.008)	(0.708)	(0.251)	
1%	1.192^a	-2.158^a	0.491^a	2.708	0.097^b	2404.579
	(0.172)	(0.173)	(0.008)	(2.223)	(0.043)	
			Brazil			
5%	0.449^a	-2.120^a	0.745^a	0.253^a	24.541	4404.834
	(0.106)	(0.418)	(0.009)	(0.095)	(310.601)	
2.5%	0.462^a	-2.113^a	0.745^a	1.218^b	0.076^a	4405.906
	(0.105)	(0.414)	(0.009)	(0.488)	(0.012)	
1%	0.465^a	-2.121^a	0.745^a	1.784^c	0.118^a	4404.861
	(0.104)	(0.408)	(0.009)	(0.979)	(0.031)	
			Chile			
5%	0.450^a	-2.399^a	0.636^a	0.656^b	0.398^a	5270.988
	(0.114)	(0.433)	(0.007)	(0.278)	(0.135)	
2.5%	0.452^a	-2.419^a	0.635^a	1.193^a	0.382^b	5268.813
	(0.107)	(0.417)	(0.007)	(0.429)	(0.184)	
1%	0.444^a	-2.373^a	0.635^a	1.689^b	0.433^b	5269.273
	(0.120)	(0.419)	(0.007)	(0.852)	(0.190)	

Standard errors are in parentheses; estimates with a, b, c are significant at the 1%, 5%, 10% levels respectively.

Table 3 (continued). Estimates of the RLS Model with Time Varying Probabilities

Threshold	σ_η	p	σ_e	ϕ	γ_1	γ_2	Likelihood
			Colombia				
5%	0.337^a	-2.018^a	0.640^a	0.057^a	1.722^b	0.276^b	5345.784
	(0.092)	(0.474)	(0.007)	(0.016)	(0.737)	(0.134)	
2.5%	0.311^a	-1.904^a	0.640^a	0.057^a	2.647	0.371^a	5345.966
	(0.0974)	(0.506)	(0.007)	(0.016)	(3.661)	(0.137)	
1%	0.335^a	-1.897^a	0.641^a	0.062^a	3.195	0.135^a	5351.935
	(0.125)	(0.587)	(0.007)	(0.017)	(15.966)	(0.036)	
			Mexico				
5%	0.993^a	-2.909^a	0.670^a	0.072^a	1.116^a	0.706^a	5886.841
	(0.011)	(0.340)	(0.007)	(0.015)	(0.288)	(0.242)	
2.5%	0.924^a	-2.859^a	0.668^a	0.068^a	1.660^a	0.088^a	5885.390
	(0.005)	(0.304)	(0.007)	(0.015)	(0.413)	(0.011)	
1%	1.009^a	-2.800^a	0.669^a	0.070^a	1.678^a	0.532^b	5889.191
	(0.012)	(0.250)	(0.007)	(0.015)	(0.642)	(0.266)	
			Peru				
5%	0.553^a	-2.148^a	0.476^a	0.062^a	0.015^a	258.096	3206.984
	(0.074)	(0.230)	(0.007)	(0.021)	(0.0004)	(10428.507)	
2.5%	0.516^a	-2.076^a	0.478^a	0.071^a	2.242^b	0.212^b	3215.909
	(0.075)	(0.241)	(0.007)	(0.022)	(0.934)	(0.083)	
1%	0.586^a	-2.096^a	0.477^a	0.067^a	5.631	0.110^a	3211.692
	(0.072)	(0.193)	(0.006)	(0.020)	(118.490)	(0.005)	

Standard errors are in parentheses; estimates with a, b, c are significant at the 1%, 5%, 10% levels respectively.

Table 4. Estimates of the RLS Model with Mean Reversion

	σ_η	α	σ_e	ϕ	β	Likelihood
Argentina	0.965^a	0.016^a	0.496^a		-0.738^a	2403.952
	(0.067)	(0.004)	(0.008)		(0.123)	
Brazil	0.106	0.065^b	0.744^a		-0.223^a	4403.7421
	(0.072)	0.030)	(0.009)		(0.015)	
Chile	0.156^a	0.035^c	0.633^a		-0.209^a	5264.194
	(0.054)	(0.020)	(0.007)		(0.015)	
Colombia	0.052	0.084^a	0.632^a		-0.288^a	5341.479
	(0.126)	(0.021)	(0.007)		(0.016)	
Mexico	0.907^a	0.003^a	0.669^a	0.069^a	-0.362^a	5892.114
	(0.128)	(0.001)	(0.007)	(0.015)	(0.062)	
Peru	0.141^a	0.109^a	0.473^a		-0.310^a	3213.794
	(0.025)	(0.023)	(0.006)		(0.014)	

Standard errors are in parentheses; estimates with a, b, c are significant at the 1%, 5%, 10% levels respectively.

Table 5. Estimates of the RLS Model with a Time Varying Probability of Shifts and Mean Reversion, Threshold: 1%

	σ_η	p	σ_e	ϕ	γ_1	γ_2	β	Likelihood
Argentina	0.912^a	-2.120^a	0.491^a		2.271	0.050^a	-0.598^a	2394.529
	(0.139)	(0.178)	(0.008)		(1.514)	(0.004)	(0.093)	
Brazil	0.134^a	-1.447^a	0.743^a		0.507^c	0.344^a	-0.184^a	4397.301
	(0.048)	(0.392)	(0.009)		(0.297)	(0.089)	(0.012)	
Chile	0.173^a	-1.859^a	0.633^a		0.455^b	0.245^a	-0.207^a	5263.683
	(0.060)	(0.514)	(0.007)		(0.182)	(0.063)	(0.016)	
Colombia	0.060^a	-1.400^a	0.632^a		0.228^a	0.992^b	-0.292^a	5339.754
	(0.087)	(0.179)	(0.007)		(0.077)	(0.503)	(0.013)	
Mexico	0.925^a	-2.770^a	0.669^a	0.068^a	1.396^b	0.091^a	-0.292^a	5887.014
	(0.261)	(0.348)	(0.007)	(0.015)	(0.570)	(0.031)	(0.044)	
Peru	0.166^a	-1.346^a	0.471^a		1.276^a	0.502^c	-0.340^a	3205.012
	(0.035)	(0.163)	(0.006)		(0.393)	(0.296)	(0.015)	

Standard errors are in parentheses; estimates with a, b, c are significant at the 1%, 5%, 10% levels respectively.

Table 6. Estimated Parameters of ARFIMA(0,d,0) and ARFIMA(1,d,1) models

	d	AR	MA	d	AR	MA	d	AR	MA
	Argentina			Brazil			Chile		
Volatility	0.291			0.201			0.197		
	(0.000)			(0.000)			(0.000)		
	0.508	0.349	-0.650	0.392	0.078	-0.390	0.436	0.277	-0.625
	(0.000)	(0.000)	(0.000)	(0.000)	(0.227)	(0.000)	(0.000)	(0.000)	(0.000)
C_t	-0.068			-0.067			-0.041		
	(0.000)			(0.000)			(0.000)		
	-0.775	0.789	-0.043	-0.783	0.865	-0.154	-0.919	0.922	-0.037
	(0.000)	(0.000)	(0.219)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.236)
	Colombia			Mexico			Peru		
Volatility	0.230			0.248			0.264		
	(0.000)			(0.000)			(0.000)		
	0.423	0.266	-0.547	0.491	0.243	-0.589	0.421	0.275	-0.497
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
C_t	-0.013			0.020			0.006		
	(0.309)			(0.000)			(0.688)		
	-0.844	0.887	-0.049	-0.864	0.925	-0.037	-0.779	0.841	-0.038
	(0.000)	(0.000)	(0.087)	(0.000)	(0.000)	(0.237)	(0.000)	(0.000)	(0.183)

p-values are reported in parenthesis.

Table 7. Estimates: GARCH, FIGARCH and CGARCH models

	β_r	β_σ	ρ	φ	d	β_r	β_σ	ρ	ψ	d
	Argentina					Brazil				
GARCH	0.187	0.837				0.140	0.861			
	(0.000)	(0.000)				(0.000)	(0.000)			
FIGARCH	-0.056	0.155			0.539	0.077	0.613			0.680
	(0.722)	(0.361)			(0.000)	(0.210)	(0.000)			(0.000)
CGARCH	0.288	0.632	0.998	0.025		0.061	0.792	0.999	0.109	
	(0.000)	(0.000)	(0.000)	(0.135)		(0.005)	(0.000)	(0.000)	(0.000)	
CGARCH (BP)	0.044	0.016	0.509	0.048		0.042	0.017	0.510	0.056	
	(0.236)	(0.963)	(0.000)	(0.179)		(0.369)	(0.968)	(0.000)	(0.213)	
CGARCH (SM)	0.061	0.022	0.519	0.169		0.034	0.018	0.512	0.080	
	(0.000)	(0.847)	(0.000)	(0.000)		(0.155)	(0.947)	(0.000)	(0.000)	

BP denotes Bai and Perron (2003) to obtain $\hat{\tau}_t$; SM denotes a smoothed estimated of $\hat{\tau}_t$; p value in parenthesis.

Table 7 (continued). Estimates: GARCH, FIGARCH and CGARCH models

	β_r	β_σ	ρ	ψ	d	β_r	β_σ	ρ	ψ	d
			Chile					Colombia		
GARCH	0.112	0.900				0.248	0.811			
	(0.000)	(0.000)				(0.000)	(0.000)			
FIGARCH	0.254	0.735			0.632	0.163	0.501			0.557
	(0.000)	(0.000)			(0.000)	(0.012)	(0.000)			(0.000)
CGARCH	0.072	0.750	1.000	0.073		0.173	0.782	1.000	0.020	
	(0.000)	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)	
CGARCH (BP)	-0.092	0.073	-0.006	0.136		-0.002	0.020	0.475	0.077	
	(0.992)	(0.994)	(0.963)	(0.988)		(0.961)	(0.999)	(0.000)	(0.039)	
CGARCH (SM)	-0.073	0.032	0.481	0.221		0.048	0.017	0.503	0.073	
	(0.073)	(0.921)	(0.000)	(0.000)		(0.003)	(0.913)	(0.000)	(0.000)	

BP denotes Bai and Perron (2003) to obtain $\hat{\tau}_t$; SM denotes a smoothed estimated of $\hat{\tau}_t$; p value in parenthesis.

Table 7 (continued). Estimates: GARCH, FIGARCH and CGARCH models

	β_r	β_σ	ρ	ψ	d	β_r	β_σ	ρ	ψ	d
	Mexico					Peru				
GARCH	0.233	0.839				0.330	0.797			
	(0.000)	(0.000)				(0.000)	(0.000)			
FIGARCH	0.067	0.841			0.946	0.286	0.557			0.566
	(0.042)	(0.000)			(0.000)	(0.004)	(0.000)			(0.000)
CGARCH	0.065	0.818	1.000	0.101		0.175	0.634	1.000	0.113	
	(0.000)	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)	
CGARCH (BP)	-1.027	1.787	0.776	1.205		0.040	0.016	0.500	0.040	
	(0.675)	(0.470)	(0.000)	(0.623)		(0.000)	(0.884)	(0.000)	(0.000)	
CGARCH (SM)	0.043	0.016	0.502	0.047		0.041	0.016	0.501	0.043	
	(0.000)	(0.853)	(0.000)	(0.000)		(0.008)	(0.928)	(0.000)	(0.003)	

BP denotes Bai and Perron (2003) to obtain $\hat{\tau}_t$; SM denotes a smoothed estimated of $\hat{\tau}_t$; p value in parenthesis.

Table 8. Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Argentina			
Basic RLS	0.145	1.362	3.125	8.429	61.963	118.029
	(0.024)	(0.825*)	(1.000*)	(0.000)	(0.018)	(0.000)
Threshold 1% RLS	0.120	1.473	4.030	7.242	62.242	114.337
	(1.000*)	(0.222*)	(0.097)	(0.276*)	(0.000)	(0.000)
Mean Reversion RLS	0.131	1.339	3.230	7.163	57.806	97.508
	(0.024)	(1.000*)	(0.771*)	(0.561*)	(1.000*)	(1.000*)
Modified RLS	0.122	1.430	3.672	7.092	63.492	109.518
	(0.568*)	(0.327*)	(0.097)	(1.000*)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.232	2.899	9.279	31.849	167.211	607.624
	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	3.238	77.485	307.187	1197.914	7055.237	26250.370
	(0.000)	0.000)	(0.000)	(0.000)	(0.000)	(0.000)
			Brazil			
Basic RLS	0.507	3.912	10.846	36.467	250.632	1191.939
	(0.001)	(0.060)	(0.003)	(0.000)	(0.000)	(0.022)
Threshold 1% RLS	0.492	3.858	10.813	36.469	253.275	1226.363
	(1.000*)	(0.116*)	(0.005)	(0.000)	(0.000)	(0.011)
Mean Reversion RLS	0.493	3.778	10.400	34.551	222.385	1060.620
	(0.837*)	(1.000*)	(0.301*)	(0.002)	(0.007)	(0.054)
Modified RLS	0.499	3.792	10.360	34.074	219.278	1038.370
	(0.150*)	(0.253*)	(1.000*)	(1.000*)	(1.000*)	(0.054)
ARFIMA(0,d,0)	0.667	6.319	18.895	61.216	298.521	937.055
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000*)
ARFIMA(1,d,1)	0.674	6.510	19.642	64.132	315.047	1006.433
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.054)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a $(*)$ indicates that the model is within the 10% MCS using all comparisons.

Table 8 (continued). Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Chile			
Basic RLS	0.408	2.715	7.678	26.857	160.769	689.341
	(0.000)	(0.123*)	(0.119*)	(0.394*)	(0.022)	(0.000)
Threshold 1% RLS	0.393	2.634	7.397	26.221	163.373	691.651
	(1.000*)	(1.000*)	(1.000*)	(0.879*)	(0.008)	(0.000)
Mean Reversion RLS	0.402	2.668	7.513	26.191	154.837	638.048
	(0.000)	(0.420*)	(0.432*)	(0.879*)	(1.000*)	(1.000*)
Modified RLS	0.400	2.657	7.472	26.155	155.744	641.944
	(0.000)	(0.423*)	(0.441*)	(1.000*)	(0.098)	(0.081)
ARFIMA(0,d,0)	0.614	6.741	22.600	80.008	434.454	1595.243
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.594	6.251	20.640	72.153	384.979	1395.665
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
			Colombia			
Basic RLS	0.312	3.341	10.432	38.244	272.898	1380.787
	(1.000*)	(1.000*)	(0.343*)	(0.272*)	(0.002)	(0.000)
Threshold 1% RLS	0.373	3.421	10.213	36.325	255.193	1290.177
	(0.000)	(0.013)	(1.000*)	(0.900*)	(0.044)	(0.000)
Mean Reversion RLS	0.412	3.496	10.362	36.261	241.960	1144.643
	(0.000)	(0.013)	(0.579*)	(0.900*)	(1.000*)	(1.000*)
Modified RLS	0.412	3.507	10.399	36.163	243.052	1168.833
	(0.000)	(0.013)	(0.579*)	(1.000*)	(0.548*)	(0.003)
ARFIMA(0,d,0)	0.813	11.046	38.431	140.423	801.526	3018.886
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.846	11.869	41.733	153.675	886.334	3365.760
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a $(*)$ indicates that the model is within the 10% MCS using all comparisons.

Table 8 (continued). Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Mexico			
Basic RLS	0.299	2.715	8.258	28.893	197.425	992.964
	(1.000*)	(1.000*)	(0.224*)	(0.002)	(0.000)	(0.000)
Threshold 1% RLS	0.342	2.867	8.255	27.352	183.932	900.678
	(0.000)	(0.015)	(0.140*)	(0.101*)	(0.002)	(0.000)
Mean Reversion RLS	0.354	2.848	7.994	25.936	173.080	852.005
	(0.000)	(0.016)	(1.000*)	(1.000*)	(1.000*)	(0.649*)
Modified RLS	0.346	2.845	8.090	26.342	174.064	847.066
	(0.000)	(0.020)	(0.394*)	(0.379*)	(0.689*)	(1.000*)
ARFIMA(0,d,0)	0.741	9.569	33.552	123.001	712.049	2679.897
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.722	9.091	31.640	115.342	663.589	2484.965
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
			Peru			
Basic RLS	0.185	2.411	8.473	31.146	179.363	766.564
	(1.000*)	(0.025)	(0.000)	(0.000)	(0.000)	(0.000)
Threshold 1% RLS	0.203	2.547	9.197	34.789	204.453	870.528
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean Reversion RLS	0.224	2.344	8.050	29.699	167.649	702.526
	(0.000)	(1.000*)	(1.000*)	(1.000*)	(1.000*)	(1.000*)
Modified RLS	0.214	2.441	8.780	33.834	196.975	830.357
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.397	5.034	16.498	54.349	273.831	953.812
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.398	5.062	16.609	54.823	277.155	969.622
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a $(*)$ indicates that the model is within the 10% MCS using all comparisons.

Table 9. Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t r^2_{t+\tau}]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Argentina			
Basic RLS	0.000	0.002	0.005	0.006	0.006	0.007
	(0.331*)	(0.578*)	(0.762*)	(0.931*)	(0.570*)	(0.570*)
Threshold 1% RLS	0.000	0.002	0.005	0.006	0.006	0.007
	(1.000*)	(1.000*)	(0.762*)	(0.912*)	(0.507*)	(0.564*)
Mean Reversion RLS	0.000	0.002	0.005	0.006	0.006	0.007
	(0.331*)	(0.578*)	(0.762*)	(0.945*)	(0.531*)	(0.572*)
Modified RLS	0.000	0.002	0.005	0.006	0.006	0.007
	(0.331*)	(0.578*)	(0.762*)	(0.938*)	(0.490*)	(0.567*)
GARCH(1,1)	0.001	0.003	0.005	0.006	0.006	0.007
	(0.149*)	(0.065)	(0.011)	(0.063)	(1.000*)	(1.000*)
FIGARCH(1,1)	0.001	0.003	0.005	0.006	0.007	0.007
	(0.149*)	(0.065)	(0.003)	(0.000)	(0.020)	(0.000)
ARFIMA(0,d,0)	0.000	0.002	0.005	0.007	0.014	0.043
	(0.149*)	(0.298*)	(0.011)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.000	0.002	0.005	0.006	0.007	0.011
	(0.156*)	(0.578*)	(1.000*)	(1.000*)	(0.020)	(0.000)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a $(*)$ indicates that the model is within the 10% MCS using all comparisons.

Table 9 (continued). Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t r^2_{t+\tau}]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Brazil			
Basic RLS	0.000	0.000	0.000	0.002	0.013	0.044
	(0.563*)	(0.011)	(0.000)	(0.000)	(0.002)	(0.000)
Threshold 1% RLS	0.000	0.000	0.000	0.001	0.012	0.042
	(0.563*)	(0.011)	(0.000)	(0.000)	(0.004)	(0.000)
Mean Reversion RLS	0.000	0.000	0.000	0.001	0.012	0.043
	(0.331*)	(0.011)	(0.000)	(0.350*)	(0.004)	(0.000)
Modified RLS	0.000	0.000	0.000	0.001	0.012	0.043
	(1.000*)	(1.000*)	(1.000*)	(1.000*)	(0.004)	(0.000)
GARCH(1,1)	0.000	0.000	0.001	0.002	0.011	0.030
	(0.006)	(0.000)	(0.000)	(0.000)	(1.000*)	(1.000*)
FIGARCH(1,1)	0.000	0.001	0.002	0.008	0.041	0.140
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.000	0.001	0.002	0.007	0.040	0.148
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,1)	0.000	0.000	0.001	0.005	0.025	0.087
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a $(*)$ indicates that the model is within the 10% MCS using all comparisons.

Table 9 (continued). Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t r^2_{t+\tau}]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Chile			
Basic RLS	0.000	0.000	0.000	0.001	0.005	0.014
	(0.124*)	(0.051)	(0.202*)	(0.026)	(0.000)	(0.000)
Threshold 1% RLS	0.000	0.000	0.000	0.001	0.005	0.014
	(1.000*)	(1.000*)	(1.000*)	(0.002)	(0.000)	(0.000)
Mean Reversion RLS	0.000	0.000	0.000	0.001	0.005	0.013
	(0.144*)	(0.055)	(0.252*)	(0.635*)	(1.000*)	(1.000*)
Modified RLS	0.000	0.000	0.000	0.001	0.005	0.013
	(0.146*)	(0.057)	(0.305*)	(1.000*)	(0.000)	(0.000)
GARCH(1,1)	0.000	0.000	0.000	0.001	0.006	0.021
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
FIGARCH(1,1)	0.000	0.000	0.000	0.001	0.007	0.021
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.000	0.000	0.001	0.002	0.007	0.023
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.000	0.000	0.001	0.002	0.008	0.026
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a $(*)$ indicates that the model is within the 10% MCS using all comparisons.

Table 9 (continued). Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t r^2_{t+\tau}]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Colombia			
Basic RLS	0.000	0.000	0.001	0.003	0.013	0.045
	(0.410*)	(0.034)	(0.009)	(0.357*)	(0.997*)	(0.394*)
Threshold 1% RLS	0.000	0.000	0.001	0.004	0.015	0.047
	(0.410*)	(0.041)	(0.002)	(0.003)	(0.001)	(0.000)
Mean Reversion RLS	0.000	0.000	0.001	0.003	0.014	0.046
	(0.410*)	(0.977*)	(1.000*)	(1.000*)	(0.505*)	(0.235*)
Modified RLS	0.000	0.000	0.001	0.003	0.014	0.046
	(1.000*)	(1.000*)	(0.019)	(0.357*)	(0.196*)	(0.195*)
GARCH(1,1)	0.000	0.000	0.001	0.004	0.013	0.044
	(0.127*)	(0.004)	(0.000)	(0.000)	(1.000*)	(1.000*)
FIGARCH(1,1)	0.000	0.000	0.001	0.005	0.020	0.068
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.000	0.000	0.001	0.005	0.019	0.062
	(0.008)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,1)	0.000	0.000	0.001	0.005	0.020	0.063
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a $(*)$ indicates that the model is within the 10% MCS using all comparisons.

Table 9 (continued). Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t r^2_{t+\tau}]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Mexico			
Basic RLS	0.000	0.000	0.001	0.005	0.022	0.070
	(0.549*)	(0.619*)	(0.034)	(0.001)	(0.000)	(0.000)
Threshold 1% RLS	0.000	0.000	0.001	0.005	0.023	0.070
	(1.000*)	(1.000*)	(0.064)	(0.001)	(0.000)	(0.000)
Mean Reversion RLS	0.000	0.000	0.001	0.005	0.022	0.069
	(0.380*)	(0.619*)	(1.000*)	(1.000*)	(1.000*)	(0.000)
Modified RLS	0.000	0.000	0.001	0.005	0.023	0.069
	(0.795*)	(0.740*)	(0.178*)	(0.002)	(0.000)	(0.000)
GARCH(1,1)	0.000	0.000	0.002	0.006	0.024	0.064
	(0.380*)	(0.619*)	(0.009)	(0.001)	(0.000)	(1.000*)
FIGARCH(1,1)	0.000	0.001	0.002	0.008	0.034	0.090
	(0.894*)	(0.418*)	(0.007)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.000	0.001	0.002	0.007	0.027	0.079
	(0.006)	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.000	0.001	0.002	0.006	0.025	0.069
	(0.029)	(0.046)	(0.000)	(0.000)	(0.000)	(0.001)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a (∗) indicates that the model is within the 10% MCS using all comparisons.

Table 9 (continued). Forecast Evaluations $[\hat{y}_{t+\tau|t} = E_t r^2_{t+\tau}]$

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
				Peru		
Basic RLS	0.000	0.000	0.000	0.000	0.000	0.001
	(0.370*)	(0.032)	(1.000*)	(1.000*)	(0.299*)	(0.000)
Threshold 1% RLS	0.000	0.000	0.000	0.000	0.000	0.001
	(0.864*)	(0.007)	(0.092)	(0.000)	(0.379*)	(1.000*)
Mean Reversion RLS	0.000	0.000	0.000	0.000	0.000	0.001
	(0.853*)	(1.000*)	(0.363*)	(0.001)	(0.287*)	(0.118*)
Modified RLS	0.000	0.000	0.000	0.000	0.000	0.001
	(1.000*)	(0.084)	(0.194*)	(0.001)	(0.287*)	(0.048)
GARCH(1,1)	0.000	0.000	0.000	0.000	0.000	0.001
	(0.045)	(0.000)	(0.000)	(0.000)	(1.000*)	(0.000)
FIGARCH(1,1)	0.000	0.000	0.000	0.000	0.001	0.004
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.000	0.000	0.000	0.000	0.000	0.002
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.000	0.000	0.000	0.000	0.000	0.002
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The MSFEs are reported in the main entires; the MCS p-values are in parenthesis; a (∗) indicates that the model is within the 10% MCS using all comparisons.

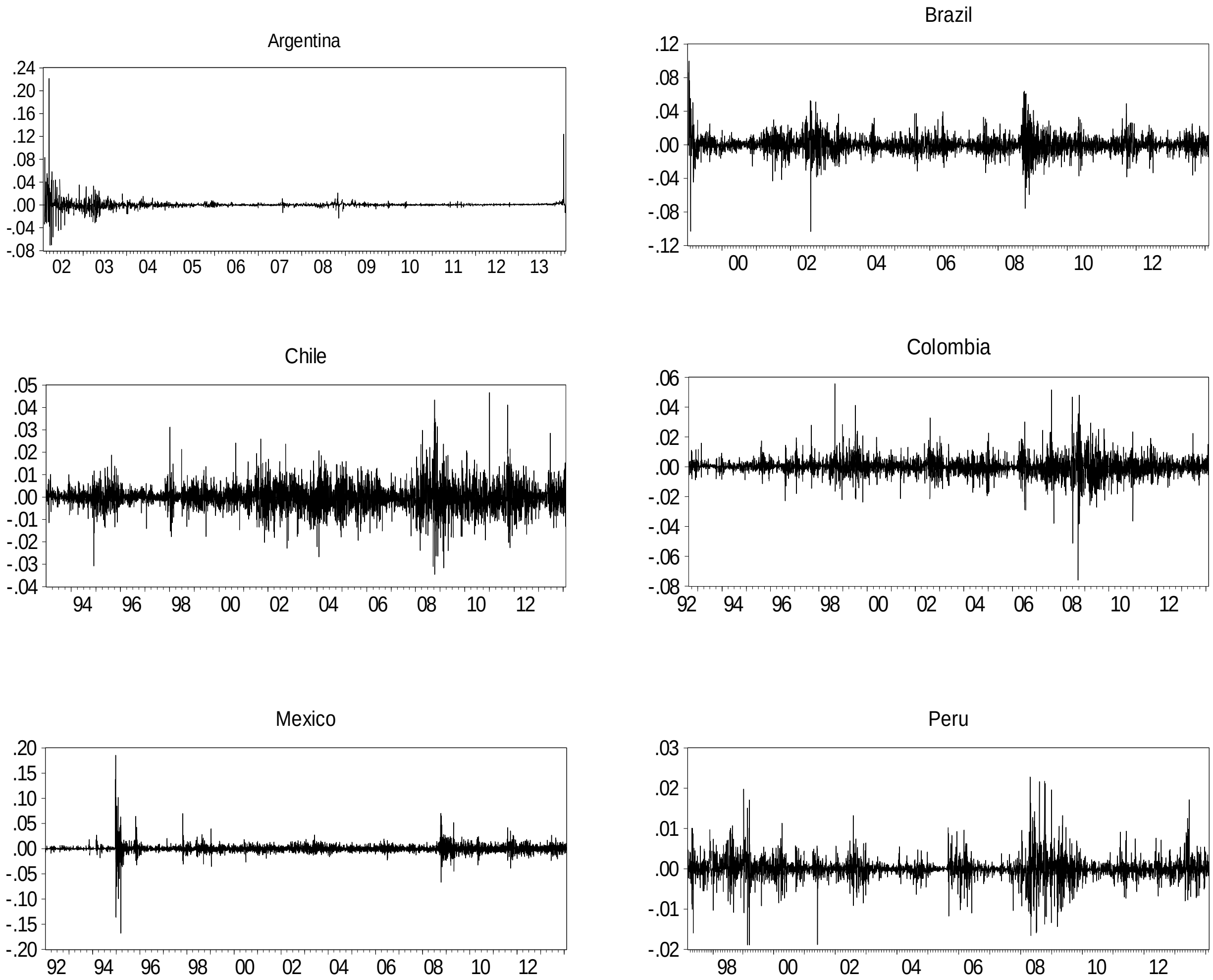

Figure 1. Daily Forex Returns

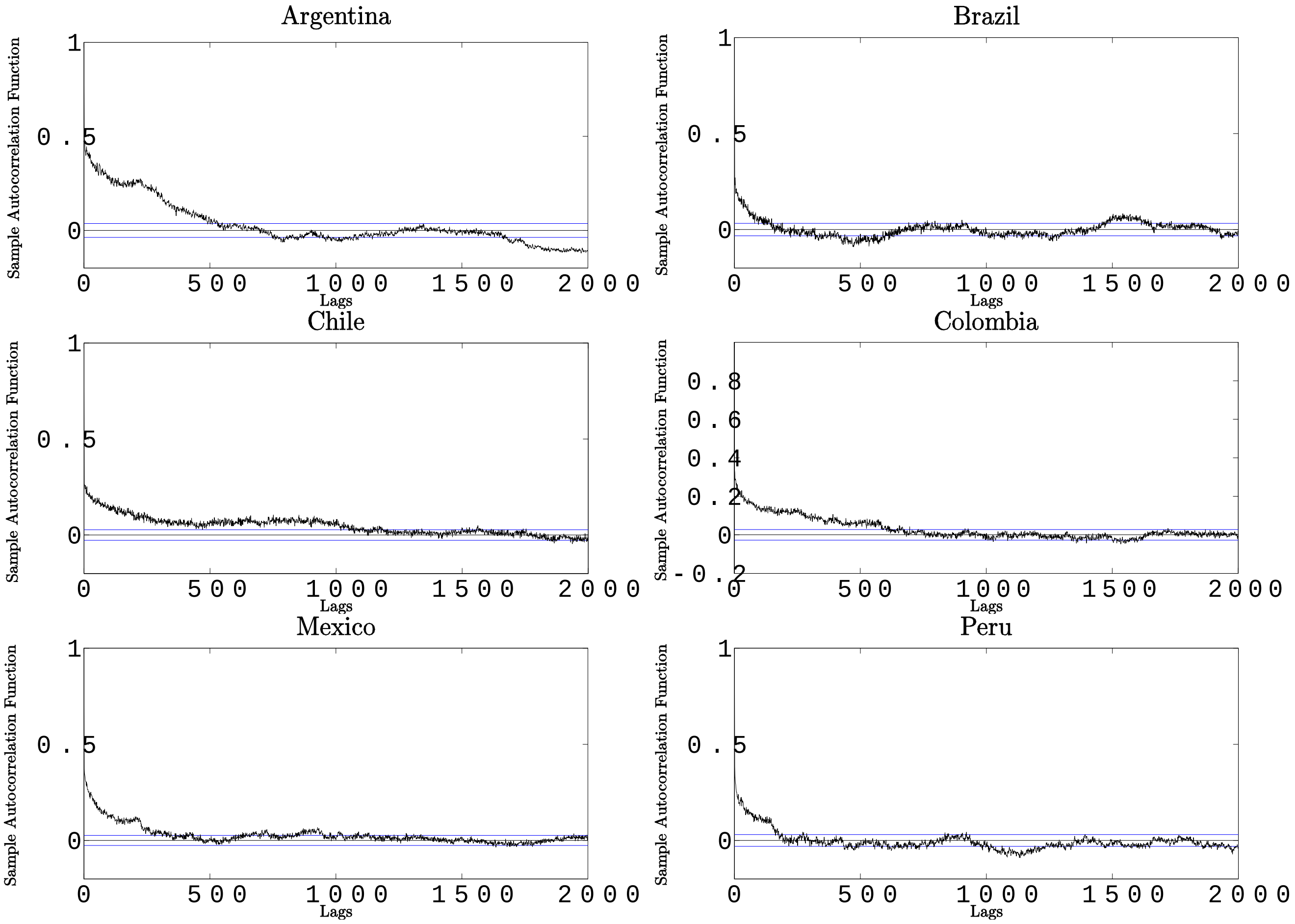

Figure 2. Sample ACF of Forex Returns Volatility

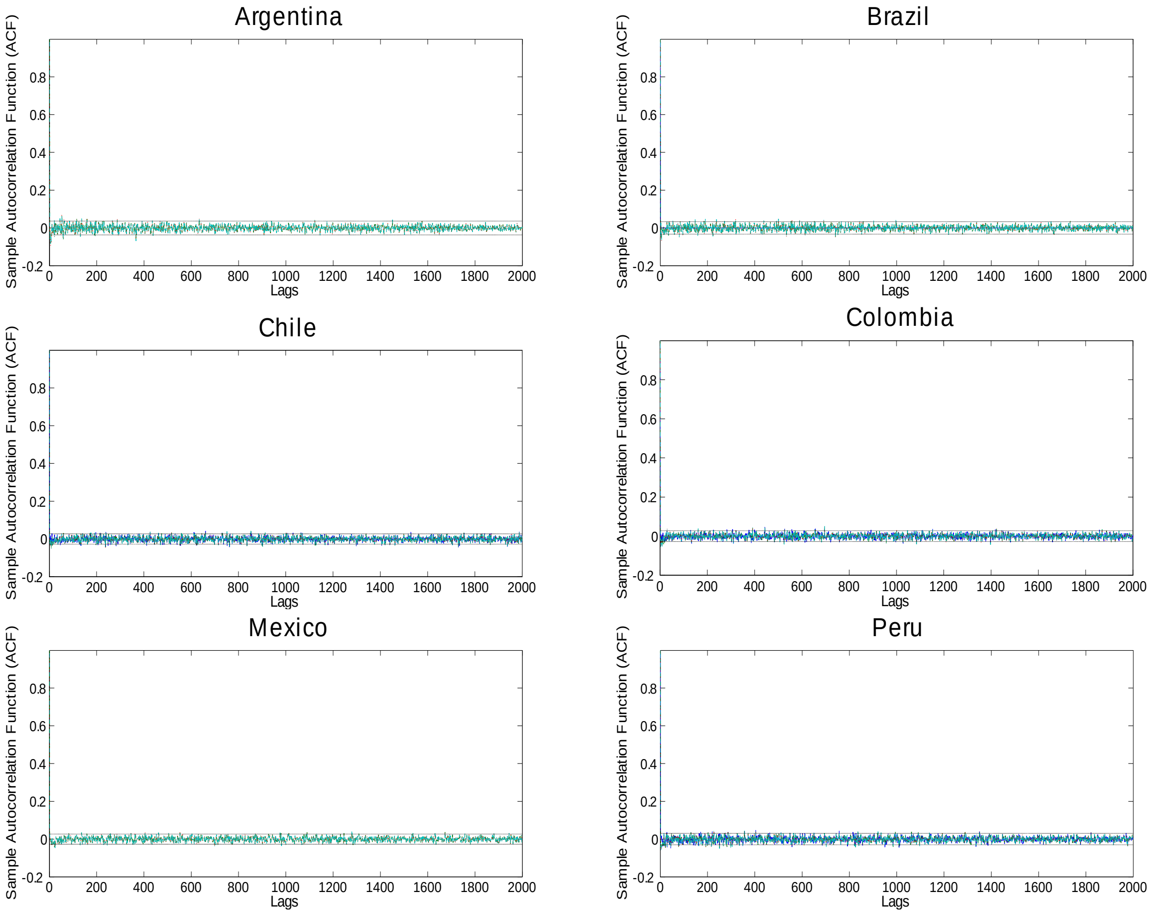

Figure 3. Sample ACF of the Short Memory Components: Volatility minus Smoothed Level Shift Component from Basic RLS (Solid line), Threshold 1% RLS (Dashed line), Mean Reversion RLS (Dotted line) and from Modified RLS (Dash-Dot line).

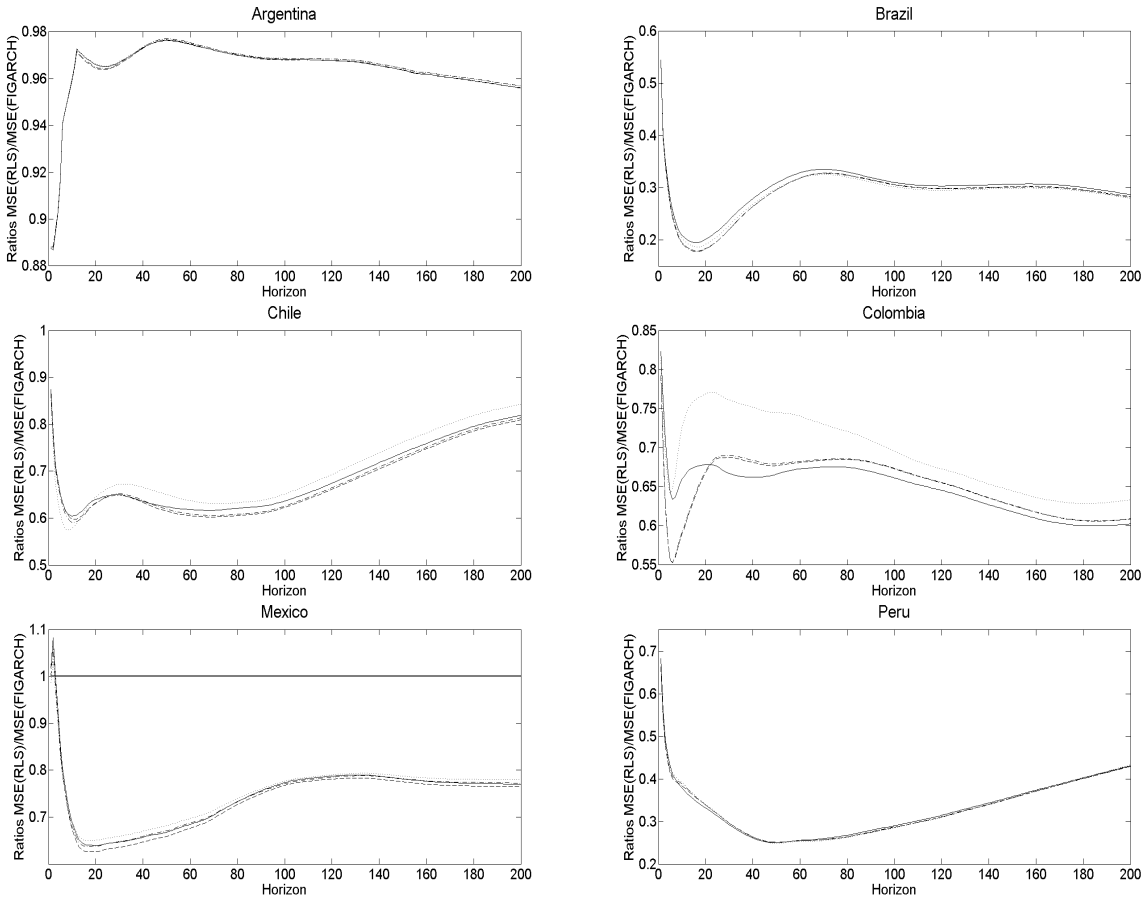

Figure 4. In sample RLS versus out-of-sample FIGARCH forecasts: Ratios MSE("i" RLS Model)/MSE(FIGARCH); i = Basic, solid line; i = 1% Threshold, dotted line; i = Mean Reversion, dashed line; i = Modified, dash-dot line.

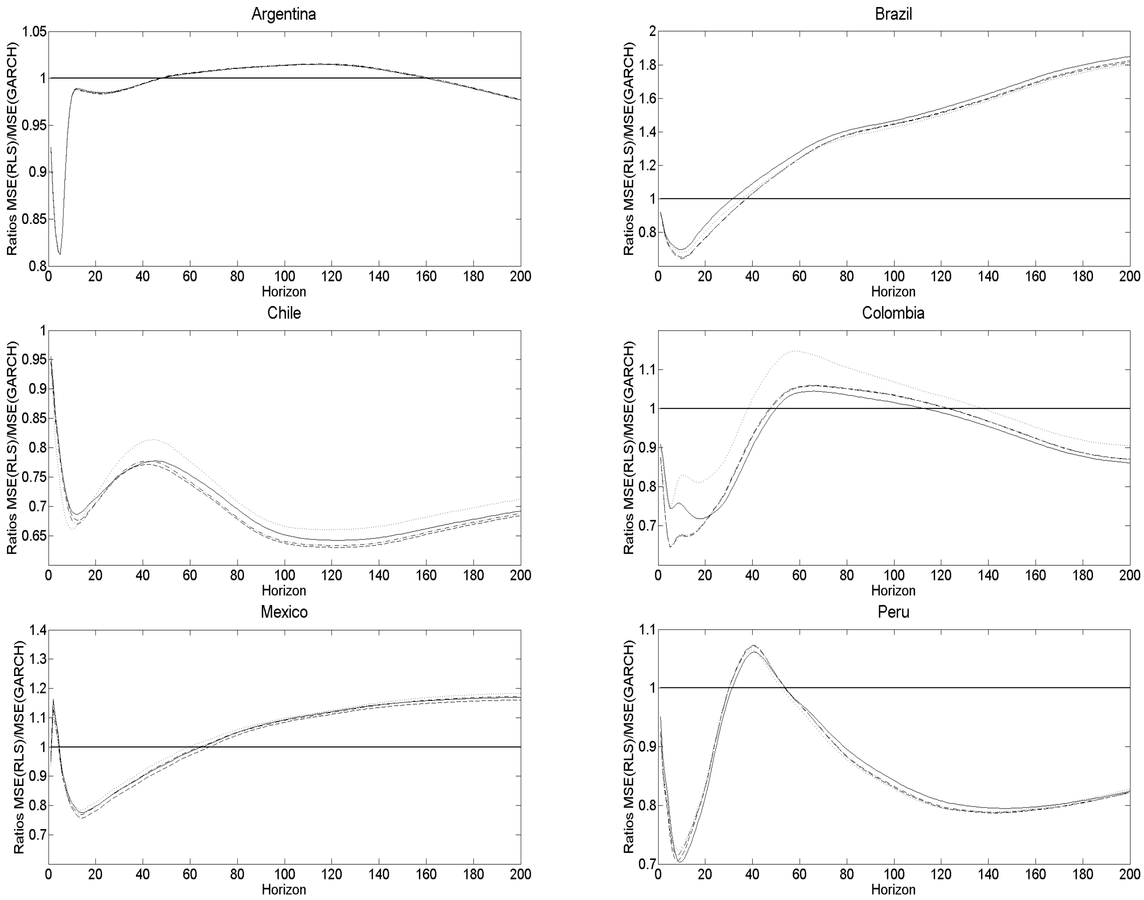

Figure 5. In sample RLS versus out-of-sample GARCH forecasts: Ratios MSE("i" RLS Model)/MSE(GARCH); i = Basic, solid line; i = 1% Threshold, dotted line; i = Mean Reversion, dashed line; i = Modified, dash-dot line.

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