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Abstract

Following Varneskov and Perron (2014), I apply the RLS-ARFIMA(0,d,0) and the RLS-ARFIMA (1,d,1) models to the daily stock and Forex market returns volatility of Argentina, Brazil, Chile, Mexico and Peru. It is a parametric state-space model with an estimation framework that combines long memory and level shifts by decomposing the underlying process into a simple mixture model and ARFIMA dynamics. The full sample parameters estimates show that level shifts are rare but they are present in all series. A genuine long-memory component is present in volatility of some countries and the results suggest that the remaining short-memory component is nearly uncorrelated once the level shifts are accounted for. I compare the results with four RLS models as in Xu and Perron (2014) and applied in Rodríguez (2016) for same Latin-American series. An out-of-sample forecasting comparison is also performed using the approach of Hansen et al. (2011). The RLS-ARFIMA models presents better performance for some horizons while the other four RLS models are better for other horizons. In none horizon of forecasting, simple ARFIMA models are selected or belong to the 10% of the MCS of Hansen et al. (2011).

JEL Clasification: C22, C52, G12.

Keywords: Long Memory, Random Level Shifts, ARFIMA Models, GARCH Effects, Stock Markets, Latin-America, Volatility.

Resumen

Siguiendo Varneskov y Perron (2014), yo aplico los modelos RLS-ARFIMA (0,d,0) y RLS-ARFIMA (1,d,1) a datos diarios de las volatilidades de los mercados bursátiles en Argentina, Brasil, Chile, México y Perú. Este modelo es de tipo paramétrico en forma espacio estado y su estimación combina una mezcla de un proceso de larga memoria con cambios de nivel. Esto se hace descomponiendo el proceso subyacente en un modelo de mezcla simple con dinámica ARFIMA. Las estimaciones de parámetros muestran que los cambios de nivel son raros, pero ellos están presentes en todas las series analizadas. Un componente de genuina larga memoria es encontrado en la volatilidad de algunos países y los resultados sugieren que el componente de corta memoria remanente es prácticamente no correlacionado una vez que los cambios de nivel son tomados en cuenta. Los resultados son comparados con aquellos obtenidos en cuatro modelos propuestos en Xu y Perron (2014) y también usados en Rodríguez (2016) para las mismas series de América Latina. Una comparación de predicción fuera de muestra también es realizada usando el enfoque de Hansen et al. (2011). Los modelos RLS-ARFIMA presentan mejor performance para algunos horizontes, mientras que los otros cuatro modelos RLS son mejores para otros horizontes. En ningún horizonte de predicción se seleccionan los modelos ARFIMA, o en ningún caso pertenecen al 10% del MCS propuesto por Hansen et al. (2011).

Clasificación JEL: C22, C52, G12.

Palabras Claves: Larga Memoria, Cambios de Nivel Aleatorios, Modelos ARFIMA, Efectos GARCH, Mercados Bursátiles, América Latina, Volatilidad.

Modeling Latin-American Stock and Forex Markets Volatility: Empirical Application of a Model with Random Level Shifts and Genuine Long Memory[1]

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1 Introduction

Typically, the volatility of financial time series exhibits long-term dependence or long memory. This property is represented in the domain of time by the behavior of its autocorrelation function (ACF), which presents significantly different values from zero up to a large number of lags, indicating hyperbolic decay. In the domain of frequencies, a higher weight of the low frequencies in the spectral density is observed, as is a rapid growth in this function as the frequencies approach the origin. Several authors document this characteristic; see Taylor (1986), Ding et al. (1993), Dacorogna et al. (1993), Robinson (1994), among others. There are several possible formalizations for this definition; see McLeod and Hipel (1978), Beran (1994), Robinson (1994), and Baille (1996), among others. I follow the definitions presented in Perron and Qu (2010). Let $\{x_t\}_{t=1}^{T}$ be a stationary time series with spectral density function $f_x(\omega)$ at frequency ω, so x_t has long memory if $f_x(\omega) = g(\omega)\omega^{-2d}$, for $\omega \to 0$, where $g(\omega)$ is a function of smooth variation in a vicinity of the origin, which means that for all real numbers t, it is verified that $g(t\omega)/g(\omega) \to 1$ for $\omega \to 0$. The spectral density function increases for frequencies increasingly close to the origin, depending on the value of the parameter d. The divergent infinite rate depends on the value of the parameter d. On the other hand, let $\gamma_x(\tau)$ be the ACF of x_t, so x_t has long memory if $\gamma_x(\tau) = c(\tau)\tau^{2d-1}$, for $\tau \to \infty$, where $c(\tau)$ is a function of smooth variation. When $0 < d < 1/2$ the ACF decays at a slow rate that depends on the value of parameter d[3]. Another way to formalize the concepts is to say that $x_t \sim I(d)$, where $x_t = (1-L)^d e_t$ with $e_t = C(L)\epsilon_t$, $\epsilon_t \sim i.i.d.$ $(0, \sigma_\epsilon^2)$ and $E|\epsilon_t|^r < \infty$ for some $r > 2$ to be a short memory process with lag polynomial $C(L) = \sum_{i=0}^{\infty} c_i L^i$ satisfying $\sum_{i=0}^{\infty} i|c_i| < \infty$ and $C(1) \neq 0$.

Granger and Joyeux (1980) and Hosking (1981) introduced the ARFIMA(p,d,q) model as a parametric way of capturing long memory dynamics. There is also literature on semiparametric estimators of the fractional parameter d where the most used estimators are the method of Geweke and Porter-Hudak (1983) using the log-periodogram; see also Robinson (1995a). In addition, there is the local Whittle estimator of Kunsch (1987) and Robinson (1995b); see also Andersen et al. (2003). Another way to capture the long-memory behavior is by mixing it with GARCH effects, as in the Fractional Integrated GARCH (FIGARCH) model proposed by Baillie et al. (1996).

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[3] These definitions in the domain of frequency and time are equivalent if certain general conditions are verified, in accordance with the findings in Beran (1994).

Recently, the literature has focused increasingly on the possibility of long memory behavior being confused with a short memory process contaminated by level shifts[4]. Important and interesting references in respect of this are Lobato and Savin (1998), Diebold and Inoue (2001), Granger and Hyung (2004), and Perron and Qu (2007, 2010), among others. For instance, Diebold and Inoue (2001) argue that long memory and structural changes are related through the following models: the simple mixture permanent stochastic breaks model of Engle and Smith (1999) and the Markov-Switching model of Hamilton (1989). The authors show analytically that stochastic regime shifts are easily confused with long memory, even asymptotically, provided that the probabilities of structural breaks are small. The Monte Carlo simulations attest to the relevance of the finite samples theory, and make it clear that confusion is not only a theoretical matter, but a real possibility in empirical economic and financial applications. Granger and Hyung (2004), for their part, show that the slow decay in the ACF and other properties of the fractionally integrated models are caused by occasional breaks. Analytically, the authors show that not taking the breaks into consideration causes the presence of long memory in the ACF and that the fractional parameter estimated using the method of Geweke and Porter-Hudak (1983) is biased.

Other references are Teverovsky and Taqqu (1997). Using the daily returns of the Center for Research in Security Prices (CRSP) for the period 1962-1987, they present a method for distinguishing between the effects of level shifts and the effects of long memory. Gourieroux and Jasiak (2001) evaluate the relationship between the presence of infrequent breaks and long memory based on the correlogram estimation instead of estimating the fractional parameter. The authors find that non-linear time series with infrequent breaks could have long memory. Therefore, these series and not the fractionally integrated processes with *i.i.d.* innovations would cause the hyperbolic decay of the ACF. Other references are Mikosch and Stărică (2004a, 2004b), and Stărică and Granger (2005). The principal finding of this branch of the literature is that if a short memory process is contaminated by level shifts, the time series will display many of the same properties as a genuine long-memory process.

A recent study on the analysis of long memory and level shifts, or structural shifts, is that of Perron and Qu (2010). The authors present a method of distinguishing between long memory and level shifts using the ACF, the periodogram, and the estimates of the fractional integration parameter d. Perron and Qu (2010) propose a simple mixture model that combines a short memory process and a component that reflects the level shifts determined by an occurrence variable following a Bernouilli process. Applying this method to the log-squared returns of four indices (S&P 500, NASDAQ, AMEX and Dow Jones), they conclude that the model that best describes the volatility of the returns is that which considers a short memory process with a random level shifts component (denoted by RLS). Lu and Perron (2010), and Li and Perron (2013) use the RLS model to model the volatility of stock market and exchange rate returns, respectively. For Latin American stock and Forex markets we have studies such as Gonzáles Tanaka and Rodríguez (2016), Ojeda Cunya and Rodríguez (2016), Pardo Figueroa and Rodríguez (2014), Rodríguez (2016), and Rodríguez and Tramontana Tocto (2015).

Other attempts to mix both RLS and short memory dynamics are Chen and Tiao (1990), and McCulloch and Tsay (1993), all of whom reach the conclusion that the long-memory properties of the data are spurious. Similar conclusions have been reached by another branch of the literature

[4] This distinction is an important methodological aspect to discern. Impacts of shocks on volatility are transitory or long-term if it is shown that volatility is a short memory or long memory process, respectively. This has consequences for practical or empirical terms. For instance, the impact of different measures of the Central Bank or the agents in the stock market is different depending on the behavior of volatility.

dealing with semiparametric estimation of the fractional parameter in the presence of structural breaks; see Smith (2005), McCloskey and Hill (2013), and McCloskey and Perron (2013). A drawback of this approach is that level shifts are not identified, making it unsuitable for forecasting[5].

From the above-mentioned results it is clear that the presence of level shifts may cause spurious detection of genuine long memory behavior, and consequently result in a misspecification. However, some of the literature argues the reverse; that is, that genuine long memory behavior may also cause spurious detection of random level shifts. This issue is proposed by Nunes et al. (1995), and Granger and Hyung (2004). A solution to this dual problem has been advocated by Varneskov and Perron (2014), who propose a parametric model that allows for both random level shifts and long memory; that is, by modelling the latent volatility process as a combination of a random level shifts component and ARFIMA dynamics (denoted as the RLS-ARFIMA (p,d,q) model). This approach is close to that of Ray and Tsay (2002) in that both model structural changes in the presence of genuine long memory. However, these studies are otherwise markedly different. Ray and Tsay (2002) use a Markov-Switching approach while Varneskov and Perron (2014) use the RLS methodology. This fundamental change allows for the fact that level changes are random; that is, at any time t a level shift can occur regardless of whether this occurred at time $t-1$. This allows level shifts to be independent of past observations. Further, the state-space model of Varneskov and Perron (2014) nests a RLS model with the short memory ARMA dynamics of Lu and Perron (2013). The estimation procedure is similar to the one used in Lu and Perron (2010) with significant changes and adequacy due to the presence of ARFIMA dynamics. The base of the procedure is taken from Wada and Perron (2006), and Perron and Wada (2009).

I apply the RLS-ARFIMA(0,d,0) and RLS-ARFIMA (1,d,1) models to the daily stock and Forex markets returns volatility of Argentina, Brazil, Chile, Mexico, and Peru. The full sample parameters estimates show that level shifts are rare but present in all series. I compare the results with four RLS models, as in Xu and Perron (2014), Rodríguez (2016), and Gonzáles Tanaka and Rodríguez (2016). The results suggest that estimates of the fractional parameter using daily data are very small. These results are in accordance with Varneskov and Perron (2014). They find that estimates of the fractional parameter for high frequency data are higher, around 0.40. However, estimates of this parameter using daily frequency data are smaller and in many cases close to zero. Our estimates using daily data suggest the same observation. Therefore, Varneskov and Perron (2014) suggest that residual dynamics extracted using daily data may be characterized as a combination of short memory dynamics and measurement errors. Given the small values of the estimates of the fractional parameter, I argue that the daily volatility series I am using are better modeled as a short memory process contaminated by rare level shifts, and only in a some cases is there genuine long memory. In this regard my results corroborate the findings in similar markets of Rodríguez and Tramontana Tocto (2015), Gonzáles Tanaka and Rodríguez (2016), Ojeda Cunya and Rodríguez (2016), and Rodríguez (2016). An out-of-sample forecasting comparison is also performed using the approach of Hansen et al. (2011). The RLS-ARFIMA models is seen to perform better for some horizons while the other four RLS models are better for others. This is a logical consequence of the small magnitude of the estimates of the fractional parameter in the RLS-ARFIMA models. In no forecasting horizon are simple ARFIMA (p,d,q) selected, and nor do they belong to the 10% of the MCS of Hansen et al. (2011).

[5] Still more similar conclusions are obtained from the branch of the literature that focuses on testing for spurious long memory against the alternative of a short-memory process contaminated by level shifts; see Ohanissian et al. (2008), Perron and Qu (2010) and Qu (2011). See also an empirical application to Latin American stock markets in Pardo and Rodríguez (2014).

This study is structured as follows. Section 2 briefly describes the RLS-ARFIMA model, other RLS models, and the estimation method. Section 3 deals with the results, with a description of the data, estimations of parameters, and a forecasting exercise. Finally, Section 4 presents the conclusions.

2 Methodology

In this Section, I briefly describe the RLS-ARFIMA (p,d,q) model proposed by Varneskov and Perron (2014). Some details regarding the estimation method are established. Other RLS models are also presented.

2.1 The RLS Model with a Long Memory Process

Using the notation of Varneskov and Perron (2014), the RLS-ARFIMA model is specified as:

$$\begin{aligned} y_t &= a + \tau_t + h_t, \\ \tau_t &= \tau_{t-1} + \delta_{T,t}, \\ \delta_{T,t} &= \pi_t \eta_t, \end{aligned} \tag{1}$$

where the level shifts component τ_t is a random walk with innovations $\delta_{T,t}$ that obey a mixture of two Normally distributed processes according to $\delta_{T,t} = \pi_{T,t}\eta_{1t} + (1 - \pi_{T,t})\eta_{0t}$ with $\eta_{jt} \sim i.i.d.$ $N(0, \sigma^2_{\eta_j})$ for $j = 0, 1$. The variable $\pi_t \sim Bernoilli(\gamma/T)$ for some $\gamma \in [0, T]$. Furthermore, π_t, η_t and h_t are mutually independent. The Bernoulli probability of the random level shift process is dependent on the sample size, T, to make the expected number of level shifts constant for a given series. This is needed to model structural changes in the mean (or rare events), which affect the properties of the series until the next shift (event) occurs. Following Varneskov and Perron (2014), I impose the restrictions $\sigma^2_{\eta_1} = \sigma^2_\eta$ and $\sigma^2_{\eta_0} = 0$.

The long-memory component h_t may be written as an $AR(\infty)$ process, $h_t = \sum_{i=1}^{\infty} \psi_i h_{t-i} + \epsilon_t$ where $\sum_{i=1}^{\infty} \psi_i L^i = \frac{\phi(L)}{\theta(L)}(1-L)^d$. Since $d \in [0, 1/2)$, the roots of $\phi(x) = 0$ and $\theta(x) = 0$ are outside the unit circle, $\phi(L)$ and $\theta(L)$ do not to have common roots, and h_t has a unique and stationary solution[6]. The contribution of the fractional difference filter may be written as a binomial expansion $(1-L)^d = \sum_{i=0}^{\infty} \pi_i L^i$ with $\pi_i = \Gamma(i-d)/\Gamma(i+1)\Gamma(-d)$ where $\Gamma(.)$ is the Gamma function. Using this representation, we may write Δy_t as an infinite dimensional difference process $\Delta y_t = h_t - h_{t-1} + \delta_{T,t}$ for $t = 2, ..., T$. The basic principle behind the estimation procedure is to augment the probability of states through the realizations of the mixture of normally distributed processes at time t, and apply the Kalman filter to construct the likelihood function conditional on the realization of states. Unfortunately, Δy_t does not have finite dimensional state-space representation unless $d = 0$ and p, $q < \infty$ which is an issue similarly faced by Chan and Palma (1998) and Beran (1995). Varneskov and Perron (2014), following the literature, suggest approximating the $AR(\infty)$ process by an AR(M) where M is chosen depending on the length of the modeled series[7]. Hence, the approximate state-space representation of Δy_t in matrix form is $\Delta y_t = FH_t + \delta_{T,t}$, $H_t = GH_{t-1} + E_t$ where $F = [1, -1, 0, ...0]$, $H_t = [h_t, h_{t-1}, ..., h_{t-M+1}]'$, and $E_t = [\epsilon_t, 0, ..., 0]'$ are $M \times 1$ vectors where $E_t \sim i.i.d.$ $N(0_{M\times 1}, Q)$ and $0_{M\times 1}$ denotes a $M \times 1$ vector of zeros. Here, G and Q are both $M \times M$ matrices of

[6] See Brockwell and Davis (1991).

[7] Further and specific details are in Varneskov and Perron (2014).

parameters and identifying terms, $G = \begin{bmatrix} \Psi_{M-1} & \psi_M \\ I_{M-1} & 0_{(M-1)\times 1} \end{bmatrix}$ and $Q = \begin{bmatrix} \sigma_\epsilon^2 & 0_{1\times(M-1)} \\ 0_{(M-1)\times 1} & 0_{(M-1)\times(M-1)} \end{bmatrix}$, where $\Psi_M = [\psi_1, \psi_2, ..., \psi_M]$ is $1 \times M$ and I_M is M dimensional identity matrix.

Let the available information up to time t be denoted by the vector $Y_t = [\Delta y_1, \Delta y_2, ..., \Delta y_T]$, and let the parameter vector be denoted by $\Sigma = [\sigma_\eta, p, \sigma_\epsilon, d, \phi(L), \theta(L)]$. Then, we can express the conditional log-likelihood function as $\ln(L) = \sum_{t=1}^{T} \ln f[\Delta y_t|Y_{t-1};\Sigma]$, $f[\Delta y_t|Y_{t-1};\Sigma] = \sum_{i=0}^{1}\sum_{j=0}^{1} f[\Delta y_t|s_{t-1} = i, s_t = j, Y_{t-1};\Sigma] \times \Pr[s_{t-1} = i, s_t = j;\Sigma]$ where s_t is an indicator for the particular state at time t, which is independent of past realizations. If a level shift occurs $\pi_{T,t} = 1$, then $s_t = 1$, and similarly $s_t = 0$ if a level shift does not occur, $\pi_{T,t} = 0$[8]. The two possible states cause the number of estimates for the state vector and its conditional variance to grow exponentially over time with a factor of t^2. A solution to this, suggested in Harrison and Stevens (1976), is to re-collapse $H_{t|t}^{ij}$ and $P_{t|t}^{ij}$ to ensure they are unaffected by the history of states before time $t-1$. By increasing the estimation complexity relative to Markov regime-switching models, we gain the modeling flexibility of allowing for shifts of unknown timing, frequency and magnitude.

The added challenge relative to the long-memory state-space framework of Chan and Palma (1998) arises from the shift-dependent error in the measurement equation, whereas relative to Lu and Perron (2010) it is the presence of $(1-L)^d/\theta(L)$ in the representation of h_t potentially with $d \in [0, 1/2)$ - such that no finite state space representation exists - that causes additional difficulties. The modeling strategy is similar to the approach suggested by Ray and Tsay (2002). However, Varneskov and Perron (2014) introduce an estimation methodology that augments the Bayesian approach in Ray and Tsay (2002) in three different directions, by allowing for a short memory ARMA process, by allowing level shifts to occur at each time t, and not in blocks, and finally, extending their analysis by providing a forecasting framework. The methodology is able to capture short-term changes in mean as well as rare shifts, and it can be used for out-of-sample forecasting. This specification also has the advantage of making level shifts random events that do not depend on past realizations of the data.

2.2 Other RLS Models

The DGP in (1) nests all other types of RLS, ARMA and ARFIMA models with simple variations of the parameters d, γ/T or σ_η. For instance, if we let $d = 0$, we retrieve the basic RLS model proposed by Lu and Perron (2010) where the short memory component is modeled as an ARMA (0,0) model with $\gamma/T = \alpha$. On the other hand, if $\gamma = 0$ or $\sigma_\eta = 0$ we retrieve the ARFIMA model. In the case of the basic RLS model, as the probability of level shifts is constant ($\gamma/T = \alpha$), we can retrieve the dates of these level shifts using the approach of Bai and Perron (2003). In this method, the number and dates of breaks are selected by minimizing the squared sum of residuals: $\sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [y_t - \mu_i]^2$, where m is the number of breaks, T_i $(i = 1, 2, ...; m)$ are the dates of the breaks with $T_0 = 0$, and $T_{m+1} = T$ and μ_i $(i = 1, 2, ..., m+1)$ are the means (averages) inside each regime. This method is efficient and can manage a long number of observations; see Bai and

[8] As such, the likelihood function resembles its counterpart for Markov regime switching models -see e.g. Hamilton (1994a)- but it has two more complexities. First, the mean and the variance of the conditional density are nonlinear functions of the past realizations and the fundamental parameters. Hence, we cannot separate all elements and apply a standard EM algorithm. Second, the conditional probability of being in a given regime is not separable from the conditional density.

Perron (2003) for further details[9].

There are three extensions to the basic RLS model, according to Xu and Perron (2014). In the first extension, constant probabilities of level shifts are changed by time-varying probabilities, depending on past extreme returns based on different threshold levels. The level shifts usually occur in clusters in certain periods of time related to financial crisis. This phenomenon of clustering indicates that level shifts are not *i.i.d.*, but that the probability of these shifts varies in accordance with economic, political, and social conditions in the countries. Following on from the notation used in Xu and Perron (2014), the probability of level shift is defined as $p_t = f(p, x_{t-1})$, where p is a constant and x_{t-1} are the covariates that help to better predict the probability of level shifts. According to the study by Martens et al. (2004), there is a strong relationship between current volatility and past returns, also known as the leverage effect. This effect has been modeled through the news impact curve proposed by Engle and Ng (1993). In the current framework it is expressed as $\log(\sigma_t^2) = \beta_0 + \beta_1 \mathbf{1}(r_{t-1} < 0) + \beta_2 |r_{t-1}| \mathbf{1}(r_{t-1} < 0)$, where σ_t^2 represents the volatility and $\mathbf{1}(A)$ is the indicator function that takes the value of one when the event A occurs. In the current case, the variable x_{t-1} is represented by extreme past returns that are below a threshold κ%. Therefore, I employ the returns that belong to 1%, 2.5% and 5% of the distribution of the returns ($\kappa = 1.0\%, 2.5\%, 5.0\%$). Thus, the probability of level shifts is given by $f(p, x_{t-1}) = \left\{ \begin{array}{l} \Phi(p + \gamma_1 \mathbf{1}\{x_{t-1} < 0\} + \gamma_2 \mathbf{1}\{x_{t-1} < 0\} |x_{t-1}|) \text{ for } |x_{t-1}| > \kappa \\ \Phi(p) \quad \text{other cases,} \end{array} \right\}$, where $\Phi(.)$ is the Normal accumulated distribution, with which we ensure that $f(p, x_{t-1})$ is between 0 and 1. This model is denoted as the Threshold κ% RLS model.

The second extension of the basic RLS model is the introduction of a mean reversion mechanism (with the constant probability of level shifts remaining). The level shifts occur around a mean; that is, each time a level shift occurs and the volatility of the series increases, a similar change occurs in the opposite direction, which makes the mean of the volatility remains at a given value. This process of mean reversion is modeled as follows: $\eta_{1t} = \beta(\tau_{t|t-1} - \overline{\tau}_t) + \widetilde{\eta}_{1t}$, where $\widetilde{\eta}_{1t}$ is distributed Normally with mean 0 and variance $\sigma^2_{\widetilde{\eta}_1}$, $\tau_{t|t-1}$ is the estimated level shift component at time t, and $\overline{\tau}_t$ is the mean of all level-shift components estimated from the start of the sample to time t. The process of mean reversion occurs when $\beta < 0$ and this parameter represents the velocity at which the volatility returns to its mean. This model is denoted as the Mean Reversion RLS model. The third extension is a combination of the above two modifications and is denoted as the Modified RLS model.

The estimation method for the basic RLS model is proposed by Lu and Perron (2010) while the three other RLS models are described in Xu and Perron (2014). All methods are based on the approach of Wada and Perron (2006), as well as Perron and Wada (2009). The method is subject to similar difficulties as for the RLS-ARFIMA model. Further and specific details may be found in Lu and Perron (2010), Li and Perron (2013), and Xu and Perron (2014).

2.3 Forecasting

The state space structure of the RLS-ARFIMA model allows τ-step-ahead forecasts to be obtained by combining results from the state-space and Markov switching literature; see Hamilton (1994b), and Gabriel and Martins (2004). This approach allows for progression beyond the Lu and Perron's

[9] Note that since the model allows consecutive level changes, the minimum length of a segment is set to only one observation.

(2010) state-space level shift framework. The forecasting of the basic RLS model is embedded in the framework of Varneskov and Perron (2014). The Proposition 2 in Varneskov and Perron (2014) provides the elements to justify their approach; see Varneskov and Perron (2014) for specific details.

3 Empirical Results

Three issues are discussed in this Section: first, a brief description of the data used in the estimations. Second, an analysis of the results obtained from the estimation of the RLS-ARFIMA models and other competitors models. Third, an analysis of the forecasting performance of the RLS-ARFIMA in comparison with the four other models in the family of RLS and standard ARFIMA models.

3.1 The Data

I apply the above-mentioned models to two kinds of financial series: stock and Forex return volatilities, using daily frequency and including five Latin American countries. In the case of the stock market, we have Argentina, covering the period 08/04/1988 to 06/13/2013 (6142 observations); Brazil, from 01/02/1992 to 06/13/2013 (5303 observations); Chile, from 01/02/1989 to 13/06/2013 (6096 observations); Mexico, from the 01/19/1994 to 13/06/2013 (4839 observations); and Peru, from 01/03/1990 to 06/13/2013 (5831 observations). For the Forex series we have Argentina, from 02/01/2002 to 02/07/2014 (2958 observations); Brazil, from 04/01/1999 to 02/07/2014 (3785 observations); and Chile, Colombia, Mexico, and Peru, from 01/04/1993 (5282 observations), 08/20/1992 (5259 observations), 01/02/1992 (5636 observations, and 01/03/1997 (4251 observations), respectively, all of which ended on 02/07/2014.

The returns are constructed as $r_t = \ln(P_t) - \ln(P_{t-1})$, where P_t are the values of each of the indexes or the exchange rates depending on the market analyzed. Following recent literature (see Lu and Perron (2010), Li and Perron (2010), Xu and Perron (2010), among others), we model log-absolute returns[10]. When returns are zero or close to it, the log-absolute transformation implies extreme negative values. Using the estimation method described above, these outliers would be attributed to the level shifts component and thus bias the probability of shifts upward. To avoid this problem, I bound absolute returns away from zero by adding a small constant; that is, I use $y_t = \log(|r_t| + 0.001)$, a technique introduced to the stochastic volatility literature by Fuller (1996). The results are robust to alternative specifications; for example, using another value for this so-called offset parameter, deleting zero observations, or replacing them with another small value.

Another important comment is the fact that I use daily returns as opposed to realized volatility series constructed from intra-daily high-frequency data, which have become popular of late. It is true that realized volatility series are less noisy measure of volatility, but their use in the current context would be problematic for the following reasons: (i) these series are typically available for a short span. Given the fact that the level shifts will be relatively rare, it is imperative to have a long

[10] This measure has two advantages: (i) it does not suffer from a non-negativity constraint as do, for example, absolute or squared returns. In fact, it uses a similar argument to the EGARCH(1,1) model proposed by Nelson (1991): the dependent variable is $\log(\sigma_t^2)$ in order to avoid the problems of negativity when the dependent variable is σ_t^2 as in the standard GARCH models and other relative models; and (ii) there is no loss related to the use of square returns in identifying level shifts since log-absolute returns are a monotonic transformation. It is true that log-absolute returns are quite noisy but this is not problematic since the algorithm used is robust to the presence of noise.

span of data in order to make reliable estimates of the probability of level-shift occurrence; and (ii) they are available only for specific assets as opposed to market indices. Because the aim of the RLS model is to allow for particular events affecting overall markets, using a specific asset would confound such market-wide events with idiosyncratic ones associated with the particular asset used.

Table 1 shows the main descriptive statistics of returns and volatility of the series in both markets. Figures 1a and 1b show the behavior of the returns. We can see that these series move very close to zero mean and have clusters or groupings in their distribution in time. These clusters in the series support the use of model-level changes combined with a long-memory process, because they are the representation of shocks that have long-term effects within each regime. On the other hand, volatility asymmetry is very small and is in the range of -0.259 and -0.027. The kurtosis in all series is very close to 3 (2.578-2.827) but without presenting a Normal distribution. Further details on the stylized facts in the stock and Forex markets in Peru may be found in Humala and Rodríguez (2013). The ACF of the volatility series are presented in Figures 2a and 2b. The long memory behavior can be clearly appreciated for all the series, but the evidence appears to be stronger for the case of the Forex markets.

3.2 Results

The parameters estimated using the RLS-ARFIMA (0,d,0) and the RLS-ARFIMA (1,d,1) models are presented in Table 2a and 2b for the Stock and Forex markets, respectively. In addition, by way of comparison, I add the estimates of the Basic RLS, ARFIMA (0,d,0) and ARFIMA (1,d,1) models. In order to save space, I do not include estimates for the other three RLS models, but the results are similar and are available upon request.

With respect to Table 2a, the following observations are obtained: (i) estimates of the fractional parameter from the ARFIMA models are high, especially in the case of the ARFIMA (1,d,1) model. This characteristic is observed in all five countries analyzed. In some cases, such as Argentina and Mexico, the estimates of the fractional parameter are higher than 0.41; that is, they are relatively close to the nonstationarity border (0.5); (ii) in all cases, the estimates of the fractional parameter are statistically significant at 1.0%. This is consistent with what was observed in Figure 2a, which shows that the volatility series exhibit long-memory behavior; that is, shocks that affect the volatility of returns do not disappear in the short term; (iii) the phenomenon changes when the RLS-ARFIMA models are estimated. In general, all estimates of fractional parameters reduce in magnitude very significantly. In some cases, such as Brazil and Peru, the estimates of the fractional parameter are zero, while in many cases the significance is reduced at 10.0% of significance; and (iv) comparing with the basic RLS model, the estimates of σ_η are found to be higher in the cases of the RLS-ARFIMA models. This is particularly clear in the cases of Argentina and Peru. However, in the case of Chile the opposite result can be observed. In the other countries, the estimates of σ_η are similar or relatively high in relation to the basic RLS model, indicating the importance of the level shift component. It is only in the case of Chile that the result indicates greater importance of the short-memory process as compared to the level shifts component.

In conclusion, estimates from the RLS-ARFIMA models suggest a different outcome. When there is a significant estimate of the fractional parameter, it is small. Therefore, according to these models, there is genuine long memory but its magnitude is reduced.

The same is true of Table 2b where the results correspond to the Forex markets. For Argentina, the estimate of the fractional parameter is found to be extremely high (0.981) in the case of the ARFIMA (1,d,1) model. Chile and Mexico are cases where the estimates are close to the nonstationarity borderline. However, estimates obtained from the RLS-ARFIMA models give different results: the fractional parameter is small.

Figures 3a and 3b show the ACF of the residuals extracted from the volatility series minus the level shifts component when a RLS-ARFIMA (0,d,0) and RLS-ARFIMA (1,d,1) are used in the stock markets, respectively. The same information appears in Figures 4a and 4b but in relation to the Forex markets. The same conclusion may be obtained from all four figures: no evidence of long memory behavior is found. The differences are extremely clear in comparison with Figures 2a and 2b.

In summary, the results suggest that estimates of the fractional parameter using daily data are very small or in some cases, not statistically signifcant. These results are in accordance with Varneskov and Perron (2014). They find that estimates of the fractional parameter for high frequency data are higher, around 0.40. However, estimates of this parameter using daily frequency data are smaller and in many cases close to zero. My estimates using daily data result in the same observation. Therefore, Varneskov and Perron (2014) suggest that residual dynamics extracted using daily data may be characterized as a combination of short memory dynamics and measurement errors. Given the small values of the estimates of the fractional parameter, I argue that the daily volatility series employed here are better modeled as a short memory process contaminated by rare level shifts, and that only in some cases is there genuine long memory. In this regard my results corroborate the findings from similar markets of Rodríguez and Tramontana Tocto (2015), Gonzáles Tanaka and Rodríguez (2016), Ojeda Cunya and Rodríguez (2016), and Rodríguez (2016).

3.3 Forecasting

I consider out-of-sample forecasting of the last $T_{out} = 1800$ days of all samples. The parameters are estimated once, without the last 1800 days, and the forecast calculation is conditional to these estimates[11]. I consider direct τ-step-ahead forecasting for six different horizons $\tau = 1, 5, 10, 20, 50, 100$. Let the cumulative direct τ-step-ahead forecast be defined as $\overline{y}_{t+\tau,i|t} = \sum_{s=1}^{\tau} \widehat{y}_{t+s,i|t}$ for model $i \in M^0$ where M^0 is the initial, finite set of models, and similarly let the cumulative volatility proxy be denoted as $\overline{\sigma}^2_{t,\tau} = \sum_{s=1}^{\tau} y_{t+s}$. For out-of-sample evaluation, I use the mean square forecast error (MSFE) criterion, defined as $MSFE_{\tau,i} = \frac{1}{T_{out}} \sum_{t=1}^{T_{out}} (\overline{\sigma}^2_{t,\tau} - \overline{y}_{t+\tau,i|t})^2$, which is shown in Hansen and Lunde (2006) and Patton (2011) to be robust to noise in the volatility proxy[12].

To facilitate model comparison, Varneskov and Perron (2014) define the relative performance of models $i, j \in M^0$ at time t as $d_{ij,t} = (\overline{\sigma}^2_{t,\tau} - \overline{y}_{t+\tau,i|t})^2 - (\overline{\sigma}^2_{t,\tau} - \overline{y}_{t+\tau,j|t})^2$. Then, it is assumed that $d_{ij,t}$ satisfies the following conditions: for some $r > 2$ and $\gamma > 0$, it holds that $E|d_{ij,t}|^{r+\gamma} < \infty$, $\forall i, j \in M^0$ and that $\{d_{ij,t}\}_{i,j \in M^0}$ are strictly stationary with $var(d_{ij,t}) > 0$ and α-mixing of order $-r/(r-2)$. Under the above conditions on $\{d_{ij,t}\}$, the relative forecast accuracy may be assessed using the 10% Model Confidence Set (MCS) of Hansen et al. (2011)[13]. It is important for our application that the MCS is based on a bootstrap implementation, which is robust to comparing nested models when the parameters are estimated once using the same in-sample estimation period for all models; see for example the discussions in Giacomini and White (2006), and Hansen et al. (2011). In terms of notation, I use a letter (a) when the model is the best in forecasting performance

[11] This approach is taken due to the heavy calculation task of re-estimating parameters in each step. For robustness, both recursive and rolling window estimation have been used on the series. The numerical results were similar, and the model rankings were identical. This is explained by the fact that the parameter estimates are robust to the choice of the estimation window.

[12] The results were qualitatively the same using the mean absolute forecast errors.

[13] See Appendix A.2 of this reference for a review.

(which means a p-value=1.000). I also use a letter (b) when the model corresponds to the 10% MCS of Hansen et al. (2011).

Eight models have been estimated in order to compare their forecasting performance: the Basic RLS, the Threshold 1% RLS, the Mean Reversion RLS, the Modified RLS, the RLS-ARFIMA (0,d,0), the RLS-ARFIMA (1,d,1), the ARFIMA (0,d,0) and the ARFIMA (1,d,1). Table 3a presents results for the stock markets while the Table 3b shows the results for the Forex markets. Let me to consider the result in Table 3a. In the case of Argentina the RLS-ARFIMA(0,d,0) shows the best performance for $\tau = 1$. After it, for $\tau = 5, 10, 20$, the best model is the Mean Reversion RLS model. For $\tau = 50, 100$, the RLS-ARFIMA(1,d,1) is the best model. In no case are the simple or typical ARFIMA (p,d,q) models the best, or do they correspond to the 10% MCS of Hansen et al. (2011). Other models corresponding to the 10% MCS are any variations or members of the RLS model family.

In the case of Brazil, a similar observation is obtained. For $\tau = 1$ the RLS-ARFIMA(0,d,0) is the best model selected. In the cases of $\tau = 5, 10, 20$, the Basic RLS models are selected as the best. Finally, as in the case of Argentina for $\tau = 50, 100$ the RLS-ARFIMA(1,d,1) exhibits the best performance. Again, no standard ARFIMA models are included within the 10% MCS.

In Chile, we find some variations in the selection of the best models. For $\tau = 1$ the Basic RLS model has the best performance. In the horizons $\tau = 5, 10$, the Modified RLS model is preferred. For $\tau = 20$, the Mean Reversion RLS model is selected. Finally, for $\tau = 50, 100$, the RLS-ARFIMA(0,d,0) performs best. In the case of that country, the standard ARFIMA models are selected as corresponding to the 10% MCS (p-values around 0.21 and 0.363) jointly with other RLS models.

The case of Mexico is the only one where no RLS-ARFIMA model is selected as the best or as corresponding to the 10% MCS. In fact, for $\tau = 1$ the Basic RLS model is selected. For $\tau = 5, 10, 20$ the Mean Reversion RLS model is preferred. For $\tau = 50$ and $\tau = 100$ the selected models are the Modified RLS models and the Threshold 1% RLS model, respectively. In this country no standard ARFIMA models correspond to the 10% of the MCS. The same aspect is observed for the RLS-ARFIMA models. This is explained by the fact that for this country, the estimate of the fractional parameter is not significant for the RLS-ARFIMA(1,d,1) while the $\widehat{d}$ is too short for the RLS-ARFIMA(0,d,0). In consequence, the contribution and performance of the RLS-ARFIMA models is limited or absent.

In the case of Peru, the preferred model is the Mean Reversion RLS model for $\tau = 1, 5$. The Modified RLS model is selected as the best model for $\tau = 10, 20$. For $\tau = 50, 100$ the RLS-ARFIMA(0,d,0) is the model with the best forecasting performance. In this case, no ARFIMA models corresponds to the 10% MCS. Interestingly, no RLS-ARFIMA (1,d,1) is selected in the 10% of the MCS either, as in the case of Mexico. And as with Mexico, this can be explained. It should be recalled that the estimate of d in the RLS-ARFIMA(1,d,1) was zero or statistically insignificant (see Table 2a). However, for the RLS-ARFIMA(0,d,0) we obtained $\widehat{d} = 0.155$, which is significant at 1.0%. This is why the RLS-ARFIMA(0,d,0) is present in the good performance for Peru but it is absent in Mexico, because we obtained no significance of the estimate of $\widehat{d}$ for both RLS-ARFIMA models.

The results can be summarized through the following observations: (i) it is interesting that for Chile, Mexico, and Peru (unlike Argentina and Brazil) the best forecasting performance is given by the Basic RLS model or other models and not RLS-ARFIMA models. This is consistent with or explained by the fact that in these three countries the short-memory component is successfully modeled using an AR(1) process; see Rodríguez (2016); (ii) Mexico is the only country where no RLS-ARFIMA model is selected as the preferred model, and, indeed, where none of these models correspond to the 10% MCS. Estimates of the fractional parameter d support these results. However, no standard ARFIMA models perform better. The other RLS models are the best in terms of forecasting; (iii) for long horizons, the RLS-ARFIMA(1,d,1) model is the best for Argentina and Brazil; (iv) for short horizons, the RLS-ARFIMA(0,d,0) performs well for Argentina and Brazil. For Mexico and Chile, the Basic RLS model is preferred, and for Peru the Mean Reversion RLS model is selected; (v) in comparison with previous results, such as Rodríguez (2016), some countries modeled as AR(1) processes show no evidence of ARFIMA behavior.

The five countries and six horizons used here would imply that we have 30 cases to consider for each model. With respect to the RLS-ARFIMA (0,d,0), it is found to correspond to the 10% MCS in 12 out of the 30 cases. As regards the RLS-ARFIMA (1,d,1), this is true in 12 of 30 cases. The Mean Reversion RLS model appears in 16 of 30 cases. The Basic RLS occurs in seven of 30 cases. In the cases of Chile, Mexico, and Peru, the best models are more clearly selected. In other words, not many models are found to belong to the 10% MCS.

Very similar observations result from Table 3b, which shows the forecasting comparison in the Forex markets: (i) the ARFIMA models are never selected, except for in one single case out of all possible ones; namely, Brazil for $\tau = 100$; (ii) the RLS-ARFIMA (0,d,0) is found in 14 of the 30 possible cases. The RLS-ARFIMA (1,d,1) appears in 14 of 30 cases; (iii) if the best model of all (p-value=1.000) is to be selected, then the RLS family of models performs very well. The dominance of the RLS-ARFIMA models is not observed. But other RLS models appear to perform very well or to dominate the RLS-ARFIMA models in some cases and vice versa. For instance, in the case of the Forex market of Argentina, the RLS-ARFIMA (1,d,1) is the best model for $\tau = 1$. No other model is seen to correspond to the 10% MCS. For $\tau = 5$, the Mean Reversion RLS models are the best selected, followed by the Basic RLS, Modified RLS, Threshold RLS, and the two RLS-ARFIMA models. A similar observation is made for $\tau = 10$. For other horizons the Mean reversion RLS model appears to dominate, though this changes depending on the country analyzed. The conclusion is that the family of RLS models in all their versions dominates the simple ARFIMA models.

In summary, the RLS-ARFIMA models exhibits a better performance for some horizons, while the other four RLS models are better for other horizons. This is a logical consequence of the small magnitude of the estimates of the fractional parameter in the RLS-ARFIMA models. In no horizon of forecasting are simple ARFIMA models selected or do they belong to the 10% MCS of Hansen et al. (2011).

4 Conclusions

Following Varneskov and Perron (2014), I apply the RLS-ARFIMA(0,d,0) and RLS-ARFIMA (1,d,1) models to the daily stock and Forex market returns volatility of Argentina, Brazil, Chile, Mexico, and Peru. This model is a parametric state-space model with an estimation framework that combines long memory and level shifts by decomposing the underlying process into a simple mixture model with ARFIMA dynamics.

The full sample parameter estimates show that level shifts are rare but present in all series. I compare the results with four RLS models as in Xu and Perron (2014). The results suggest that estimates of the fractional parameter using daily data are very small. These results are in accordance with Varneskov and Perron (2014). They find that estimates of the fractional parameter for high frequency data are higher, around 0.40. However, estimates of this parameter using daily frequency data are smaller and in many cases close to zero. My estimates using daily data suggest a similar observation. Therefore, Varneskov and Perron (2014) suggest that residual dynamics extracted using daily data may be characterized as a combination of short memory dynamics and measurement errors. Given the small values of the estimates of the fractional parameter, I argue that the daily volatility series used here are better modeled as a short-memory process contaminated by rare level shifts, with genuine long memory in some cases. In this regard, my results corroborate the findings in similar markets of Rodríguez and Tramontana Tocto (2015), Gonzáles Tanaka and Rodríguez (2016), Ojeda Cunya and Rodríguez (2016), and Rodríguez (2016).

An out-of-sample forecasting comparison is also performed using the approach of Hansen et al. (2011). The RLS-ARFIMA models perform better for some horizons while the other four RLS models are better for others. This is a logical consequence of the small magnitude of the estimates of the fractional parameter in the RLS-ARFIMA models. In no horizon of forecasting are simple ARFIMA models selected, or do they belong to the 10% MCS of Hansen et al. (2011).

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Table 1. Summary Descriptive Statistics of Returns and Volatility Series

	Mean	SD	Maximum	Minimum	Skewness	Kurtosis	Sample
Stock Market Returns							
Argentina	0.002	0.032	0.330	-0.757	-0.862	62.476	6142
Brazil	0.002	0.028	0.345	-0.395	-0.039	30.609	5303
Chile	0.001	0.012	0.118	-0.077	0.182	8.696	6096
Mexico	0.001	0.016	0.122	-0.143	-0.019	9.595	4839
Peru	0.001	0.017	0.143	-0.132	0.519	11.094	5831
Stock Market Volatility							
Argentina	-4.397	1.060	-0.277	-6.908	-0.235	2.808	6142
Brazil	-4.383	0.975	-0.927	-6.908	-0.259	2.827	5303
Chile	-4.993	0.845	-2.128	-6.908	-0.151	2.538	6096
Mexico	-4.797	0.895	-1.937	-6.908	-0.161	2.604	4839
Peru	-4.858	0.951	-1.931	-6.907	-0.027	2.622	5831
Forex Market Returns							
Argentina	0.000	0.008	0.221	-0.070	9.962	283.372	2957
Brazil	0.000	0.011	0.100	-0.103	0.332	14.113	3784
Chile	0.000	0.006	0.047	-0.035	0.394	8.988	5281
Mexico	0.000	0.009	0.186	-0.168	1.495	97.508	5635
Peru	0.000	0.003	0.023	-0.019	0.451	14.330	4250
Forex Market Volatility							
Argentina	-6.026	0.739	-1.503	-6.908	1.473	5.838	2957
Brazil	-5.145	0.857	-2.259	-6.908	0.065	2.569	3784
Chile	-5.616	0.736	-3.044	-6.908	0.245	2.440	5281
Mexico	-5.576	0.817	-1.679	-6.908	0.440	3.148	5635
Peru	-6.148	0.597	-3.738	-6.908	0.940	3.524	4250

Table 2a. Parameter Estimates: Stock Markets

	a	ϕ	θ	d	σ_η	α or γ/T	σ_ϵ	$AR(1)$
				Argentina				
Basic RLS					0.679^a	0.008^c	0.937^a	
					(0.189)	(0.004)	(0.009)	
ARFIMA (0,d,0)	-4.382^a			0.178^a				
	(0.054)			(0.008)				
ARFIMA (1,d,1)	-4.493^a	0.252^a	-0.620^a	0.428^a				
	(0.196)	(0.031)	(0.044)	(0.035)				
RLS_ARFIMA (0,d,0)				0.041^a	0.875^a	0.003^a	0.945^a	
				(0.014)	(0.104)	(0.001)	(0.009)	
RLS_ARFIMA (1,d,1)		0.228^a	0.397^a	0.203^a	1.211^a	0.001^b	0.953^a	
		(0.084)	(0.097)	(0.035)	(0.186)	(0.000)	(0.009)	
				Brazil				
Basic RLS					0.425^a	0.010^c	0.881^a	
					(0.118)	(0.006)	(0.009)	
ARFIMA (0,d,0)	-4.351^a			0.155^a				
	(0.044)			(0.008)				
ARFIMA (1,d,1)	-4.128^a	0.174^a	-0.579^a	0.409^a				
	(0.159)	(0.033)	(0.048)	(0.034)				
RLS_ARFIMA (0,d,0)				0.000	0.425^a	0.010^c	0.881^a	
				(0.000)	(0.119)	(0.006)	(0.009)	
RLS_ARFIMA (1,d,1)		0.104^c	0.261^a	0.121^a	0.427^a	0.008	0.883^a	
		(0.061)	(0.087)	(0.042)	(0.130)	(0.005)	(0.009)	

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 2a (continued). Parameter Estimates: Stock Markets

	a	ϕ	θ	d	σ_η	α or γ/T	σ_e	$AR(1)$
Chile								
Basic RLS					0.612^a	0.008^c	0.778^a	0.080^a
					(0.150)	(0.004)	(0.007)	(0.014)
ARFIMA (0,d,0)	-4.977^a			0.181^a				
	(0.045)			(0.009)				
ARFIMA (1,d,1)	-4.907^a	0.356^a	-0.565^a	0.340^a				
	(0.110)	(0.048)	(0.059)	(0.033)				
RLS_ARFIMA (0,d,0)				0.101^a	0.074^a	0.268^b	0.786^a	
				(0.014)	(0.022)	(0.136)	(0.008)	
RLS_ARFIMA (1,d,1)		0.371^a	0.553^a	0.307^a	0.044	0.074	0.797^a	
		(0.057)	(0.067)	(0.037)	(0.047)	(0.144)	(0.007)	
Mexico								
Basic RLS					0.520^a	0.006^c	0.830^a	0.025^c
					(0.157)	(0.004)	(0.009)	(0.015)
ARFIMA (0,d,0)	-4.792^a			0.152^a				
	(0.042)			(0.009)				
ARFIMA (1,d,1)	-4.686^a	0.338^a	-0.682^a	0.412^a				
	(0.145)	(0.032)	(0.039)	(0.042)				
RLS_ARFIMA (0,d,0)				0.027^c	0.537^a	0.005^c	0.832^a	
				(0.015)	(0.150)	(0.003)	(0.009)	
RLS_ARFIMA (1,d,1)		0.560^a	0.635^a	0.104	0.541^a	0.005^c	0.832^a	
		(0.116)	(0.128)	(0.109)	(0.154)	(0.003)	(0.009)	

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 2a (continued). Parameter Estimates: Stock Markets

	a	ϕ	θ	d	σ_η	α or γ/T	σ_e	$AR(1)$
Peru								
RLS					0.875^a	0.0045^a	0.842^a	0.115^a
					(0.128)	(0.0016)	(0.008)	(0.015)
ARFIMA (0,d,0)	-4.843^a			0.222^a				
	(0.069)			(0.009)				
ARFIMA (1,d,1)	-4.886^a	0.205^b	-0.378^a	0.335^a				
	(0.132)	(0.089)	(0.104)	(0.028)				
RLS_ARFIMA (0,d,0)				0.155^a	1.859^a	0.000^b	0.853^a	
				(0.012)	(0.710)	(0.000)	(0.008)	
RLS_ARFIMA (1,d,1)		0.761^a	0.631^a	0.000	1.373^a	0.001^b	0.850^a	
		(0.050)	(0.055)	(0.000)	(0.484)	(0.000)	(0.008)	

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 2b. Parameter Estimates: Forex Markets

	a	ϕ	θ	d	σ_η	α or γ/T	σ_e	$AR(1)$
				Argentina				
RLS					1.309[a]	0.015[a]	0.496[a]	
(SD)					(0.198)	(0.003)	(0.008)	
ARFIMA (0,d,0)	-5.792[a]			0.291[a]				
(SD)	(0.093)			(0.011)				
ARFIMA (1,d,1)	29.505	0.128[a]	-0.920[a]	0.981[a]				
(SD)	(71.000)	(0.037)	(0.013)	(0.039)				
RLS_ARFIMA (0,d,0)				0.048[c]	1.420[a]	0.011[a]	0.504[a]	
(SD)				(0.025)	(0.225)	(0.003)	(0.009)	
RLS_ARFIMA (1,d,1)		0.985[a]	1.000[a]	0.066[c]	1.401[a]	0.012[a]	0.503[a]	
(SD)		(0.019)	(0.002)	(0.034)	(0.230)	(0.003)	(0.009)	
				Brazil				
RLS					0.535[a]	0.016[b]	0.745[a]	
(SD)					(0.120)	(0.007)	(0.009)	
ARFIMA (0,d,0)	-5.134[a]			0.201[a]				
(SD)	(0.061)			(0.010)				
ARFIMA (1,d,1)	-5.033[a]	0.085	-0.401[a]	0.396[a]				
(SD)	(0.167)	(0.065)	(0.087)	(0.036)				
RLS_ARFIMA (0,d,0)				0.035[c]	0.432[a]	0.018[c]	0.750[a]	
(SD)				(0.019)	(0.105)	(0.010)	(0.009)	
RLS_ARFIMA (1,d,1)		-0.0.73[c]	0.023	0.118[a]	0.421[a]	0.015[c]	0.754[a]	
(SD)		(0.041)	(0.029)	(0.031)	(0.107)	(0.009)	(0.009)	

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 2b (continued). Parameter Estimates: Forex Markets

	a	ϕ	θ	d	σ_η	α or γ/T	σ_e	$AR(1)$
Chile								
RLS					0.477^a	0.009^b	0.636^a	
(SD)					(0.128)	(0.004)	(0.007)	
ARFIMA (0,d,0)	-5.653^a			0.197^a				
(SD)	(0.045)			(0.008)				
ARFIMA (1,d,1)	-5.781^a	0.279^a	-0.629^a	0.437^a				
(SD)	(0.139)	(0.033)	(0.045)	(0.036)				
RLS_ARFIMA (0,d,0)				0.045^a	0.516^a	0.005^c	0.641^a	
(SD)				(0.015)	(0.153)	(0.003)	(0.007)	
RLS_ARFIMA (1,d,1)		0.316^c	0.399^c	0.123^c	0.517^a	0.005^c	0.642^a	
(SD)		(0.167)	(0.211)	(0.066)	(0.164)	(0.003)	(0.007)	
Mexico								
RLS					1.072^a	0.003^a	0.670^a	0.071^a
(SD)					(0.072)	(0.001)	(0.007)	(0.015)
ARFIMA (0,d,0)	-5.655^a			0.248^a				
(SD)	(0.070)			(0.008)				
ARFIMA (1,d,1)	-5.956^a	0.244^a	-0.591^a	0.491^a				
(SD)	(0.224)	(0.034)	(0.047)	(0.035)				
RLS_ARFIMA (0,d,0)				0.088^a	1.216^a	0.002^a	0.674^a	
(SD)				(0.014)	(0.240)	(0.001)	(0.007)	
RLS_ARFIMA (1,d,1)		0.261	0.308	0.130^a	1.238^a	0.002^a	0.675^a	
(SD)		(0.210)	(0.207)	(0.042)	(0.234)	(0.001)	(0.007)	

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 2b (continued). Parameter Estimates: Forex Markets

	a	ϕ	θ	d	σ_η	α or γ/T	σ_e	$AR(1)$
Peru								
RLS					0.513^a	0.017^a	0.490^a	0.104^a
(SD)					(0.084)	(0.005)	(0.006)	(0.020)
ARFIMA (0,d,0)	-6.120^a			0.264^a				
(SD)	(0.062)			(0.010)				
ARFIMA (1,d,1)	-6.080^a	0.276^a	-0.498^a	0.420^a				
(SD)	(0.133)	(0.062)	(0.076)	(0.035)				
RLS_ARFIMA (0,d,0)				0.145^a	0.460^a	0.009^a	0.500^a	
(SD)				(0.019)	(0.112)	(0.004)	(0.006)	
RLS_ARFIMA (1,d,1)		0.408^b	0.499^a	0.236^a	0.444^a	0.008^b	0.502^a	
(SD)		(0.160)	(0.168)	(0.018)	(0.116)	(0.003)	(0.006)	

Standard errors are reported in parentheses. Estimates with ^{a,b,c} are significant at the 1%, 5% or 10% level, respectively.

Table 3a. Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Stock Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Argentina			
Basic RLS	0.753	4.728	12.475	37.205	209.381	860.698
	(0.000)	(0.091)	(0.088)	(0.164^{b})	(0.001)	(0.000)
Threshold 1% RLS	0.758	4.739	12.456	36.995	208.492	861.245
	(0.000)	(0.089)	(0.088)	(0.297^{b})	(0.001)	(0.000)
Mean Reversion RLS	0.750	4.668	12.204	35.878	198.130	804.210
	(0.000)	(1.000^{a,b})	(1.000^{a,b})	(1.000^{a,b})	(0.022)	(0.003)
Modified RLS	0.752	4.710	12.394	36.976	207.835	846.003
	(0.000)	(0.091)	(0.088)	(0.480^{b})	(0.003)	(0.000)
RLS-ARFIMA(0,d,0)	0.726	4.704	12.337	36.333	200.368	819.478
	(1.000^{a,b})	(0.361^{b})	(0.341^{b})	(0.878^{b})	(0.007)	(0.000)
RLS-ARFIMA(1,d,1)	0.758	4.914	12.664	35.945	182.388	723.798
	(0.000)	(0.003)	(0.088)	(0.953^{b})	(1.000^{a,b})	(1.000^{a,b})
ARFIMA(0,d,0)	0.991	8.634	26.376	86.607	455.305	1630.945
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.942	7.403	21.462	67.010	332.064	1139.459
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

Table 3a (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Stock Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Brazil			
Basic RLS	0.691	3.915	10.044	31.052	175.247	773.826
	(0.000)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.342^{b})	(0.000)	(0.000)
Threshold 1% RLS	0.702	3.967	10.122	30.964	170.932	740.975
	(0.000)	(0.003)	(0.178^{b})	(0.621^{b})	(0.005)	(0.000)
Mean Reversion RLS	0.694	3.931	10.082	31.075	173.849	765.744
	(0.000)	(0.190^{b})	(0.337^{b})	(0.342^{b})	(0.002)	(0.000)
Modified RLS	0.702	3.960	10.813	30.901	170.627	740.144
	(0.000)	(0.007)	(0.000)	(0.827^{b})	(0.018)	(0.000)
RLS-ARFIMA(0,d,0)	0.691	3.915	10.044	31.051	175.232	773.728
	($1.000^{a,b}$)	(0.223^{b})	(0.735^{b})	(0.342^{b})	(0.000)	(0.000)
RLS-ARFIMA(1,d,1)	0.763	4.012	10.163	30.882	169.430	731.034
	(0.000)	(0.000)	(0.157^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)
ARFIMA(0,d,0)	0.925	8.368	26.763	92.738	498.011	1792.570
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.879	7.215	22.140	74.181	383.240	1341.148
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

Table 3a (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Stock Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Chile			
Basic RLS	0.452	4.077	11.906	40.709	260.675	1002.936
	(1.000^{a,b})	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Threshold 1% RLS	0.452	4.078	11.907	40.709	260.569	1002.177
	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean Reversion RLS	0.470	3.868	10.993	36.616	230.168	901.681
	(0.000)	(0.218^{b})	(0.339^{b})	(1.000^{a,b})	(0.011)	(0.000)
Modified RLS	0.490	3.853	10.952	36.768	235.372	940.135
	(0.00)	(1.000^{a,b})	(1.000^{a,b})	(0.276^{b})	(0.007)	(0.000)
RLS-ARFIMA(0,d,0)	0.462	4.335	12.513	40.314	215.802	686.443
	(0.000)	(0.000)	(0.000)	(0.000)	(1.000^{a,b})	(1.000^{a,b})
RLS-ARFIMA(1,d,1)	0.456	4.244	12.327	40.754	235.514	808.098
	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)
ARFIMA(0,d,0)	0.707	6.273	18.482	58.157	263.983	751.092
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.213^{b})
ARFIMA(1,d,1)	0.703	6.183	18.120	56.704	255.185	719.778
	(0.000)	(0.000)	(0.000)	(0.000)	(0.018)	(0.363^{b})

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

Table 3a (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Stock Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Mexico			
Basic RLS	0.570	4.073	11.897	40.191	231.507	886.526
	($1.000^{a,b}$)	(0.000)	(0.000)	(0.000)	(0.000)	(0.052)
Threshold 1% RLS	0.593	4.368	12.831	42.601	232.507	850.812
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	($1.000^{a,b}$)
Mean Reversion RLS	0.602	3.653	10.301	35.287	220.786	928.209
	(0.000)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.217^{b})	(0.002)
Modified RLS	0.620	3.843	10.884	36.655	217.713	869.037
	(0.000)	(0.000)	(0.000)	(0.001)	($1.000^{a,b}$)	(0.263^{b})
RLS-ARFIMA(0,d,0)	0.584	3.836	11.078	38.177	236.765	965.243
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
RLS-ARFIMA(1,d,1)	0.581	3.835	11.084	38.186	236.547	963.322
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(0,d,0)	0.788	7.066	22.669	77.332	392.473	1312.289
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.785	6.992	22.359	75.996	383.109	1272.077
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

Table 3a (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Stock Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Peru			
Basic RLS	0.509	5.510	16.675	54.607	308.784	1148.305
	(0.162^b)	(0.000)	(0.000)	(0.011)	(0.006)	(0.000)
Threshold 1% RLS	0.505	5.454	16.511	54.525	320.168	1219.345
	(0.451^b)	(0.001)	(0.000)	(0.011)	(0.001)	(0.000)
Mean Reversion RLS	0.504	5.247	15.997	53.400	322.688	1247.681
	(1.000^{a,b})	(1.000^a)	(0.885^b)	(0.091)	(0.002)	(0.000)
Modified RLS	0.509	5.285	15.983	52.805	320.776	1248.114
	(0.033)	(0.147^b)	(1.000^{a,b})	(1.000^{a,b})	(0.003)	(0.000)
RLS-ARFIMA(0,d,0)	0.517	5.989	18.350	58.061	276.216	908.350
	(0.002)	(0.000)	(0.000)	(0.011)	(1.000^{a,b})	(1.000^{a,b})
RLS-ARFIMA(1,d,1)	0.547	5.716	17.288	55.719	290.903	1017.086
	(0.000)	(0.000)	(0.000)	(0.011)	(0.015)	(0.000)
ARFIMA(0,d,0)	0.921	9.602	30.121	97.585	464.321	1511.350
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.932	9.877	31.224	102.056	493.114	1630.218
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

Table 3b. Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Forex Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Argentina			
Basic RLS	0.145	1.362	3.125	8.429	61.963	118.029
	(0.012)	(0.825^{b})	($1.000^{a,b}$)	(0.004)	(0.018)	(0.000)
Threshold 1% RLS	0.120	1.473	4.030	7.242	62.242	114.337
	(0.012)	(0.577^{b})	(0.177^{b})	(0.280^{b})	(0.000)	(0.000)
Mean Reversion RLS	0.131	1.339	3.230	7.163	57.806	97.508
	(0.014)	($1.000^{a,b}$)	(0.771^{b})	(0.556^{b})	($1.000^{a,b}$)	($1.000^{a,b}$)
Modified RLS	0.122	1.430	3.672	7.092	63.492	109.518
	(0.012)	(0.577^{b})	(0.177^{b})	($1.000^{a,b}$)	(0.000)	(0.000)
RLS-ARFIMA(0,d,0)	0.104	1.482	4.356	7.162	62.135	113.666
	(0.014)	(0.577^{b})	(0.177^{b})	(0.451^{b})	(0.000)	(0.000)
RLS-ARFIMA(1,d,1)	0.102	1.480	4.353	7.123	62.279	113.265
	($1.000^{a,b}$)	(0.577^{b})	(0.177^{b})	(0.556^{b})	(0.001)	(0.000)
ARFIMA(0,d,0)	0.232	2.899	9.279	31.849	167.211	607.624
	(0.004)	(0.011)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	3.238	77.485	307.187	1197.914	7055.237	26250.370
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

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Table 3b (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Forex Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Brazil			
Basic RLS	0.507	3.912	10.846	36.467	250.632	1191.939
	(0.000)	(0.187^{b})	(0.010)	(0.000)	(0.000)	(0.021)
Threshold 1% RLS	0.492	3.858	10.813	36.469	253.275	1226.363
	(0.000)	(0.256^{b})	(0.010)	(0.000)	(0.000)	(0.001)
Mean Reversion RLS	0.493	3.778	10.400	34.551	222.385	1060.620
	(0.000)	(1.000^{a,b})	(0.301^{b})	(0.002)	(0.007)	(0.054)
Modified RLS	0.499	3.792	10.360	34.074	219.278	1038.370
	(0.000)	(0.303^{b})	(1.000^{a,b})	(1.000^{a,b})	(1.000^{a,b})	(0.054)
RLS-ARFIMA(0,d,0)	0.466	3.829	10.749	36.608	253.662	1206.028
	(1.000^{a,b})	(0.303^{b})	(0.014)	(0.000)	(0.000)	(0.015)
RLS-ARFIMA(1,d,1)	0.468	3.859	10.797	36.497	246.949	1137.718
	(0.204^{b})	(0.265^{b})	(0.013)	(0.000)	(0.000)	(0.037)
ARFIMA(0,d,0)	0.667	6.319	18.895	61.216	298.521	937.055
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000^{a,b})
ARFIMA(1,d,1)	0.674	6.510	19.642	64.132	315.047	1006.433
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.054)

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Table 3b (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Forex Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Chile			
Basic RLS	0.408	2.715	7.678	26.857	160.769	689.341
	(0.000)	(0.043)	(0.041)	(0.677^{b})	(0.006)	(0.000)
Threshold 1% RLS	0.393	2.634	7.397	26.221	163.373	691.651
	(0.000)	(1.000^{a,b})	(1.000^{a,b})	(0.906^{b})	(0.031)	(0.000)
Mean Reversion RLS	0.402	2.668	7.513	26.191	154.837	638.048
	(0.000)	(0.420^{b})	(0.432^{b})	(0.906^{b})	(0.368^{b})	(1.000^{a,b})
Modified RLS	0.400	2.657	7.472	26.155	155.744	641.944
	(0.000)	(0.423^{b})	(0.441^{b})	(1.000^{a,b})	(0.205^{b})	(0.209^{b})
RLS-ARFIMA(0,d,0)	0.378	2.708	7.681	26.478	154.643	662.363
	(1.000^{a,b})	(0.177^{b})	(0.150^{b})	(0.843^{b})	(0.205^{b})	(0.105^{b})
RLS-ARFIMA(1,d,1)	0.387	2.733	7.719	26.372	152.009	650.954
	(0.000)	(0.069)	(0.066)	(0.906^{b})	(1.000^{a,b})	(0.209^{b})
ARFIMA(0,d,0)	0.614	6.741	22.600	80.008	434.454	1595.243
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.594	6.251	20.640	72.153	384.979	1395.665
	(0.000)	(0.000)	(0.000)	(0.000)	(0.018)	(0.000)

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Table 3b (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Forex Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Mexico			
Basic RLS	0.299	2.715	8.258	28.893	197.425	992.964
	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.224^{b})	(0.000)	(0.000)	(0.000)
Threshold 1% RLS	0.342	2.867	8.255	27.352	183.932	900.678
	(0.000)	(0.015)	(0.013)	(0.020)	(0.008)	(0.130^{b})
Mean Reversion RLS	0.354	2.848	7.994	25.936	173.080	852.005
	(0.000)	(0.016)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	(0.945^{b})
Modified RLS	0.346	2.845	8.090	26.342	174.064	847.066
	(0.000)	(0.020)	(0.394^{b})	(0.379^{b})	(0.689^{b})	($1.000^{a,b}$)
RLS-ARFIMA(0,d,0)	0.370	2.935	8.255	26.839	177.629	853.895
	(0.000)	(0.010)	(0.013)	(0.022)	(0.063)	(0.916^{b})
RLS-ARFIMA(1,d,1)	0.340	2.929	8.362	27.321	178.656	848.270
	(0.000)	(0.014)	(0.000)	(0.004)	(0.123^{b})	(0.945^{b})
ARFIMA(0,d,0)	0.741	9.569	33.552	123.001	712.049	2679.897
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.722	9.091	31.640	115.342	663.589	2484.965
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

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Table 3b (continued). Forecast Evaluations ($\widehat{y}_{t+\tau|t} = E_t \ln(|r_{t+\tau}| + 0.001)$): Forex Markets

	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 20$	$\tau = 50$	$\tau = 100$
			Peru			
Basic RLS	0.185	2.411	8.473	31.146	179.363	766.564
	(0.062)	(0.090)	(0.141^{b})	(0.000)	(0.000)	(0.000)
Threshold 1% RLS	0.203	2.547	9.197	34.789	204.453	870.528
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
Mean Reversion RLS	0.224	2.344	8.050	29.699	167.649	702.526
	(0.000)	($1.000^{a,b}$)	(0.798^{b})	(0.000)	(0.000)	(0.000)
Modified RLS	0.214	2.441	8.780	33.834	196.975	830.357
	(0.000)	(0.019)	(0.028)	(0.000)	(0.000)	(0.000)
RLS-ARFIMA(0,d,0)	0.184	2.414	8.044	27.539	150.180	652.568
	($1.000^{a,b}$)	(0.110^{b})	(0.710^{b})	(0.000)	(0.000)	(0.000)
RLS-ARFIMA(1,d,1)	0.186	2.422	8.004	27.093	146.319	638.442
	(0.000)	(0.090)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)	($1.000^{a,b}$)
ARFIMA(0,d,0)	0.397	5.034	16.498	54.349	273.831	953.812
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARFIMA(1,d,1)	0.398	5.062	16.609	54.823	277.155	969.622
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Threshold 1% RLS is the RLS Model with time-varying probabilities, mean reversion RLS is the RLS Model with mean reversion and Modified RLS is the RLS model with time varying probability of shifts and mean reversion. MSFEs are reported in the main entries; MCS p-values are in parentheses. An (^{a}) indicates that the model is the best according to the MSFE. A (^{b}) indicates that the model is within the 10% MCS using all comparisons.

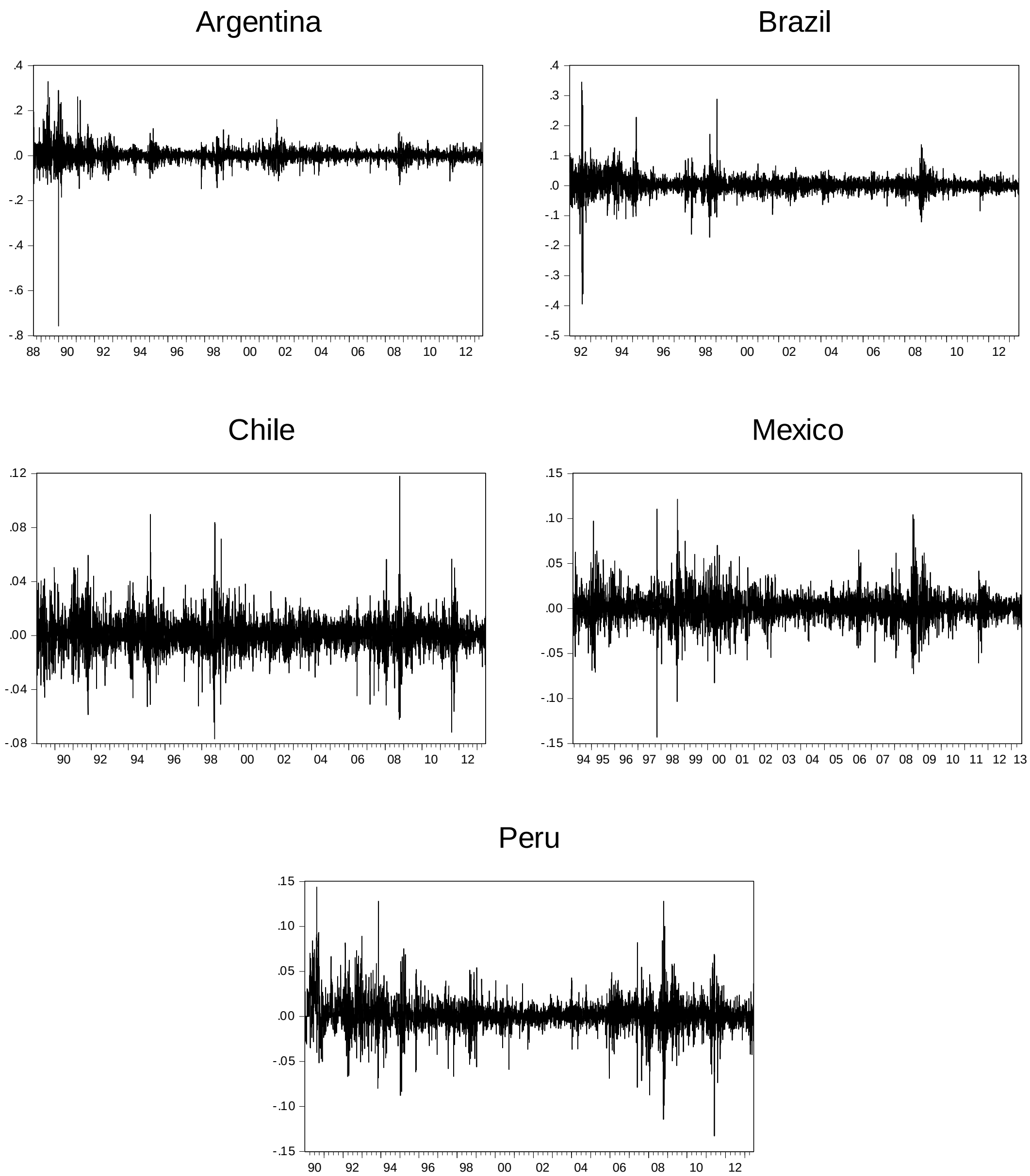

Figure 1a. Daily Stock Returns

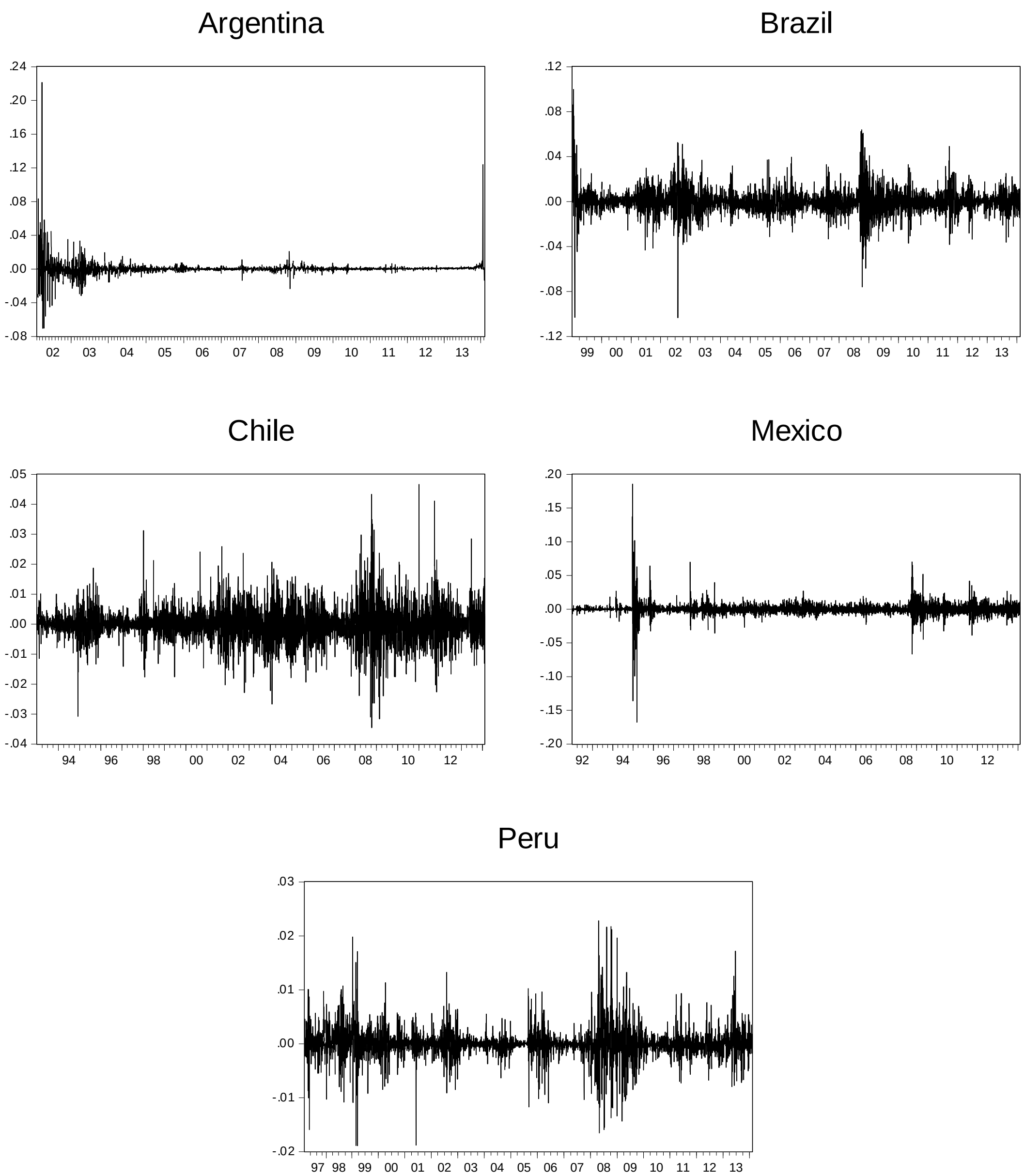

Figure 1b. Daily Forex Returns

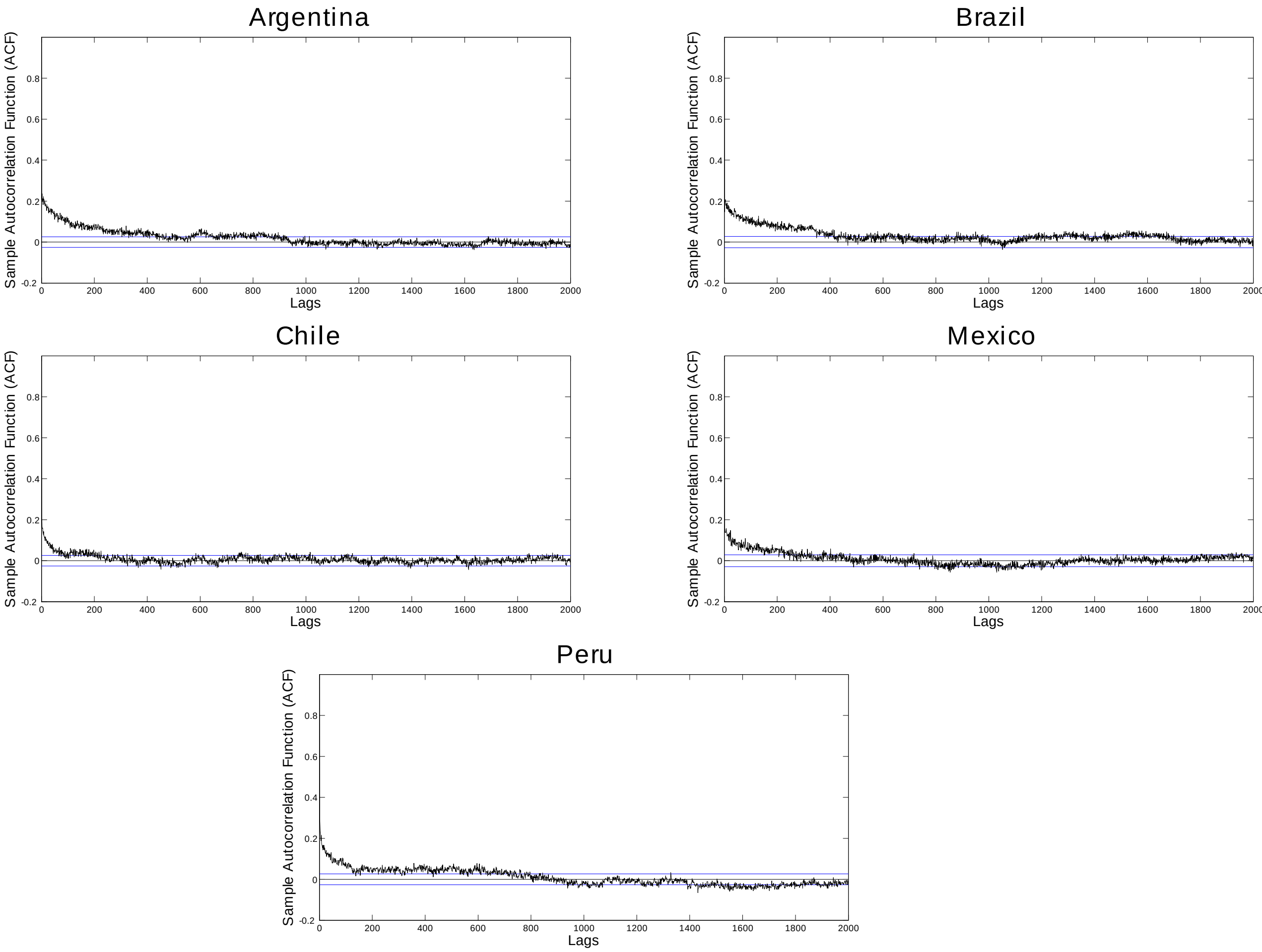

Figure 2a. Sample ACFs of Volatility: Stock Markets

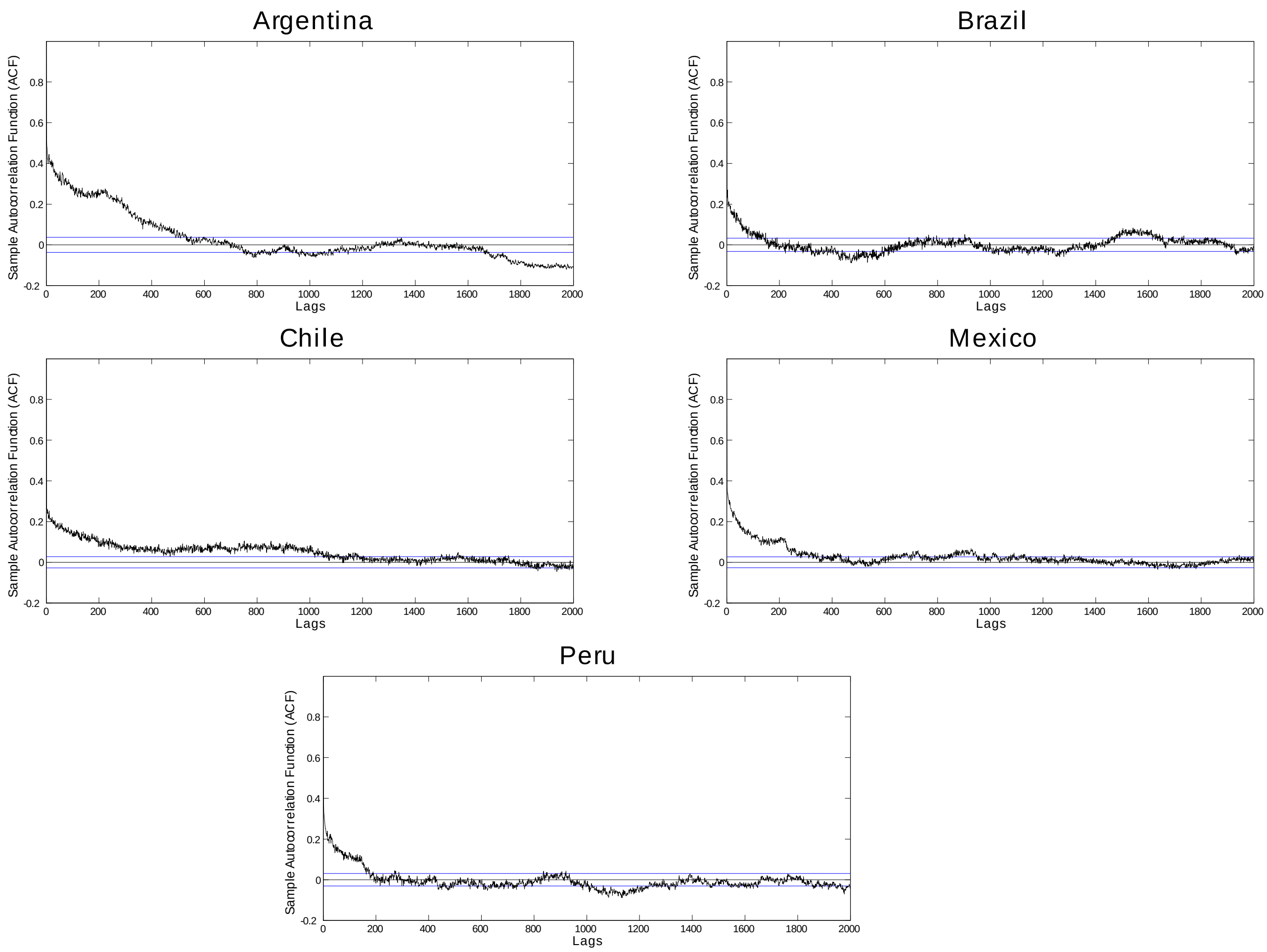

Figure 2b. Sample ACFs of Volatility: Forex Markets

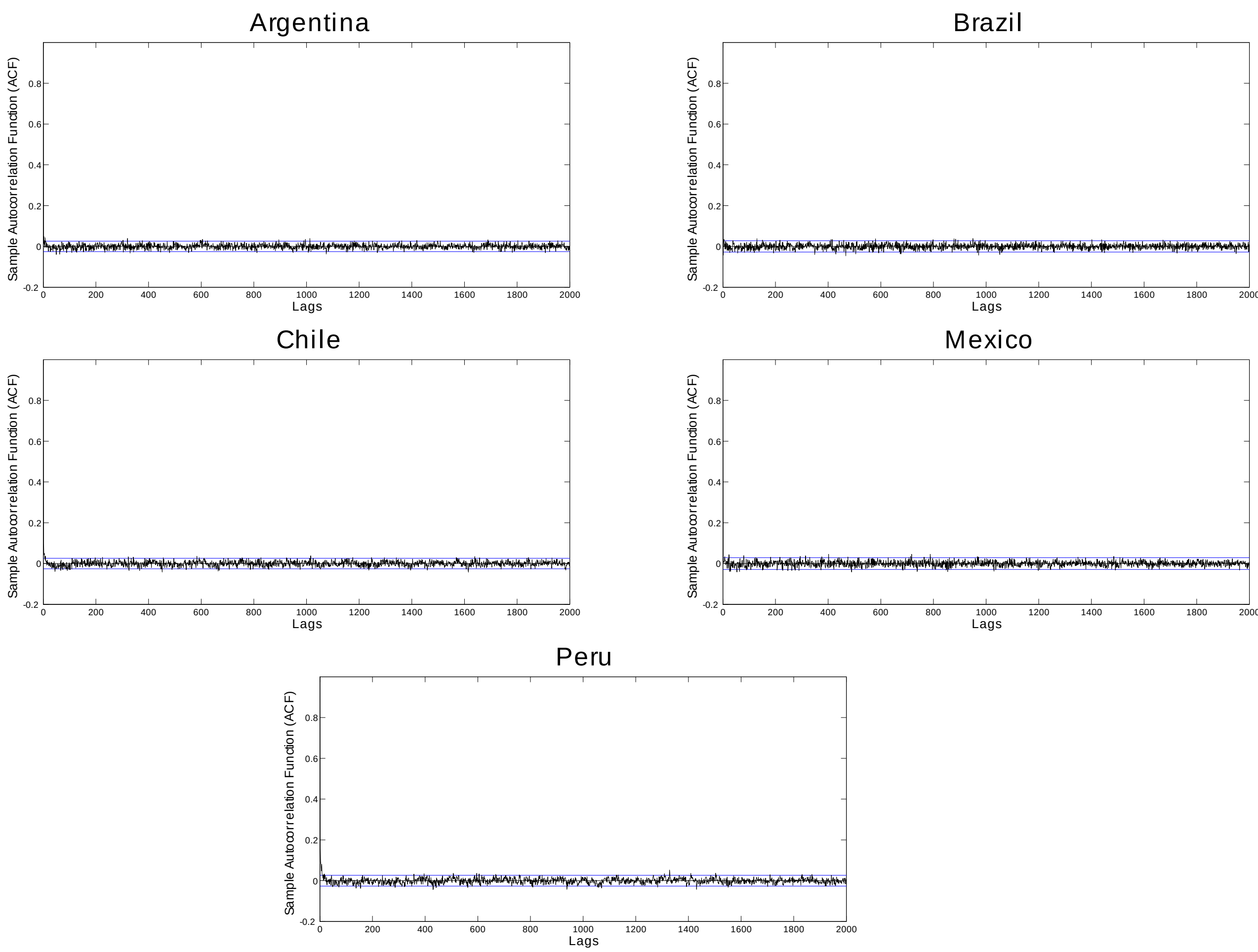

Figure 3a. Sample ACFs of Residuals from RLS-ARFIMA (0,d,0) Model: Stock Markets

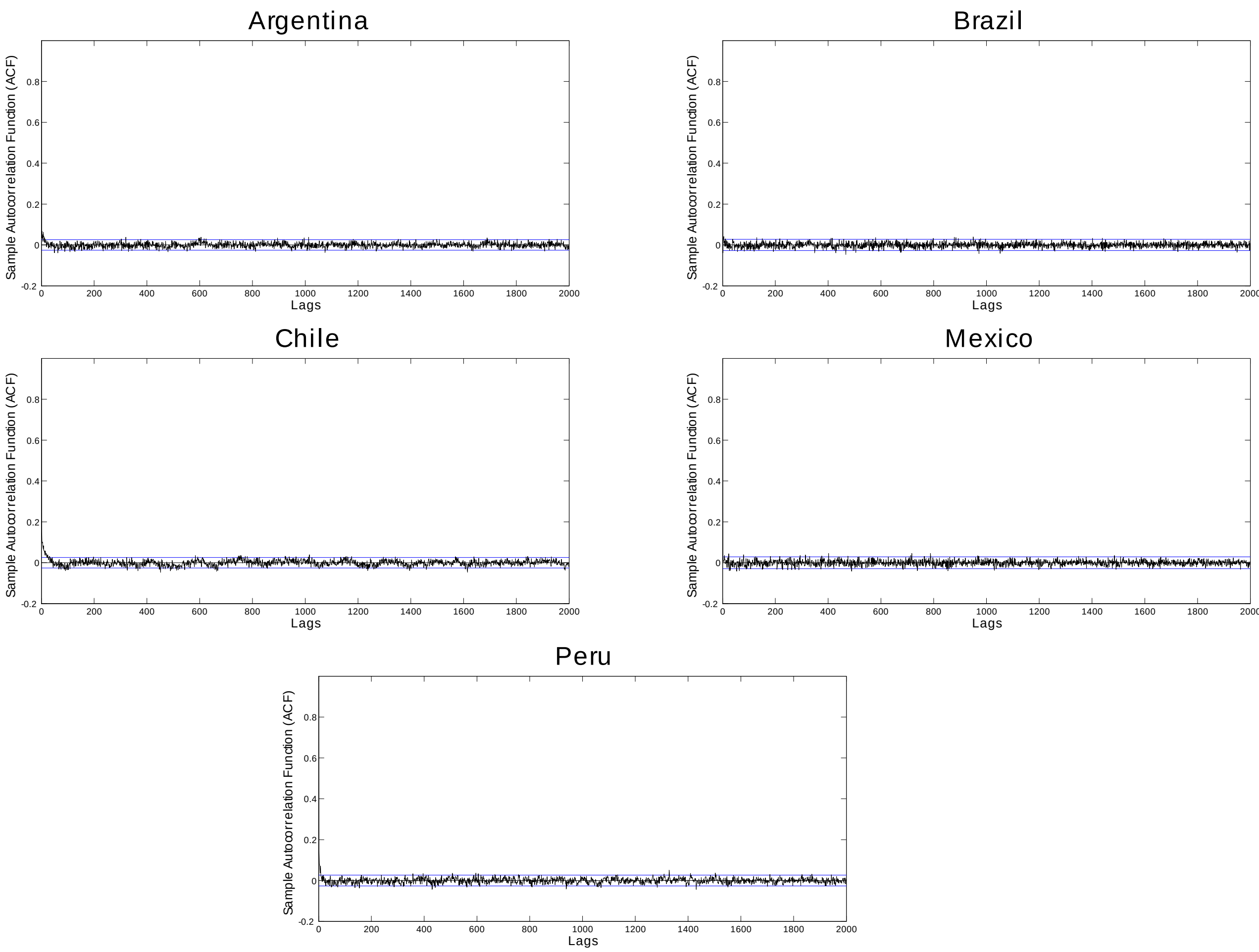

Figure 3b. Sample Autocorrelations of Residuals from RLS-ARFIMA (1,d,1) Model: Stock Markets

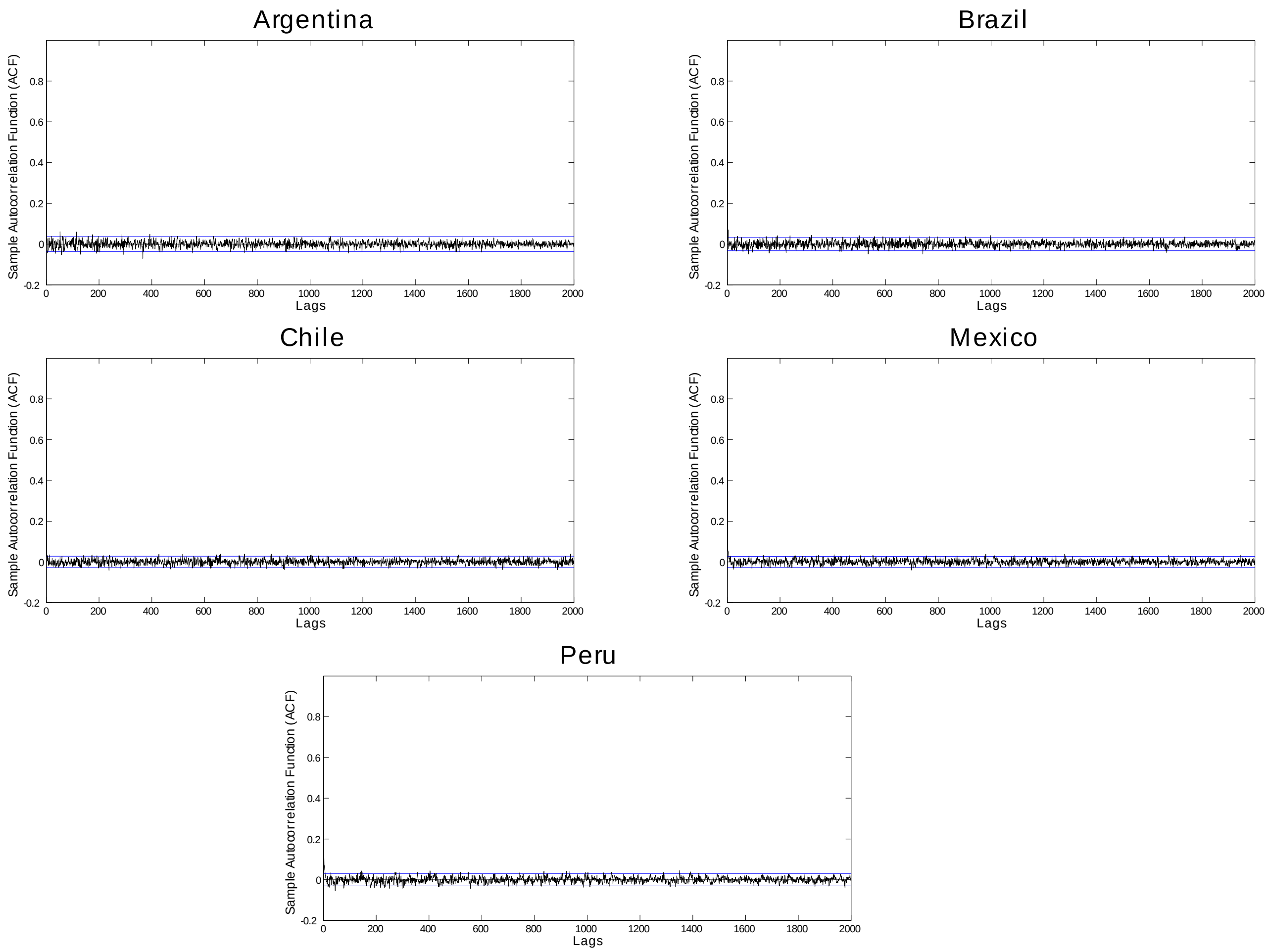

Figure 4a. Sample ACFs of Residuals from RLS-ARFIMA (0,d,0) Model: Forex Markets

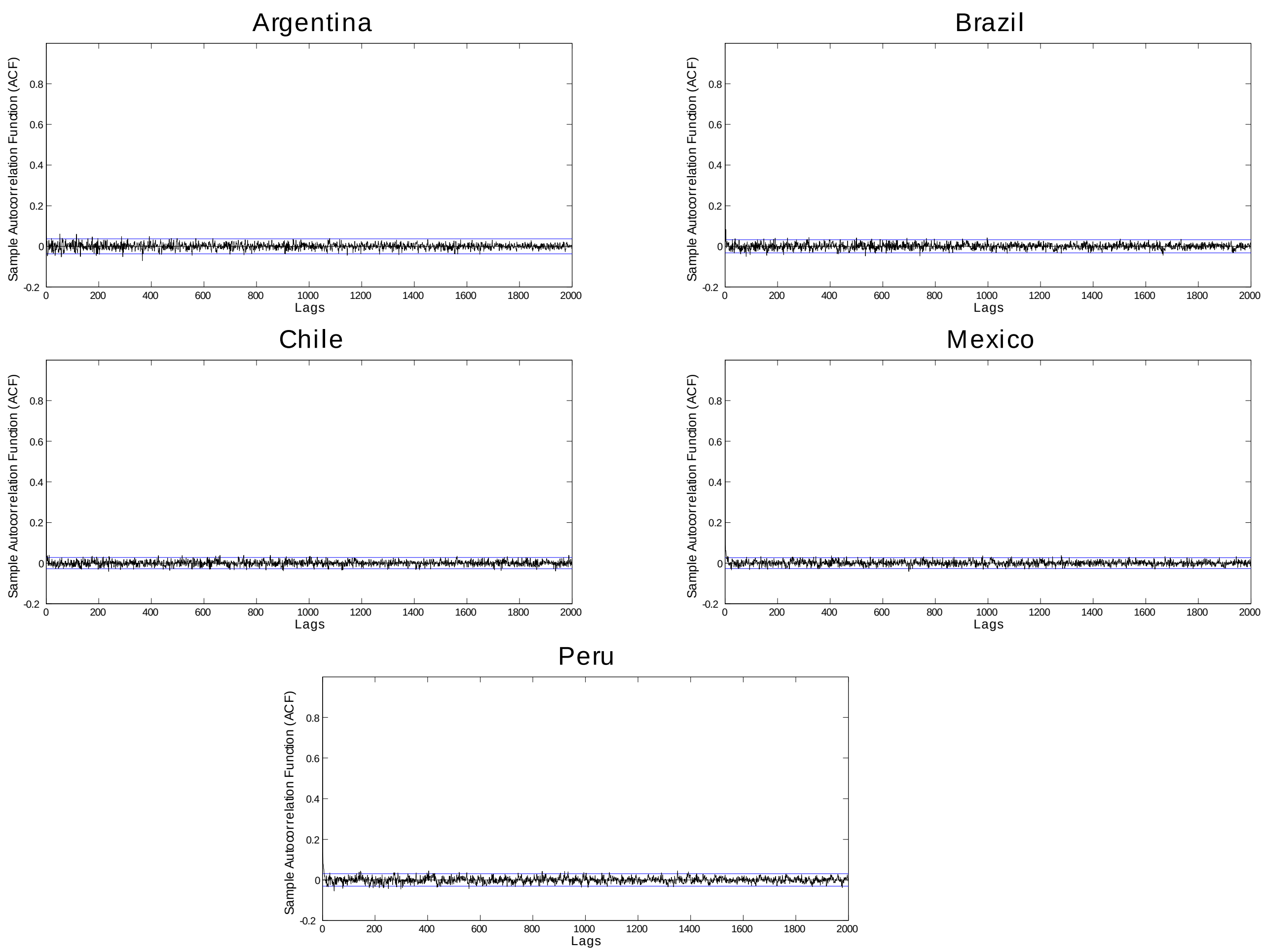

Figure 4b. Sample ACFs of Residuals from RLS-ARFIMA (1,d,1) Model: Forex Markets

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