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Abstract

Using a sample of weekly frequency of the stock and Forex markets returns series, we estimate a set of Markov-Switching-Generalized Autoregressive Conditional Heterocedasticity (MS-GARCH) models to a set of Latin American countries (Brazil, Chile, Colombia, Mexico and Peru) with an approach based on both the Monte Carlo Expectation-Maximization (MCEM) and Monte Carlo Maximum Likelihood (MCML) algorithms. The estimates are compared with a standard GARCH, MS and other models. The results show that the volatility persistence is captured differently in the MS and MS-GARCH models. The estimated parameters with a standard GARCH model exacerbates the volatility in almost double compared to MS-GARCH model and a lower likelihood with the other model than MS-GARCH model. There is different behavior of the coefficients and the variance according the two regimes (high and low volatility) by each model in the Latin American stock and Forex markets. There are common episodes related to global international crises and also domestic events producing the different behavior in the volatility of each time series.

JEL Classification: C22, C52, C53. **Keywords:** MS-GARCH Models, GARCH Models, Returns, Volatility, Latin-American Stock market, Latin-American Forex market.

Resumen

Usando una muestra de frecuencia semanal de las series de retornos de los mercados bursátiles y cambiarios, estimamos un conjunto de modelos de heterocedasticidad condicional autorregresiva generalizada Markov-Switching (MS-GARCH) para un conjunto de países Latinoamericanos (Brasil, Chile, Colombia, México y Perú) con un enfoque basado tanto en los algoritmos de maximización de expectativas de Monte Carlo (MCEM) como en los de máxima verosimilitud de Monte Carlo (MCML). Las estimaciones se comparan con un modelos estándares de tipo GARCH, MS y otros. Los resultados muestran que la persistencia de la volatilidad se captura de forma diferente en los modelos MS y MS-GARCH. Los parámetros estimados con un modelo GARCH estándar exacerban la volatilidad en casi el doble en comparación con el modelo MS-GARCH y una menor verosimilitud con el otro modelo comparado con el modelo MS-GARCH. Hay un comportamiento diferente de los coeficientes y la varianza según los dos regímenes (alta y baja volatilidad) por cada modelo en los mercados bursátiles y cambiarios de América Latina. Hay episodios comunes relacionados con las crisis internacionales globales y también con los acontecimientos internos que producen los diferentes comportamientos en la volatilidad de cada serie temporal.

Clasificación: JEL: C22, C52, C53. **Palabras Claves:** Modelos MS-GARCH, Modelos GARCH, Retornos, Volatilidad, Mercados Bursátiles de América Latina, Mercado Cambiario de América Latina.

Empirical Modeling of Latin American Stock and Forex Markets Returns and Volatility using Markov-Switching GARCH Models[1]

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1 Introduction

The volatility of the Stock and Foreign exchange (Forex) markets rate plays a very important role for a country's economic growth and the stability of its financial markets. Analysis of the characteristics of the stock market returns and volatility of Latin-American countries has been inspired by the crucial role they play in a crisis, such as, or instance, the global financial crisis of 2008-2009. An important ingredient during a crisis is the possibility of modeling and estimating volatility under a reasonable level of accuracy. Moreover, Forex rate variations have an effect on inflation, since imported goods are also included to measure the general price level; on the balance of goods and services, as they affect the competitiveness of sectors that produce and sell tradable goods and services; and on the valuation of assets and liabilities through currency mismatches (balance sheet effect). Therefore, modeling the returns and volatility of Forex rates would be useful for private agents and policy makers alike. For the former, it gives valuable information for better options contracts that allow hedging under great uncertainty, and for the latter, it would aid in a better understanding of business cycles given the correlation between Forex rate fluctuations, capital inflows and investment expectations.

Stock and Forex rate returns and volatility exhibit sudden jumps due not only to structural breaks in the real economy, but also to changes in expectations or different information about the future. These market returns are affected by shocks that never persist for a long time, rendering their behavior mean-reverting. A good estimation of returns and volatility models should capture the change of mean returns and volatility according to the regimes of low or high volatility, and according to these shocks.

Time series of stock market returns have four typical stylized facts, according to Franses and Van Dijk (2000): *i*) large returns occur more often than expected (leptokurtosis or fat tails), which implies that the kurtosis is much larger than 3, or the tails of the distributions are fatter than the tails of the normal distribution; *ii*) large Forex and stock market returns are often negative (negative skewness), which implies that the left tail of the distribution is fatter than the right tail, or that large negative returns tend to occur more often than large positive ones; *iii*) large returns

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tend to occur in clusters, which implies that relatively volatile periods, characterized by large price changes (large returns) alternate with quieter periods in which prices remain more or less stable (small returns); *iv*) high volatility often follows large negative stock and Forex market returns, which implies that periods of high volatility tend to be triggered by a large negative return (this stylized fact is also called the "leverage effect"). These features of stock and Forex market returns require nonlinear models, simply because linear models would not be able to generate data with these features[3].

The most popular and widely used nonlinear financial models in the modeling of volatility models are generalized autoregressive conditional heteroskedasticity (GARCH), Engle (1982), Bollerslev (1986); and regime change models such as Markov Switching models (MS), Hamilton (1989), and Threshold Autoregressive models (TAR), Tong (1983), Tong (1993). Because of the popularity of presenting GARCH models by allowing explicit modeling of volatility, and the ability of the MS models to model the distribution of returns under the regime type (or state of the economy) conducted by an unobservable Markov chain, it is interesting to combine and consider a single MS-GARCH model, which can be understood as a GARCH model in which the parameters depend on an unobservable regime (periods of high or low volatility of returns on financial assets)[4].

Because an exact calculation of the likelihood of MS-GARCH models is unfeasible in practice - since the estimation thereof is dependent on the path - several alternative methods have emerged in the literature to estimate them. In this paper we choose the method presented by Augustyniak (2014), who estimates the maximum likelihood estimator (MLE) of the MS-GARCH model using Monte Carlo Expectation-Maximization (MCEM) and Monte Carlo Maximum Likelihood (MCML) algorithms, and also obtains an approximation of the asymptotic standard errors of the MLE.

The objective of this research is to estimate the MS-GARCH parameters of the volatility of the stock returns of the following Latin American stock and Forex markets: Brazil, Colombia, Chile, Mexico and Peru, in order to discern episodes of high and low volatility undergone by each economy with more accuracy, and to recognize some common behavior pattern during financial turmoils. All these MS-GARCH models are compared with standard GARCH models in terms of their ability to estimate volatility, with MS models in terms of their ability to capture the volatility persistence, and with other models in terms of maximum likelihood. The estimation performances of the competing models are evaluated using weekly time frequency of Latin American stock and Forex market returns.

The results show that for all Latin-American countries analyzed in this paper, the volatility persistence is captured differently in MS and MS-GARCH models. The adjustment of the MS-GARCH model in Latin-American countries is superior to the standard GARCH model according to the estimated parameters. The empirical evidence shows that a standard GARCH model exacerbates the volatility almost twice as much as a MS-GARCH model (to compare the long term mean value parameter of the GARCH and MS-GARCH models) in all the time series considered. The fit of

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[3]For a review of stylized facts in the stock market of Peru, see Humala and Rodríguez (2013).

[4]Lamoureux and Lastrapes (1990) justify this compact model, while Mikosch and Starica (2004) show that the high persistence observed in the variance of financial returns can be explained by time-varying GARCH parameters.

the MS-GARCH model is superior to other models, such as Gray's model, in estimating the mean of low volatility for the data sets considered. For all countries surveyed, according to BIC the best model for estimating Forex rate and stock markets returns and volatility is the MS-GARCH model; the second best is the MS model; the third is a standard GARCH model; and the last is Gray's model (only used for comparative terms). In Peru, according to the terms of maximum likelihood the best model is an MS-GARCH; the second is Gray's model; the third is a standard GARCH; and the last is a MS model. After the crisis, periods of high turbulence are more correlated to Forex rate and stock markets. The temporal correlations between countries show that since the international financial crisis, correlations have tended to be positive, revealing a kind of positive interdependence during episodes of financial turmoil.

The rest of the paper is organized as follows. Section 2 presents the literature review. Section 3 presents the methodology for estimating the standard GARCH models, MS-GARCH models, and the path dependence problem. Section 4 describes and analyzes the data and shows empirical results of the models. Conclusions are presented in Section 5. In the Appendix, the MCEM-MCML algorithm proposed by Augustyniak (2014) is set out.

2 Literature Review

So far in the literature, many volatility models have been put forward, but the most successful are Engle (1982) who formally introduces an autoregressive conditional heteroskedasticity model (ARCH) to explain the dynamic of inflation in the United Kingdom, on the basis of which a series of extensions are developed. For instance, Bollerslev (1986) presents a generalization of the ARCH (GARCH) process by allowing past conditional variances to be incorporated as regressors within the current conditional variance equation. GARCH models are popular because of their ability to capture many of the typical stylized facts of financial time series, such as time-varying volatility, persistence and volatility clustering.

MS-GARCH models begins with Hamilton and Susmel (1994), who are the first to apply simultaneously Hamilton's (1988) seminal idea of endogenous regime-switching parameters into an ARCH specification to account for the possible presence of structural breaks. However, they use an ARCH specification instead of a GARCH to overcome the problem of infinite path-dependence, i.e. to avoid the conditional variance at time t depending on the entire sample path. Hamilton and Susmel (1994) note that estimation using a path dependent model is a challenging task because exact computation of the likelihood is infeasible in practice. Given this impasse, Gray (1996) , Dueker (1997), Klaassen (2002), and Haas et al. (2004), among others, propose variants of the MS-GARCH model to avoid the problem of path dependency with maximum likelihood, while others suggested alternative estimation methods.

The first to suggest a method where the conditional distribution of returns is independent of the regime path was Gray (1996). He suggests integrating out the unobserved regime path in the GARCH equation using the conditional expectation of the past variance. His model can be regarded as the first MS-GARCH.

Following this line, Klaassen (2002) generalizes the regime-switching ARCH models of Cai (1994) and Hamilton and Susmel (1994) by allowing GARCH dynamics and computing multi-period-ahead volatility forecasts through a first-order recursive procedure that enables the use of all available information, instead of only part of it like Gray (1996). He uses data on the three major U.S. dollar exchange rates, finding that the variance dynamics differ across regimes, and obtains a better fit with his model.

As an empirical application, Moore and Wang (2007) investigate the volatility in the stock markets of five new European Union (EU) member states - the Czech Republic, Hungary, Poland, Slovenia and Slovakia - over the sample period 1994–2005, using a Markov switching model. Their model detects that there are two or three volatility states for emerging stock markets. The results reveal a tendency among emerging stock markets to move from the high volatility regime in the earlier period of transition to the low volatility regime as they enter the EU. They find that joining the EU is associated with signs of the stabilization of emerging stock markets in the form of a reduction in their volatility.

Considering the Markov Switching GARCH(1,1) model with time varying transition probabilities, Kramer (2008) obtains sufficient conditions for the square of the process to display long memory, and provides some additional intuition for the empirical observation that estimated GARCH parameters often sum to almost one.

Driven by their interest in distinguishing between two processes, one a regime-dependent stationary process and the other a non-stationary IGARCH process, Liang and Yongcheol (2008) develop an optimal testing procedure designed to possess maximal power for detecting MS-GARCH mechanisms. They consider the case in which the conditional variance follows an IGARCH process under the null while it is globally stationary under the alternative, and find strong evidence in favor of MS-GARCH models in an application to the weekly stock return data for five East Asian emerging markets.

Taking up interest in dependence on the path, Francq et al. (2008) were the first to propose an estimation method without changing the MS-GARCH model. They used the generalized method of moments (GMM) with which they avoid addressing the problem of dependence on the path not being based on the likelihood[5]. On the other hand, Bauwens et al. (2010) develop MS-GARCH models wherein the conditional mean and variance are switched in time by a hidden Markov chain from one GARCH process to another. They provide sufficient conditions for geometric ergodicity and existence of moments of the process. They were the first to estimate the MS-ARCH model using Bayesian MCMC techniques. As in Francq et al. (2008), this alternative estimation was based on the failure to obtain the maximum likelihood estimator (MLE) MS-GARCH model because the dependence of the path makes calculating the likelihood unworkable in practice.

Another empirical application of the Markov Switching approach was developed by Rim and Khemiri (2012). Its aim was to examine the relationship between exchange rates and underlying

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[5]They use a technique based on analytical expressions obtained from Francq and Zakoian (2005), incurring problems of identifiability, robustness and bias; they are unable to obtain their GMM estimator asymptotic standard errors due to numerical difficulties.

microstructural determinants. To this end, he uses a MSEGARCH (1,1) model that ensures, by construction, a non-negative conditional variance and the ability to capture asymmetry in volatility, and compares it against a MS-GARCH (1,1). Both models are estimated using the EM algorithm Hamilton (1990, 1994). He finds that the MSEGARCH model is the best fit for intraday data and a positive correlation between "trading volume" Deutsche Mark (DM)/\$ prices as well as a positive effect of order flow on returns.

Augustyniak (2014) proposes a method for the MLE of the MS-GARCH model based on the Monte Carlo expectation-maximization (MCEM) algorithm of Wei and Tanner (1990), and the Monte Carlo maximum likelihood (MCML) method of Geyer (1994, 1996). The proposed algorithm is based on simulations from the posterior distribution of the state vector and incorporates Martin A. Tanner's (1987) technique of increasing data [6]. Likewise, he proposes a method of estimating the asymptotic variance matrix and covariance matrix of the MLE. Practical implementation of the proposed model was discussed and its effectiveness is demonstrated in simulation and empirical results. He uses daily and weekly percentage log-returns on the S&P 500 price index.

3 Methodology

Let us consider a stock market index p_t and its corresponding rate of return r_t, $r_t = 100 \times [\log(p_t) - \log(p_{t-1})]$, where the index t denotes the weekly closing observations.

3.1 The Generalized ARCH (GARCH) model

The GARCH$(1,1)$ model for the series of returns r_t can be written as

$$
\begin{aligned}
r_t &= \mu + \epsilon_t = \mu + \sigma_t \eta_t, \\
\sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2,
\end{aligned}
$$

where $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ to ensure a positive conditional variance σ_t^2, $\alpha + \beta < 1$ to ensure that unconditional variance $\mathrm{var}(\epsilon_t) = \omega/(\alpha + \beta)$ is defined[7], and $\eta_t \sim i.i.d.\ \mathbb{N}(0,1)$.

3.2 The Markov Regime-Switching GARCH (MS-GARCH) model

Following Bauwens et al. (2010) and Francq et al. (2001), the MS-GARCH model can be defined by the following equations:

$$
r_t = \mu_{S_t} + \sigma_t \left(S_{1:t} \right) \eta_t, \tag{1}
$$

$$
\sigma_t^2 \left(S_{1:t} \right) = \omega_{S_t} + \alpha_{S_t} \epsilon_{t-1}^2 \left(S_{t-1} \right) + \beta_{S_t} \sigma_{t-1}^2 \left(S_{1:t-1} \right), \tag{2}
$$

$$
\epsilon_{t-1} \left(S_{t-1} \right) = r_{t-1} - \mu_{S_{t-1}} \tag{3}
$$

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[6]This method is the most frequent version of the Bayesian MCMC technique used by Bauwens et al. (2010).

[7]If $\alpha + \beta = 1$ we are facing a unit root in the variance, also called "non-stationary in variance" or "integrated GARCH (IGARCH)". Whereas if $\alpha + \beta > 1$ the conditional variance forecast will tend to infinity as the forecast horizon increases, per Brooks (2014).

The vector $(r_1, \ldots, r_T)$ represents the observations to be modeled and $\eta_t \sim i.i.d. N(0, 1)$. At each time point, the conditional mean of the observation r_t is $\mu_{S_t} = E[r_t|S_t]$ and the conditional variance is $\sigma_t^2 = \mathrm{var}(r_t|r_{1:t-1}, S_{1:t})$, where $r_{1:t-1}$ and $S_{1:t}$ are shorthand for the vectors $(r_1, \ldots, r_{t-1})$ and $(S_1, \ldots, S_t)$, respectively. The process $\{S_t\}$ is an unobserved ergodic time-homogeneous Markov chain process with N-dimensional discrete state space (i.e., S_t can take integer values from 1 to N). The $N \times N$ transition matrix of the Markov chain is defined by the transition probabilities $\{p_{ij} = \Pr[S_t = j|S_{t-1} = i]\}_{i,j=1}^N$. The vector $\theta = (\{\mu_i, \omega_i, \alpha_i, \beta_i\}_{i=1}^N, \{p_{ij}\}_{i,j=1}^N)$ denotes the parameters of the model. To ensure positivity of the variance, the following constraints are required: $\omega_i > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$, $i = 1, \ldots, N$. Since $\sum_{j=1}^N p_{ij} = 1$, for $i = 1, \ldots, N$, θ contains $(4N + N(N-1))$ free parameters. Conditions for stationarity and the existence of moments are studied by Bauwens et al. (2010), Francq et al. (2001) and Francq and Zakoian (2005).

3.3   Estimation of the MS-GARCH Model

The MS-GARCH model specified by equations (1)-(3) presents difficulties in its estimation because the conditional variance t depends on the complete path $S_{1:t}$. To simplify notation we denote $\sigma_t^2(S_{1:t})$ as σ_t^2, $r_{1:T}$ and $S_{1:T}$ as R and S respectively, and let $f(p)$ represent a probability density function. We can calculate the likelihood of the observations, $f(r|\theta)$, by integrating all the possible regime paths, obtaining $f(r|\theta) = \sum_S f(r, S|\theta) = \sum_S f(r|S, \theta) p(S|\theta) = \sum_S \left[\prod_{t=1}^T \sigma_t^{-1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{r_t - \mu_{S_t}}{\sigma_t} \right)^2 \right) \right] p(S|\theta)$. For a large T, the sum grows rapidly in N^T terms and consequently its calculation becomes unfeasible; however, an accurate estimate of the log-likelihood is obtained by Bauwens et al. (2010) by writing $\log f(r|\theta) = \log(r_1|\theta) + \sum_{t=1}^{T-1} \log f(r_{t+1}|r_{1:t}, \theta)$ and estimating $f(r_{t+1}|r_{1:t}, \theta)$, $t = 1 \ldots, T-1$, sequentially with the aid of particle filters. Log-likelihood simulation is difficult to maximize with standard optimization routines because these filters are not a continuous function of θ.

Given this deficiency, Gray (1996) proposes replacing the equations (2) and (3) in the MS-GARCH model with:

$$\sigma_t^2 = \omega_{S_t} + \alpha_{S_t} \epsilon_{t-1}^2 + \beta_{S_t} h_{t-1},$$

$$\epsilon_{t-1} = r_{t-1} - E[r_{t-1}|r_{1:t-2}],$$

where $h_{t-1} = \mathrm{var}(r_{t-1}|r_{1:t-2})$ has the effect of collapsing all of the possible conditional variances at time $t-1$ into a single value that does not depend on the regime path, allowing the conditional distribution of r_t, $f(r_t|r_{1:t-1}, S_{1:t}, \theta)$, to become independent of $S_{1:t-1}$ and the maximum likelihood estimation to be tractable, as per Hamilton (2008). However, Augustyniak (2014) shows that Gray's method does not generate consistent estimators for the MS-GARCH.

The Expectation-Maximization (EM) algorithm is a technique designed to obtain the MLE of the observed data likelihood through an iterative procedure that does not require computation of the likelihood. Instead, considering $\widetilde{\theta}$ as a given value of the parameters, it is possible to calculate and maximize $Q(\theta|\widetilde{\theta}) = E\left[\log f(r, S|\theta) | r, \widetilde{\theta} \right] = \sum_S \log f(r, S|\theta) p(S|r, \widetilde{\theta})$. McCulloch (1997)

suggests combining the EM algorithm with a Newton-Raphson method or switching to a faster method after a few EM iterations. He proposes the MCEM algorithm with the MCML approach, as per Geyer (1994, 1996). The MCML method does not work well unless θ^* is in a close neighborhood of the MLE, because the MCML algorithm makes use of importance sampling to directly maximize the log-likelihood, as per Cappé et al. (2005).

In this research the algorithm proposed by Augustyniak (2014) is used, which turns out to be a hybrid of MCEM and MCML algorithms. First, iterations of the MCEM algorithm can be performed to obtain a good estimate, θ^*, of the MLE. This estimate is then used to generate the importance sample in the MCML algorithm. The algorithms complement each other: the MCEM algorithm addresses the flaw of the MCML algorithm relating to the choice of θ^*, while the MCML method replaces many potential MCEM iterations with a single iteration, leading to a faster convergence.

See the Appendix for more details of the MCEM-MCML algorithm.

3.4   Model Specification

In order to estimate the MLE MS-GARCH model, the MCEM-MCML algorithm is used as a starting point in the approximations of the models of Gray, Dueker (1997), and Klaassen (2002). To initialize the Gibbs sampler, it takes Gray's smoothed inference model states (Hamilton, 1994) as its first state vector; and to generate the first Markov chain S_1, it assumes that the initial state S_0 is given and fixed rather than requiring be estimated.

Because the automated strategies for increasing the size of the sample through the MCEM–MCML algorithm require a certain amount of manual adjustments, and do not guarantee high reliability, Augustyniak (2014) proposes two simulations schedules: simulation schedule 1 ($m_1 = 500$, $m_2 = 1000$, $m_3 = 2500$, $m_4 = 5000$, $m^* = 10000$), which allows a quick estimate; and simulation schedule 2 ($m_{1...10} = 500$, $m_{11...28} = 1000$, $m_{29} = 2500$, $m_{30} = 5000$, $m^* = 40000$), which puts more emphasis on precision and is more robust with respect to the choice of starting points. Because accuracy gains are preferred with empirical data, schedule simulation 2 will be used in this research.

Unconstrained estimation of MS-GARCH models with empirical data can lead to the estimation of parameters on the boundary of the parameter space and result in slow convergence of the MCEM–MCML algorithm. For example, Bauwens et al. (2010) and Francq et al. (2008) fit the MS-GARCH model to the daily S&P 500 data: Bauwens et al. (2010) use the constraint $\alpha_1 = \beta_1 = 0$ in the estimation process while Francq et al.(2008) report an estimated value of α_1 very close to zero. To obtain convergence in the interior of the parameter space, Augustyniak (2014) fits a constrained MS-GARCH model by imposing $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ in the estimation process. For weekly data, both the constrained and unconstrained versions are estimated but due to slow convergence in simulation schedule 2, he concludes that the estimation of the unrestricted version is not effective.

Here, unconstrained estimation of the MS-GARCH model is performed for the Latin American countries, finding problems similar to those reported by Augustyniak (2014), α_1 estimates very close

to zero, and changes in sign and magnitude at the value of the conditional variance parameter, ω, which contradict the stylized facts of financial returns. In light of these results, we choose to carry out the constrained estimation of MS-GARCH model under the imposition of $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, obtaining results that are consistent with empirical evidence.

For these reasons, we estimate the following constrained MS-GARCH model:

$$r_t = \mu_{S_t} + \sigma_t \left(S_{1:t}\right) \eta_t, \tag{4}$$

$$\sigma_t^2 \left(S_{1:t}\right) = \omega_{S_t} + \alpha \epsilon_{t-1}^2 \left(S_{t-1}\right) + \beta \sigma_{t-1}^2 \left(S_{1:t-1}\right), \tag{5}$$

$$\epsilon_{t-1} \left(S_{t-1}\right) = r_{t-1} - \mu_{S_{t-1}}. \tag{6}$$

4 Empirical Evidence

4.1 Data and Preliminary Statistics

The weekly stock and Forex market returns series are constructed with diary data. Times series of Brazil, Chile, Colombia, Mexico and Peru are obtained from Bloomberg Financial Data. The weekly data is from Wednesday to Wednesday to avoid most public holidays[8]. Weekly data are used due to the presence of more noise with higher frequencies, such as daily data, which makes it more difficult to isolate cyclical variations and hence obscures the analysis of driving moments of switching behavior. See for instance Moore and Wang (2007). The weekly returns are constructed as the first difference of logarithmic stock index multiplied by 100, $r_t = 100 \times [\log(p_t) - \log(p_{t-1})]$, where p_t is the stock or Forex index. The volatility series are constructed as the squared of stock and Forex rate returns. Stock and Forex rate data starts from 2000:01:05 and ends at 2015:06:03 for all countries, yielding 805 observations in total each series. The criterion for the selection of the sample period 2000-2015 is based on the managed or independently floating exchange rate regimes adopted by the five Latin American countries in the sample.

The descriptive statistics of returns and volatility of stock and Forex returns are shown in Table 1 and Table 2 respectively. The first panel shows statistical data for stock returns. The average is close to zero in all cases. The asymmetry coefficient is negative for all countries in the region, with Chile presenting the greatest magnitude and Peru the least. All series display positive skewness and excess of kurtosis, a well-known stylized fact of the presence of an asymmetric distribution with heavy tails of stock markets returns. Stationarity in time series is checked by applying the Augmented Dickey Fuller (ADF) test. The results fail to reject the null of a unit root in the logarithmic stock index series, but overwhelmingly reject the null for the first difference of logarithmic stock index returns[9]. In the second panel, statistical series for the volatilities of returns are shown. The Figures of the stock and Forex returns of the countries of the region and their respective volatilities are

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[8] Given the omission of data on a Wednesday from any given week, we decided to choose some other "feasible day" in that week. The criterion for this choice was based on the construction of a ranking of missing data (from lowest to highest) on each day of the week throughout the daily series, selecting as the "first feasible" the day of the week with fewest omissions; if this did not exist, we selected the following day in the ranking of omissions as a feasible second day; and so on. In this way we built weekly series with no missing data.

[9] Results available upon request.

presented in Figures 1-4. In these figures we can see typical stylized facts as clusters, leverage effects and higher volatility during the financial crisis 2008-2009 in both markets for all countries in the sample.

4.2 Results

We performed the fit of the constrained MS-GARCH model parameters of each country using three rival models: the GARCH model, the MS model and the Gray model. The results obtained are shown in Table 3 and Table 4. The MS model is a particular case of the MS-GARCH when $\alpha = 0$ and $\beta = 0$. The preferred model is the one with the lowest BIC; nevertheless, to ensure that our MS-GARCH model (a nonlinear model) is preferred to its rivals, we adopted the Davies (1987) upper bound test. Applying this test, the null was rejected in all cases, i.e. the MS-GARCH model is prefered to its rivals[10].

As Gray's model cannot generate consistent estimators for the MS-GARCH model, the MS-GARCH log-likelihood model evaluated in Gray's MLE model is usually found to be below that obtained by the GARCH model. See Augustyniak (2014). Also, using the MLE asymptotic standard errors, we determined the levels of significance at 1%, 5% and 10%, specifying them using the letters a, b and c respectively.

This paper considers two persistent regimes based on the financial stylized facts of stock and Forex market returns. The first persistent regime is the "low volatility regime", characterized by a positive average of returns in stock markets and negative average of returns in Forex markets; and the second is the "high volatility regime", characterized by a negative average of returns in stock markets and positive average of returns in Forex markets.

The results shown in Table 3 and Table 4 reveal that the conditional mean of returns in both the MS and the MS-GARCH model, is positive and negative in the low volatility regime (μ_1) in stock and Forex markets respectively, and negative and positive in the high volatility regime (μ_2) in stock and Forex markets respectively. Likewise, we observe in the MS-GARCH models that the magnitude in absolute value of the conditional mean of the high volatility returns is higher than the returns of the low volatility regime. As to the long-term average volatility (ω) of the two regimes, we note that in all cases the MS model overestimates their value compared with the MS-GARCH model, and that the long-term average volatility of the high volatility regime (ω_2) is always positive and higher than in the low volatility regime (ω_1).

Tables 3 and 4 also show that in the GARCH models of all countries, the estimated value of the impact of past shocks to current volatility (α) is exacerbated in comparison with the value estimated by the MS-GARCH models; the opposite happens with the estimated value of the weight of lagged variance (β). Likewise, it is verified in all cases that $\alpha + \beta < 1$, i.e. there is presence of stationarity in the unconditional variance of returns. This result is statistically verified after

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[10]The Davies test uses the complete set of information, and is less computationally intensive in obtaining an upper bound for the significance level of the LR statistics under the null hypothesis consisting of the model with the lowest number of states. For more details see Appendix A of Garcia and Perron (1996).

applying an IGARCH test to each series, verifying the absence of integrated processes of order one, due to the rejection of the null hypothesis of presence of unit root in all cases[11].

The persistence of high volatility regimes (p_{22}) estimated by the MS-GARCH model is always less than that estimated by the MS model. For all countries, the estimated persistence of both regimes by MS-GARCH models turn out to be lower than that estimated by the MS and Gray models. Also, the persistence of high volatility is much lower than the persistence of low volatility under the MS-GARCH model compared with the Gray and MS models. For example, the persistence of high and low volatility estimates for Brazil for the MS-GARCH are 0.857 and 0.422, respectively, while those reported by Gray's model are 0.947 and 0.662, and those reported by the MS model are 0.961 and 0.903 in stock markets.

Finally, for each country we obtain the smoothed probabilities of being in the regime of high volatility by using the MS and MS-GARCH models. The smoothed probabilities show that the constrained MS-GARCH model refines the detection of "episodes of high volatility" (measured in weeks) that the MS model infers. The incorporation of dynamic GARCH into the MS model reduces significantly the persistence of the high volatility regime, i.e. p_{22} reduces drastically the average values of the long-term conditional variance (ω_1 and ω_2). While in an MS model persistence in volatility is explained by the persistence of the regime (i.e., long periods of high volatility can occur only when the returns remain in the regime of high volatility), in an MS-GARCH persistence is best explained by the incorporation of the dynamics of the GARCH component, where the role of the MS process is to allow jumps between regimes, as documented in the econometric literature. See Eraker et al. (2003).

In the light of these findings, the MS-GARCH model appear to be more consistent with the stylized facts of financial series than its rivals, the MS, Gray and GARCH models. In the next section we will discuss some episodes of high volatility in both markets for each country, exemplifying the differences between the MS and the MS-GARCH models and comparing inferences about some stylized facts.

4.2.1 Brazil

During the period 2002-2003, the international financial market was characterized by strong volatility and sharp risk aversion due not only to investor concerns in the face of discouraging corporate results, but also to increasingly common revelations of accounting fraud, bankruptcies and reorganizations among major businesses, particularly in the United States. On the Brazilian financial market, the situation triggered a process of exchange depreciation. The MS model infers high-volatility depreciation of around six months, from June 2002 to January 2003. However, as we can see in Figure 6, the MS-GARCH model infers high volatility during the first three weeks of June and then the two last weeks of August.

In stock markets, as regards episodes of high volatility prompted by the 2007-2008 international financial crisis, as Figure 5 shows, the MS model infers that the returns enter the high volatility

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[11]Results available upon request.

regime in three episodes: the first from the week of 08/01/07 until the week of 08/22/07 (four weeks); the second from the week of 22/21/07 until the week of 2/20/08 (14 weeks); and third since the week of 6/4/08 until the week of 1/21/09 (34 weeks) totaling 52 weeks high volatility. In turn, the MS-GARCH model infers seven episodes: the first from the week of 02/28/07, the second from the week of 08/01/07 until the week of 08/15/07 (three weeks); the third from the week of 01/16/08 until the week of 01/23/08 (2 weeks); the fourth from the week of 07/09/08 (one week); and the fifth from the week of 08/06/08 until the week of 10/22/08 (12 weeks), totaling 30 weeks of high volatility. This is also true of Forex markets, shown in Figure 6, during the global financial crisis that began in September 2008 with the bankruptcy of Lehman Brothers and the collapse of large financial institutions around the world. From September to December 2008, the MS model infers that the return process occurs in regime two. During the same period, the MS-GARCH infers that this process enters regime two at the beginning of September of 2008 and returns to regime one six weeks later.

During the period August-September 2011, businessmen and consumers' expectations were negatively affected by the worsening of the fiscal crisis in Europe and of some fiscal related issues in the U.S.A, coupled with the outlook of moderate growth in activity in these economies and its likely effects on leading mature and emerging economies. In this context, in which major European economies slowed down and the Japanese economy posted another slump, the increase in risk perception led to high volatility on financial markets. In line with the evolution of the international situation, the Brazilian economy recorded a depreciation-high volatility. The MS model infers that the return process is in regime two during September and the first three weeks of December. However, the MS-GARCH model infers that only in the second week of August and in two weeks of September is the return process in regime two. A MS model is delayed by four weeks in capturing the beginning of the return process in regime two and extends the ending of the return process in regime two by several weeks.

In August 2013, in international markets there is evidence of some accommodation of commodity prices, as well as greater volatility and a trend of appreciation of the United States dollar. Risks to global financial stability remained high, such as those associated with the deleveraging process taking place in major economic blocs and with the steep slope of the yield curve of significant mature economies. The MS model infers that the return process is in regime two for a month, from the last week of August to the last week of September. The MS-GARCH model in the Forex markets infers that the exchange rate posted a depreciation only in the last week of August. It demonstrates that an MS model exacerbates the period of exchange rate depreciation by about three weeks. The same occurs in February 2015 when risk aversion and financial-market volatility tend to react to the signaling by authorities of the beginning of the restoration process of monetary conditions in the United States within the relevant horizon for monetary policy. A MS model infers that the return process is in regime two for five weeks while a MS-GARCH model captures the exact week when depreciation occurs: the second week of February. The MS model infers that Brazil's stock returns underwent a single episode of high volatility during the week of June 12,

2013. However, the MS-GARCH model specifies that the regime of high volatility occurred over three episodes: the first during the week of January 30, 2013 (one week); the second during the week of April 17, 2013 (one week); and the third from 29 May to 19 June 2013 (four weeks), giving a total of six weeks of high volatility.

The political crisis that hit Brazil at the beginning of September 2014 due to mismanagement of economic policy and the loss of investor confidence led to a jump in volatility of its main stock market index. The MS model infers that stock returns enter the regime of high volatility from the week of September 10 and return to the regime of low volatility during the week of December 10, totaling a single episode of 14 weeks of high volatility; however, during the same year, the MS-GARCH model reveals the presence of two episodes of high volatility: the first between the weeks of September 10 to October 1 (four weeks); and the second during the week of 22 October (one week), for a total of five weeks of high volatility.

4.2.2 Chile

In July 2002, the terms of trade of the Chilean economy were seriously affected by both the deteriorating global economy as well as stagnation in Japan specifically. On the financial front, the outlook caused a deterioriation in capital flows as well as exchange rate depreciation. As we can see in Figure 8, the MS model infers that the Chilean economy was in regime two for seven weeks; however a MS-GARCH model infers that only during the first week of July was the Chilean economy in a regime of depreciation: high volatility. In January 2005, the lower growth environment of monetary policy in the United States and higher oil prices caused the dollar to strengthen and Chile recorded a depreciation. The MS model fails to capture this event, while an MS-GARCH model infers that Chile was in regime two in Forex markets in the third week of January.

During the years 2007-2008, when the international financial crisis unfolded, Chile experienced fewer episodes of high volatility compared to their counterparts in the region in stock markets. As we can see in Figure 7, the MS model infers that returns experienced the high volatility regime over three episodes: the first from the week of 1/24/07 until the week of 4/4/07 (11 weeks); the second from the week of 8/8/07 until the week of 3/12/08 (32 weeks); and the third from the week of 6/25/08 until the week of 12/10/08 (25 weeks), totaling 68 weeks of high volatility. In turn, the MS-GARCH model infers 5 episodes: the first in the week of 8/15/07 (one week); the second in the week of 11/7/07 (one week), the third in the week of 1/9/08 (one week); the fourth in the week of 07/02/08 (one week); and the fifth from the week of 10/01/08 until the week of 10/08/08 (two weeks), giving a total of six weeks of high volatility.

In October 2008, there were significant increases in risk premiums and capital outflows from the portfolio of Chile. Volatility in the foreign exchange and stock markets reached record highs. The Chilean peso depreciated, and part of this depreciation was in response to the global appreciation of the dollar, which occurred due to changes in the portfolios of US Treasuries, in pursuit of lower risk and higher liquidity. Also, pension funds in Chile inflicted placed the exchange market under further strain through substantial changes in hedging positions. In Forex markets, an MS model

infers that the economy was in regime two from March to December 2008, while the MS-GARCH model infers that in the third week of April and the first two weeks of October, Chile was in a regime two. In the last days of January 2010, Chile posted a currency depreciation due to the persistently high degree of uncertainty regarding the future development of the European financial system and the instability of foreign markets. The MS model infers that Chile was in regime two during the months of December 2009 and January 2010. The MS-GARCH model infers that only during the last week of January 2010 was Chile in regime two. In stock markets, the MS model infers that the returns experienced the regime of high volatility from the week of 6/25/08 until the week of 12/10/08, totaling a single episode of 25 weeks; however, the MS-GARCH, during the same period, specifies two episodes of high volatility, the first in the week of 7/2/08 (one week), and the second from the week of 10/1/08 until the week of 10/8/08 (two weeks), totaling three weeks of high volatility.

Also, in August-September 2011 the international stage was characterized by greater financial stress and a higher degree of risk aversion. These financial strains are related to three factors: first, the strengthening of the European financial crisis; second, uncertainty about fiscal policy in the U.S.; and third, reducing growth prospects in advanced economies and signs of slower growth in emerging economies. This increased external volatility affected the equity, currency and fixed income market in Chile. In Forex markets, a MS model infers that for two months, from August to September, Chile was in regime two. The MS-GARCH model infers that only in the second week of August and the second week of September was Chile in regime two.

During 2013, the withdrawal of monetary stimulus in the U.S.A, the economic slowdown in China, and uncertainty about a possible tax reform in Chile caused a sharp fall in the IPSA market. In stock markets, the MS model infers that the returns entering the high volatility regime occurred in two episodes: the first from the week of 5/29/13 until the week of 10/2/13 (19 weeks); and the second in the week of 11/13/13 (one week), totaling 20 weeks of high volatility; however, during the same year, the MS-GARCH provided evidence of only one episode of high volatility in the week of 6/12/13 (one week).

Finally, in March 2015, the prospects for growth in China, Russia, and particularly Latin America deteriorated, caused by idiosyncratic elements. Because of trade links and direct investment by Chilean companies, financial contagion events occur in asset prices and exchange rates. In Forex markets, the MS model cannot capture this event, while the MS-GARCH model infers that Chile was in regime two in the second week of March.

4.2.3 Colombia

During the period March-May 2006, the Colombian financial system experienced a depreciation due to declines in the value of its marketable securities. This phenomenon was associated with perceived uncertainty in international financial markets, and Colombia even underwent the steepest decline in mutual fund investments in Latin America, with a decrease of 18.3% at late may 2006. In stock markets, as we can see in Figure 9, the MS model infers that returns experienced the regime of

high volatility over two episodes: the first from the week of 2/1/06 until the week of 2/22/06 (four weeks); and the second from 5/17/06 until the week of 7/12/06 nine weeks), for a total of 13 weeks of high volatility. In turn, the MS-GARCH model, during the same period, specifies three episodes of high volatility: the first in the week of 2/8/06 (one week); the second from the week of 5/10/06 until the week of 5/17/06 (two weeks); and the third in the week of 6/14/06 (one week), giving a total of four weeks of high volatility. Also, in Forex markets, in Figure 10, the MS model infers that Colombia was in regime two for five months, from March to December 2006, while the MS-GARCH model infers that Colombia was in regime two in the third week of March and the third week of May.

In stock markets, with regard to episodes of high volatility prompted by the international financial crisis of 2007-2008, the MS model infers that the returns entered the high volatility regime over 3 episodes: the firstfrom the week of 16/01/08 until the week of 30/01/08 (three weeks); and the second from the week of 17/09/08 until the week of 29/10/08 (seven weeks); giving a total of 10 weeks of high volatility. Meanwhile, the MS-GARCH model infers six episodes: the first in the week of 28/02/07 (one week), the second in the week of 30/05/07 (one week), the third in the week of 15/08/07 (one week); the fourth from the week of 09/01/08 until the week of 16/01/08 (two weeks); the fifth in the week of 03/09/08 (one week); and the sixth in the week of 08/10/08 (one week), for a total of seven weeks of high volatility.

In Forex markets, the MS model infers that from May 2007 to January 2010, the Colombian economy was in regime two while a MS-GARCH model infers that in September 2008, because of the global financial crisis, Colombia was in regime two and then returned to regime one after two weeks. Also, in January 2014, an exchange rate depreciation was recorded in Colombia due to credit risks and a decreasing quality indicator. The MS model infers that the Colombian economy was in regime two for four months, from January to April 2014, while the MS-GARCH model infers that Colombia was in regime two only in the last week of January 2014. Finally, in December 2014 Colombia posted a currency depreciation due to expectations of a slow recovery among the countries of the euro area and less dynamic emerging economies. A MS model infers that Colombia was in regime two from December 2014 to June 2015, while a MS GARCH model infers that it was in regime two only in the first week of December.

4.2.4   Mexico

As a result of the bursting of the dot-com bubble, the terrorist attacks of 11 September 2001, and the risk of deflation by including international trade in countries with low production costs, the stock index of the Mexican stock market went through episodes of high volatility throughout 2002. In Figure 11, the MS model infers that returns underwent the regime of high volatility from the week of 5/29/01 and returned to low volatility in the week of 12/11/01, totaling 29 weeks of high volatility. However, during the same year, the MS-GARCH model reveals the presence of three episodes of high volatility, each one of a one-week duration: the first in the week of 5/29/01, the second in the week of 6/26/01, and the third in the week of 11/13/01, totaling three weeks of high

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volatility.

Likewise, during September-October 2008, the bankruptcy of Lehman Brothers led to a sharp increase in global risk perceptions and increased uncertainty about the quality of some assets held by financial institutions. The particular characteristics of the Mexican economy led these shocks to have a particularly negative effect. Mexico's foreign trade is highly concentrated in the United States, particularly with regard to the export of manufactured goods. Therefore, the decline of US economic activity had a particularly adverse effect on the economy of Mexico. In this context, capital flows to Mexico's economy contracted sharply, affecting the exchange rate and the stock market. With regard to financial contagion, rising liquidity and capital in international markets and tight credit policies of international banks in the world financing conditions. In Figure 11, the MS model infers that returns experienced the regime of high volatility in a single episode, from the week of 9/3/08 until the week of 9/11/09 (63 weeks). Meanwhile, the MS-GARCH model, during the same period, identifies two episodes of high volatility: the first from the week of 8/27/08 until the week of 10/8/08 (7 weeks), and the second in the week of 10/28/09 (one week), totaling 8 weeks of high volatility. Also, in Figure 12, the MS model infers that Mexico was in regime two for one year, from September 2008 to 2009. However, the MS-GARCH infers that Mexico was in regime two for only two months, from September to October 2008. Also the MS-GARCH model infers that in the third week of February 2009, Mexico had a half-high volatility; this is consistent with the intensification of risk aversion and the liquidation of assets by investors outside during this period, and which caused an adverse effect.

The international financial crisis of 2007-2008 generated several episodes of high volatility in the Mexican stock market. In Figure 11, the MS model infers that the returns entering the high volatility regime in 3 episodes, first since the week of 28/02/07 until the week of 21/03/07 (4 weeks), second since the week of 25/07/07 until the week of 05/03/08 (33 weeks), and third since the week of 03/09/08 until the week of 11/11/09 (63 weeks), totaling 100 weeks high volatility; while the MS-GARCH model infers 7 episodes, first in the week of 28/02/07 (1 week), second in the week of 27/06/07 (1 week), third in the week of 01/08/07 (1 week), fourth in the week of 15/08/07 (1 week), fifth in the week of 07/11/07 (1 week), sixth in the week of 30/04/08 (1 week), and seventh since the week of 27/08/08 until the week of 08/10/08 (7 weeks), totaling 13 weeks high volatility

Also, in May 2010, Mexican fears about a weakening global economy deepened when the publication of several indicators pointed to a slowdown in global trade and a delayed pace of recovery of economic growth, particularly in the USA and Japan. This led to an exchange rate depreciation. In Figure 12, the MS model infers that the currency depreciation lasted more than two months, while the MS-GARCH model infers that Mexico was in a regime two only in the first two weeks of May. In August-September 2011, the international environment deteriorated as a result of a significant slowdown in the global economy, the worsening European sovereign debt crisis, and uncertainty about the adjustment of public finances in the United States and other advanced economies. The slowdown in the US economy, due to high correlation with the Mexican economy, resulted in greater volatility in local financial markets and downward pressure on the peso. The MS model infers that

Mexico was in regime two for five months, from August to December 2011. However, the MS-GARCH model infers that Mexico was in a regime two only for the first two weeks of August. Finally, in December 2014, the international environment showed a significant decline following the occurrence of two shocks. One shock was the significant fall in the international oil price, and the other shock was the widespread appreciation of the US dollar against an adjustment of portfolio led by differences in the pace of growth and expectations of monetary positions of the major advanced economies. These events led to increased volatility in international financial markets. In line with this scenario, Mexico experienced a currency depreciation. In Figure 12, the MS model infers that Mexico underwent medium-high exchange rate depreciation, while the MS-GARCH model infers that Mexico was in regime two in the second week of December 2014.

4.2.5 Peru

In the first four months of 2002 the exchange rate showed a tendency to rise. This behavior was accentuated between July and September, given the higher demand for foreign currency, in particular the portfolio recomposition by pension management fund companies toward deposits in foreign currency, which increased by US$ 237 million, from US$ 212 million in March 2002 to US$ 449 million in September of that year. Regional factors associated with prospects for the international economy and regional political changes also influenced the decisions of agents regarding the portfolio. This led to greater coverage of financial assets in the domestic market. Demand for foreign currency was also reflected in the increase in the balance of net forward sales by banks to the public, which rose from US$ 869 million at the end of June to US$ 1,017 million at the end of September, representing an increase of US$ 148 million. Also, the shift in the position of banks increased by US$ 60 million in the same period. In a context of quickening exchange rate variability from late August, the Central Bank intervened in September through certified placements in Nuevo soles indexed to the evolution of the US dollar. Also, US$ 127 million were sold on the spot market.

In Figure 16, the MS and MS-GARCH models infer that during the month of July, Peru was in regime two. However, in September the Central Bank intervened by selling dollars to reduce excessive exchange rate volatility, and the MS model does not capture the reduced volatility, while the MS-GARCH model infers that the volatility of the exchange rate started to decline from the intervention of the Central Bank to the return to regime one in December 2002. Also, from the fourth quarter of 2005, the exchange rate behavior was influenced by electoral uncertainty. In this period, the exchange rate showed greater volatility. In the early months of 2006, this increased uncertainty was observed in certain periods, particularly during the first half of January and from late March until the completion of the first electoral round (April 9, 2006), which pushed up the exchange rate. The uncertainty about the outcome of the electoral process was reflected in the evolution of country risk. The uncertainty about the outcome of the election process raised expectations of exchange rate depreciation during the early months of 2006. These higher expectations of depreciation increased public demand for hedging against currency risk through dollar forward

transactions. Thus, the balance of net currency forwards byfbanks to the public increased from US$ 1,027 million in December 2005 to US$ 1,427 million in May 30, 2006, reaching US$ 1,650 million on April 17.

In this context of exchange rate volatility, the Central Bank intervened by buying or selling dollars and placements of certificates of deposits indexed at the exchange rate. When the upward exchange rate volatility was significant, as was the case in October 2005 and January 2006, the Central Bank curbed it by selling dollars to banks, as well as re-adjustable certificates of deposit (CDRBCRP) placements. Thus, in this period, US$ 786 million was sold and US$ 409 million was placed in CDRBCRP. Conversely, when the downward pressure on currency volatility was greater, as in February 2006, the Central Bank curbed it through purchases of foreign currency on the market, which in this period amounted to US$ 59 million. The MS-GARCH model captures all these events, which occurred from August 2005 to June 2006, and in Figure 7 we can see how the Central Bank's intervention in the exchange market reduced exchange rate volatility. Meanwhile, the MS model assumes that during the period August 2005 to June 2006, the Peruvian economy was in the regime of depreciation-high volatility.

With regard to episodes of high volatility caused by the international financial crisis 2007-2008, in Figure 15, the MS model infers that the returns entered the high volatility regime in three episodes: the first from the week of 28/03/07 until the week of 13/06/07 (12 weeks); the second from the week of 15/08/07 until the week of 05/03/08 (30 weeks); and the third from the week of 02/07/08 until the week of 03/12/08 (23 weeks), totaling 65 weeks of high volatility. Meanwhile, the MS-GARCH model likewise infers three episodes: the first since the week of 21/03/07 until the week of 09/05/07 (eight weeks): the second since the week of 24/10/07 until the week of 05/12/07 (seven weeks); and the third since the week of 02/07/08 until the week of 22/10/08 (17 weeks), totaling 32 weeks of high volatility.

In the early months of 2008, the slowdown in the U.S. economy worsened significantly: the impact of the subprime crisis on credit conditions added to high oil prices, unfavorable trends in employment, and the continuation of the adjustment in the real estate market. The international economic outlook affected the Peruvian economy through trade and financial channels. From July, particularly given the the financial turmoil, the dollar strengthened in a context of scarce dollars and negative indicators of growth in other developed economies. From the month of September 2008, the Central Bank sold foreign currency for US$ 6.8 billion and placed indexed certificates of deposits at the exchange rate (CDR) for the equivalent of US$ 3.9 billion. Likewise, the evolution of the exchange rate in 2008 was characterized by higher volatility, explained by changes in the composition of a portfolio of financial assets pertaining to both residents and non-residents. In 2008, the upward trend in the exchange rate continued. The end of February saw a depreciation of 3.9% compared to December 2008. The exchange rate volatility and its effect on agents' expectations was also reflected in the currency futures market, which went from a balance of net forward purchases of US$ 1,203 million at the end of 2007 to a balance of net forward sales of US$ 693 million at the end of 2008. This signaled a change from appreciatory expectations to depreciatory expectations. The

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Central Bank sold foreign currency worth US$ 6,843 million from September 2008 and placed CDRs totaling US$ 3 980 million. Through to the end of February, the exchange rate rose significantly, with portfolio movements in the dollar market driven by expectations of future depreciation. This trend began to reverse in the second week of March, initially explained by the increased supply of dollars in the foreign exchange market due to payments from the regulation of income tax, reinforced by the decline in risk aversion in emerging markets among non-resident investors.

During March and April, the Central Bank did not intervene in the foreign exchange market. Between September and November 2009, the average nominal exchange rate appreciated, reflecting the weakness of the dollar in international markets, as well as portfolio movements by local agents, mainly AFPs that reduced their long-standing foreign-currency positions in response to the decline in risk aversion in international financial markets. In Figure 16, the MS-GARCH model is strong enough to capture these events as well as the Central Bank interventions in the exchange market in circumstances of high volatility, compared to the MS model, which does not capture these events and overestimates volatility, as if over the entire period, from September 2008 to November 2009, the exchange rate were in regime two.

Another case was when the uncertain world economic outlook in April and May 2011 led to a volatile exchange rate. In this period, demand for dollars came from non-resident investors and Pension Administration Funds (US$ 1,274 million and US$ 544 million, respectively), which was offset by private investors' offerings (US$ 828 million). Excess demand was covered by exchange operations by the Central Bank. (US$ 1,019 million). For this reason, the exchange rate showed a distinct evolution in April with deppreciation and appreciation pressures. Figure 16 shows that the MS model fails to capture this period of appreciation in May 2011, while the MS-GARCH model does indeed capture this period of currency appreciation. September 2011 was marked by uncertainty over the pace of U.S. economic recovery and the resolution of the fiscal and banking crisis in the Eurozone. Both models capture the weeks in which the economy was in regime two. Also, in April 2012, the world uncertainties marking the world economic outlook remained as regards the pace of US growth, the debt crisis in the Eurozone, and the development of China.

The deterioration in the perception of risk in emerging countries like Peru caused an outflow of capital and increased the price volatility of financial assets. This led to a change of portfolio among the main actors involved in the exchange market and increased demand for forwards among non-resident investors in order to hedge their positions in sovereign bonds. Between May 9 and 31, the exchange rate depreciated by 2.5% and the Central Bank sold US$ 676 million and placed indexed certificates of deposit (certificates indexed to the exchange rate) totaling US$ 562 million. In June, an appreciation trend is observed. The MS model inferred that from April to August, Peru was in regime two. However, the MS-GARCH model infers that from February to March 2012, the country was in regime two, and going into regime one from May.

The aggrevation of political uncertainty prompted by the presidential victory of nationalist candidate Ollanta Humala in 2011 caused a negative performance in the Lima Stock Exchange, mainly associated with mining, due to nervousness among foreign mining investors sparked by

pre-election proposals to implement a new tax on the productive sector, as well as a number of speculative manifestations that played out across different segments. In Figure 15, the MS model infers that returns underwent the regime of high volatility from the week of 3/16/11 and returned to low volatility in the week of 10/12/11, totaling a singlr episode of 31 weeks of high volatility. Likewise, the MS-GARCH model indicates the presence of one episode of high volatility from the week of 3/9/11 until the week of 4/27/11, totaling eight weeks of high volatility.

In 2013, Lima Stock Exchange recorded a loss of 23.63%, due to the fall in international prices of raw materials (on account of lower growth in China and the debt crisis in the USA and the eurozone), the falling prices of domestic shares given the slower growth of the Peruvian economy, liquidation of positions in mining titles carried out by pension fund management companies, investment in foreign markets, and the political noise (executive attempting to buy the Peruvian assets of the Spanish-owned Repsol, and changes in the cabinet), which influenced the investment decisions of investors. In Figure 15, the MS model infers that returns experienced the regime of high volatility in two episodes, the first in the week of 4/17/13, and the second from the week of 6/12/13 until the week of 8/14/13 (10 weeks). Meanwhile, the MS-GARCH model identifies a single episode of high volatility, from the week of 4/10/13 until the week of 4/24/13 (three weeks).

Finally, from January to May 2013, signs of uncertainty were associated with lower growth prospects in the world economy; the uncertainty arising from the Euro Zone; the possible economic slowdown in China; high liquidity in international markets; and the gradual downward correction in the price of metals, including gold and copper. In this context, the Central Bank intervened in the foreign exchange market from May 24, the date that marked increased international financial volatility in anticipation of the Fed announcement that the reduction of monetary stimulus was set to begin in September 2013. First, the Central Bank intervened by placing certificates of deposit indexed to the exchange rate from May to August 2013, amounting to US$ 1,772 million. From July, the Central Bank also began to intervene through the direct sale of dollars in the exchange market, which amounted to US$ 2,990 million in the same period. the MS model infers that from April to November 2013, Peru was in regime two. However, the MS-GARCH model infers that from March to May 2013, Peru was in regime two; and in June 2013, the economy was in regime one. This is consistent with the Central Bank intervention in the foreign exchange market on May 24. Moreover, in the last quarter of 2014, the exchange rate continued its upward trend given signals that a more robust growth in the USA would lead to a reduction in monetary stimulus. Net demand for dollars in the local market increased in the last quarter. The net demand for dollars was US$ 6,617 million, higher than the US$ 2,347 million recorded in the third quarter. This difference mainly came from the increased demand for dollars by non-resident agents, amounting to US$ 4,842 million - US$ 1,657 million more than in the third quarter. In this context, the Central Bank intervened by selling dollars in the spot market for US$ 2,182 million. In this quarter, it also began to intervene through a new instrument, the swap exchange. In Figure 15, the MS model infers that in December, coinciding with the Central Bank's intervention, the exchange rate was in regime two, while the MS-GARCH model infers that in this period the exchange rate was in regime

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one.

4.2.6 Volatility spillovers and contagion between Latin American Stock and Forex Markets

Figures 15 and 16 show a bar graph of occurrences in the regime of high volatility undergone by countries during the time horizon of study in stock and forex markets, respectively. In each of the seven panels in each figure, we took account of the countries experiencing the regime of high volatility, temporarily distributed into horizons of two years for each panel, starting from the first week of January 2001 and ending in the last week of December 2014. Analyzing the smoothed transition probabilities inferred by the MS-GARCH model, we can see that for only for one week did all forex and stock markets simultaneously experience high volatility regime: that beginning Wednesday October 8, 2008, in which the international financial crisis began.

For example, we can also identify groups of four and three countries that experienced the regime of high volatility simultaneously. The regime of high volatility experienced by four countries simultaneously, occurred across three weeks: the first in the week of 8/15/07 (Brazil, Chile, Colombia and Mexico); the second in the week of 9/3/08 (Brazil, Colombia, Mexico and Peru); and the third in the week of 10/1/08 (Brazil, Colombia, Mexico and Peru). The regime of high volatility experienced by three countries simultaneously occurred in the following weeks: first, in the week of 9/12/01 (Brazil, Chile and Mexico); the second, third and fourth in the weeks of 3/16/05, 5/17/06 and 2/28/07, respectively (Brazil, Colombia and Mexico); the fifth in the week beginning 8/27/08; the sixth in the week beginning 9/10/08 until the week of 9/24/08 (Brazil, Mexico and Peru); and finally, in the week of 8/10/11 (Brazil, Colombia and Mexico), for the case of stock markets.

We can also count the number of weeks in which each country experienced high volatility for the time horizon of this research. In total, there are 183 weeks in which at least one country experienced regimes of high volatility. Considering this result, the countries by the number of weeks in which they exhibited high volatility, in descending order, were Brazil (102 weeks), Peru (55 weeks), Mexico (37 weeks), Colombia (32 weeks), and Chile (12 weeks). On the other hand, we can also count the number of episodes - groups of uninterrupted weeks - in which the regime of high volatility was experienced. Thus, the country that experienced the greatest number of episodes of high volatility was Brazil (37 episodes), followed by Mexico (27 episodes), Colombia (26 episodes), Chile (11 episodes), and Peru (nine episodes). Considering the duration of episodes of high volatility, measured from Wednesday to Wednesday, the biggest episode pertains to Peru, from the week of 7/2/08 until the week of 10/22/08 (17 weeks); followed by Brazil, from the week of 7/23/08 until the week of 10/22/08 (14 weeks); Mexico, from the week of 8/27/08 until the week of 10/8/08 (seven weeks), and Chile, from the week of 10/1/08 until the week of 10/8/08 (two weeks); while Colombia experienced six episodes, the longest lasting in the region, first from the week of 10/10/01 until the week of 10/17/01 (two weeks); the second from the week of 11/14/01 until the week of 11/21/01; the third from the week of 5/5/04 until the week of 5/12/04, the fourth from the week of 5/10/6 until the week of 5/17/06; the fifth from the week of 1/9/04 until the week of

1/16/08; and the fifth from the week of 12/3/14 until the week of 12/19/14.

Figures 15 and 16 also show bunches of uninterrupted weeks, in which one or more countries experienced the regime of high volatility. For example, in stock markets, we find one of these bunches from the week of 6/4/08 until the week of 10/22/08 (21 weeks), with a peak in 10/8/08, when all countries experienced a high volatility regime. This episode is in line with the stylized fact relating to the failures of massive financial institutions in the United States on September 16, 2008, due primarily to exposure of securities to packaged subprime loans and credit default swaps, which quickly descended into a global crisis that resulted in a number of banking failures in Europe and sharp reductions in the value of stocks and commodities worldwide.

We found another bunch from the week of 7/13/11 until the week of 8/10/11 (five weeks); the peak is on the last date, with Brazil, Colombia and Mexico experiencing a high volatility regime. This episode is in line with the stylized fact related to a sharp drop in stock prices on 8/8/11 (Black Monday) in the stock markets of the USA, Middle East, Europe and Asia, due to the United States debt ceiling crisis in 2011, which caused the reduction of its category from AAA to AA+ on 8/6/11, 2011, as well as fears of contagion from the sovereign debt crisis in Spain and Italy.

All of these joint experiences of the regime of high volatility prompted us to calculate the correlations between the smoothed probabilities of being in regime two using windows of one year (52 weeks), s year and a half (78 weeks) and two years (104 weeks). The respective figures are 17 (a,b,c) and 18 (a,b,c). Each of these figures contains five panels, one per country. In each panel, correlations have been drawn of each country versus others, and identified (with a black dot) corresponding to the maximum correlation experienced in the common study horizon week. For example, in stock markets, in the first panel, the correlations of Brazil (IBOV) versus Chile (IPSA), Colombia (IGBC), Mexico (MEXBOL) and Peru (IGBVL) is displayed, with the highest correlations corresponding to the weeks 10/01/07 (0.770), 09/05/07 (0.779), 03/11/10 (0.949), and 29/07/09 (0.856), respectively. As we can see, as the window becomes larger, correlations tend to be more stable over time. Likewise, we can see a co-movement (correlations greater than 0.5) in all windows ending in mid-2010, demonstrating the presence of systematic effects on Latin American stock markets of the international financial crisis. Also, in most panels it can be seen that after the international financial crisis, at the end of the period of analysis, correlations tend to be positive, revealing a kind of positive interdependence during episodes of financial turmoil.

The correlation between the stock returns volatility and Forex rates volatility for each country using windows of 52, 78 and 104 weeks are shown in Figures 19 a, b and c respectively. They reveal that the correlation has been positive in most of the time horizon studied, experiencing values greater than 0.5 after the outbreak of the financial crisis, and then positioned in the rest of the time horizon below this value. These results are in line with those that appear in the literature on emerging markets (Karoui, 2006). A possible extension to this research could be to analyze the evidence of co-movement of the Latin-American stock and forex market indexes during periods of financial turbulence following the international financial crisis.

5 Conclusions

An approach based on both the Monte Carlo Expectation-Maximization (MCEM) and Monte Carlo Maximum Likelihood (MCML) algorithms is used to estimate the MLE of the MS-GARCH model. The estimates are compared with standard GARCH, MS and Gray models. The last is used for comparison in terms of maximum likelihood.

For all Latin-American countries analyzed in this paper, the volatility persistence is captured differently in the MS and MS-GARCH models. When GARCH dynamics are incorporated into the MS model, the regime associated with a positive mean return and high volatility becomes much less persistent (i.e., p_{22} is significantly reduced). The fit of the MS-GARCH model in Latin-American countries is superior to the standard GARCH model according to the estimated parameters. This demostrates that the persistence of volatility is captured differently in the MS and MS-GARCH models. For the MS model, the phenomenon is directly tied to regime persistence, i.e., long periods of high volatility can only occur when the return process remains in regime two. For the MS-GARCH model, it is better explained by the GARCH dynamics of the model since the role of the MS process is now to allow for jumps in the long-term average value of volatility.

Therefore, it is not surprising that enriching the GARCH model with a MS process provides an important fit, as the presence of these jumps is well documented in the literature. Many of these jumps are related to exchange intervention of the Central Banks in the Forex markets. All time series analyzed show that the standard GARCH model exacerbates the volatility almost twofold compared to the MS-GARCH model. The fit of the MS-GARCH model is superior to Gray's model, according to the BIC for the data sets considered. The BIC reported for Gray's model is based on the log-likelihood of that model. The log-likelihood of the MS-GARCH model evaluated at the MLE of Gray's model was generally below that of the GARCH model. This implies that Gray's model can only generate a crude estimate of the MLE for the MS-GARCH model.

For all countries, according to BIC, the best model for estimating the stock and Forex markets returns is the MS-GARCH, the second best is the MS model, the third is a standard GARCH model, and the last is Gray's model (only used for comparative terms). In Peru, according to the terms of maximum likelihood, the best model for estimating the stock market and Forex returns is an MS-GARCH, the second is Gray's model, the third is a standard GARCH, and the last is a MS model.

Finally, the temporal correlations between countries show that after the international financial crisis, at the end of the period of analysis, correlations tended to be positive, revealing a kind of positive interdependence during episodes of financial turmoil. Analysis of this evidence of co-movement of the Latin-American stock and Forex market indexes in periods of financial turbulence after the international financial crisis, remains as a possible extension to this research.

Appendix: The MCEM-MCML Algorithm

Given an initial estimate $\theta^{(0)}$, the algorithm started at $r = 1$ produces a sequence of iterates $\{\theta(r)\} \geq 1$ allowing us to compute the MLE of model (1)–(3):

1. Simulate m_r samples of the state vector S from $p\left(S|r, \theta^{(r-1)}\right)$ using a single-move Gibbs sampler. The states are simulated sequentially for $t = 1, ..., T$ based on the following full conditional distribution:

$$p\left(S_t|S_{1:t-1}^{(i)}, S_{t+1:T}^{(i-1)}, r, \theta^{(r-1)}\right) \propto p_{S_{t-1}^{(i)}} p_{S_{t+1}^{(i-1)}} \Pi_{j=t}^{T} \sigma_j^{-1} \exp\left[-\frac{1}{2}\left(\frac{r_j - \mu_{S_j}}{\sigma_j}\right)^2\right]. \quad (7)$$

   To ease notation, the expression $\sigma_j(S_{1:t})$ was reduced to σ_j. In the context of (7), σ_j represents $\sigma_j(S_{1:t-1}^{(i)}, S_t, S_{t+1:j}^{(i-1)})$. It is straightforward to sample S_t from (7) since S_t can only take integer values from 1 to N. However, it should be noted that it is not possible to compute (7) numerically for each value of S_t since this will result in underflow. To avoid underflow, we can calculate the ratios of these expressions and then recover the probabilities for $S_t = 1, ..., N$ from them. The m_r simulations of the state vector S that are obtained are denoted by $\{S^{(i)}\}_{i=1}^{m_r}$. These draws form a Markov chain with $p(S|r, \theta^{(r-1)})$ as its stationary distribution (see Frühwirth-Schnatter, 2006).

2. Monte Carlo E-step: Calculate $\widehat{Q}\left(\theta|\theta^{(r-1)}\right)$, an approximation of the conventional E-step $Q\left(\theta|\theta^{(r-1)}\right)$, where

$$\begin{aligned}
\widehat{Q}\left(\theta|\theta^{(r-1)}\right) &= \frac{1}{m_r}\sum_{i=1}^{m_r}\log[f\left(r, S^{(i)}|\theta\right)] & (8) \\
&= -\frac{T\log(2\pi)}{2} - \frac{1}{2m_r}\sum_{t=1}^{T}\sum_{i=1}^{m_r}\left[\log\left(\sigma_t^{(i)}\right)^2 + \frac{\left(r_j - \mu_{S_j^{(i)}}\right)^2}{\left(\sigma_t^{(i)}\right)^2}\right] \\
&\quad + \frac{1}{m_r}\sum_{t=1}^{T}\sum_{i=1}^{m_r}\log\left(p_{S_{t-1}^{(i)}, S_t^{(i)}}\right) \\
&= term\ 1 + term\ 2. & (9)
\end{aligned}$$

   In the previous expressions, $\sigma_t^{(i)}$ is shorthand for $\sigma_t(S_{1:t}^{(i)})$.

3. M-step: Perform the following maximization:

$$\theta^{(r)} = \underset{\theta}{\arg\max}\widehat{Q}\left(\theta|\theta^{(r-1)}\right)$$

   This optimization can be split into two independent steps since terms 1 and 2 of (9) involve different subsets of the parameters. Term 1 includes the mean and GARCH parameters while

term 2 only contains transition probabilities. Maximization of term 1 must be performed numerically and is similar to a standard GARCH optimization to calculate the MLE. To improve the performance of that optimization, the gradient of term 1 with respect to the mean and GARCH parameters should be provided to the optimization routine. Maximization of term 2 can be done analytically. Term 2 is at its maximum when the transition probabilities takes the values:

$$p_{jk} = \frac{f_{jk}}{\sum\limits_{l=1}^{N} f_{jl}}, \quad j, k = 1, ..., N,$$

where f_{jk} denotes the total number of transitions from state j to state k in all of the m_r simulated state vectors[12].

4. Apply a decision rule to determine whether to switch to the MCML algorithm. If the decision is to switch, go to the step 5 and set $\theta^* = \theta^{(r)}$. Otherwise, add 1 to r and go to step 1.

5. Simulate m^* samples of the state vector S from $p(S|r, \theta^*)$ using the single-move Gibbs sampler described in step 1 of the algorithm to obtain the importance sample $\left\{ S^{(i)} \right\}_{i=1}^{m^*}$.

6. MCML-step: Perform the following maximization to obtain the MLE:

$$\hat{\theta} = \arg\max_{\theta} \left[\log \sum_{i=1}^{m^*} w_{\theta|\theta^*}^{(i)} \right]$$

In contrast to the M-step, this optimization cannot be split into two steps[13].

Using importance sampling, the final sample, $\left\{ S^{(i)}, \bar{w}_{\hat{\theta}|\theta}^{(i)} \right\}_{i=1}^{m^*}$, from $p\left(S|r, \hat{\theta} \right)$, where $\bar{w}_{\hat{\theta}|\theta}^{(i)} = w_{\hat{\theta}|\theta^*}^{(i)} / \sum\limits_{i=1}^{m^*} w_{\hat{\theta}|\theta^*}^{(i)}$, $i = 1, ..., m^*$. This sample can be used to obtain an estimate of the smoothed inference of the state at time t, $p\left(S_t = j|r, \hat{\theta} \right)$, $j = 1, ..., N$, with $\sum\limits_{i=1}^{m^*} \bar{w}_{\hat{\theta}|\theta^*}^{(i)} I_{\left\{ S_t^{(i)} = j \right\}}$ or to compute the asymptotic variance-covariance matrix of the MLE.

The MCEM-MCML algorithm requires a set of initial values, which can influence the convergence. For this purpose, Augustyniak (2014) uses the resulting estimates of the basic MS model, GARCH model and Gray's standard model, setting a value to the initial state S_0 to generate the Markov chain.

Likewise, considering that work with empirical data requires that the algorithm MCEM-MCML put more emphasis on precision and become more robust with respect to the choice of the initial values, Augustyniak (2014) recommends the use of a schedule that increases the sample size along the MCEM-MCML algorithm, rather than implementing automated schedules for each iteration.

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[12] Proof of this result can be found in Appendix B of Augustyniak (2014).
[13] Appendix C of Augustyniak (2014) provides some details related to its implementation.

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Table 1. Descriptive Statistics for Latin American Stock Markets Returns

Country	Mean	sd	Maximum	Minimum	Skewness	Kurtosis
Returns						
Brazil	0.1441	3.6971	16.1700	-25.4903	-0.4760	6.0501
Chile	0.1587	2.4670	10.1573	-21.5317	-1.0333	11.0876
Colombia	0.3205	3.1065	14.3729	-22.4525	-0.8615	9.8286
Mexico	0.2283	2.9548	11.6942	-19.4440	-0.5658	6.6148
Peru	0.2497	3.6067	22.8661	-18.5938	-0.2664	8.6979
Volatility						
Brazil	13.6721	30.4888	649.7552	0.0000	12.5928	243.2389
Chile	6.1038	19.0778	463.6153	0.0000	17.7825	414.0088
Colombia	9.7398	28.1417	504.1136	0.0000	10.4008	153.0567
Mexico	8.7724	20.3959	378.0707	0.0000	9.1279	141.5508
Peru	13.0541	35.9400	522.8599	0.0000	7.1471	73.9478

Table 2. Descriptive Statistics for Latin American Forex Returns

Country	Mean	sd	Maximum	Minimum	Skewness	Kurtosis
Returns						
Brazil	0.0671	2.3439	19.6595	-13.6840	1.3216	14.8216
Chile	0.0235	1.4313	9.4294	-6.1284	0.4372	6.2087
Colombia	0.0376	1.5218	8.7607	-6.0369	0.4237	6.8914
Mexico	0.0608	1.4365	11.7165	-7.9490	0.7692	11.1899
Peru	-0.0134	0.6496	4.7634	-4.9393	0.4004	17.5773
Volatility						
Brazil	5.4915	20.5260	386.4952	0.0000	12.1436	190.2637
Chile	2.0465	4.6856	88.9137	0.0000	9.8820	157.8903
Colombia	2.3144	5.6388	76.7502	0.0000	6.1277	57.3419
Mexico	2.0646	6.6272	137.2774	0.0000	12.9856	233.4154
Peru	0.4217	1.7156	24.3964	0.0000	9.9439	120.9263

Table 3. Estimated Parameters for weekly Latin American Stock Markets Returns

Model	μ_1	μ_2	ω_1	ω_2	α	β	p_{11}	p_{22}	BIC
				Brazil					
GARCH	0.2269^c		1.1284^b		0.0960^a	0.8224^a			2172.799
MS	0.5277^a	-0.8236^b	8.3040^a	25.8365^a			0.9614^a	0.9028^a	2165.095
Gray	0.4650^a	-1.9469	0.0000	11.8396^b	0.0219	0.8066^a	0.9474^a	0.6625^a	2216.450
MS-GARCH	0.9110^a	-2.5763^a	0.0993	5.3889^a	0.0250^c	0.8745^a	0.8576^a	0.4224^a	2157.950
				Chile					
GARCH	0.2065^a		0.1094^c		0.0847^a	0.9018^a			1824.542
MS	0.3588^a	-0.2561	2.5162^a	13.2096^a			0.9528^a	0.9024^a	1795.893
Gray	0.2916^a	-3.1276	0.0000	22.5475^b	0.1518^a	0.7000^a	0.9751^a	0.0276	1867.967
MS-GARCH	0.3217^a	-1.8813^a	0.3236^a	9.3289^a	0.0000	0.8328^a	0.9473^a	0.2815^a	1786.731
				Colombia					
GARCH	0.3282^a		1.6675^a		0.1979^a	0.6262^a			1789.305
MS	0.4565^a	-0.9994	5.4868^a	47.9785^a			0.9750^a	0.7580^a	1766.539
Gray	0.3957^a	-1.7655	0.0000	26.6445^b	0.1229^b	0.6093^a	0.9377^a	0.2509	1833.626
MS-GARCH	0.5098^a	-0.9892^a	0.6462^a	10.2839^a	0.1040^a	0.6383^a	0.8817^a	0.3841^a	1760.807
				Mexico					
GARCH	0.3284^a		0.2177^b		0.1323^a	0.8482^a			1937.977
MS	0.4047^a	-0.1220	3.8913^a	18.1237^a			0.9799^a	0.9602^a	1936.484
Gray	0.3989^a	-2.5017	0.0000	6.1426	0.1069^a	0.8281^a	0.9798^a	0.4065	1941.229
MS-GARCH	0.6639^a	-1.7675^a	0.0000	3.0723^a	0.0693^a	0.8541^a	0.8640^a	0.3427^a	1917.090
				Peru					
GARCH	0.3123^a		0.4246^a		0.1680^a	0.8102^a			2042.911
MS	0.2942^a	0.0943	5.1412^a	40.3401^a			0.9770^a	0.9197^a	2027.092
Gray	0.8187^a	-1.0518^a	0.0000	0.1252	0.2147^a	0.7492^a	0.9419^a	0.8828^a	2069.192
MS-GARCH	0.3978^a	-1.0106	0.1669^a	8.5172^a	0.0000	0.9401^a	0.9778^a	0.7010^a	2010.583

a, b, c denote significance level at 1%, 5% and 10% respectively.

Table 4. Estimated Parameters for weekly Latin American Forex Returns

Model	μ_1	μ_2	ω_1	ω_2	α	β	p_{11}	p_{22}	BIC
Brazil (BRL)									
GARCH	-0.0394		0.1920^a		0.2157^a	0.7724^a			1678.1430
MS	-0.1321^b	1.1834^b	2.2763^a	21.8092^a			0.9735^a	0.8541^a	1673.0680
Gray	-0.0849	2.7523^b	0.0000	7.9914^b	0.1526^a	0.7293^a	0.9819^a	0.4053^b	1698.3210
MS-GARCH	-0.0985^c	3.9688^a	0.1259^a	4.8457^a	0.1180^a	0.8042^a	0.9744^a	0.4700^a	1632.1000
Chile (CLP)									
GARCH	0.0235			0.0755^b	0.0980^a	0.8680^a			1384.3400
MS	-0.0453	0.4500	1.3990^a	6.0080^a			0.9880^a	0.9234^a	1382.7200
Gray	-0.0335	2.0223	0.0000	3.2789	0.0833^a	0.8306^a	0.9807^a	0.1420	1425.5280
MS-GARCH	-0.0615	1.8832^a	0.0854^b	1.6457^a	0.0516^b	0.8654^a	0.9722^a	0.4270^a	1374.8240
Colombia (COP)									
GARCH	-0.0398		0.0692^a		0.2455^a	0.7534^a			1341.5140
MS	-0.0286	0.1726	0.7467^a	5.3840^a			0.9757^a	0.9551^a	1341.0690
Gray	-0.0943^b	0.7865^a	0.0000	0.1543	0.1929^a	0.7749^a	0.9702^a	0.7915^a	1345.0670
MS-GARCH	-0.0643^b	0.6368^a	0.0008	0.7492^a	0.1062^a	0.8569^a	0.9590^a	0.8111^a	1317.4540
Mexico (MXN)									
GARCH	-0.0061		0.0707^a		0.1297^a	0.8438^a			1337.3270
MS	-0.0219	0.3435^c	0.9290^a	5.7706^a			0.9815^a	0.9381^a	1321.9950
Gray	-0.0202	1.0519^a	0.0823^c	2.6073^a	0.0156	0.7796^a	0.9883^a	0.8935^a	1345.8600
MS-GARCH	-0.0400	1.8979^a	0.0393^b	1.8141^a	0.0103	0.9223^a	0.9765^a	0.5070^a	1293.3790
Peru (PEN)									
GARCH	-0.0307^b		0.0065^a		0.2810^a	0.7509^a			522.5950
MS	-0.0435^a	0.0368	0.0539^a	1.0279^a			0.9320^a	0.8867^a	528.9300
Gray	-0.0442^a	0.3458^a	0.0000	0.0398^b	0.2663^a	0.7162^a	0.9836^a	0.8522^a	498.6430
MS-GARCH	-0.0422^a	0.1615^a	0.0007^c	0.1630^a	0.1887^a	0.7308^a	0.9181^a	0.6299^a	469.9790

a, b, c denote significance level at 1%, 5% and 10% respectively.
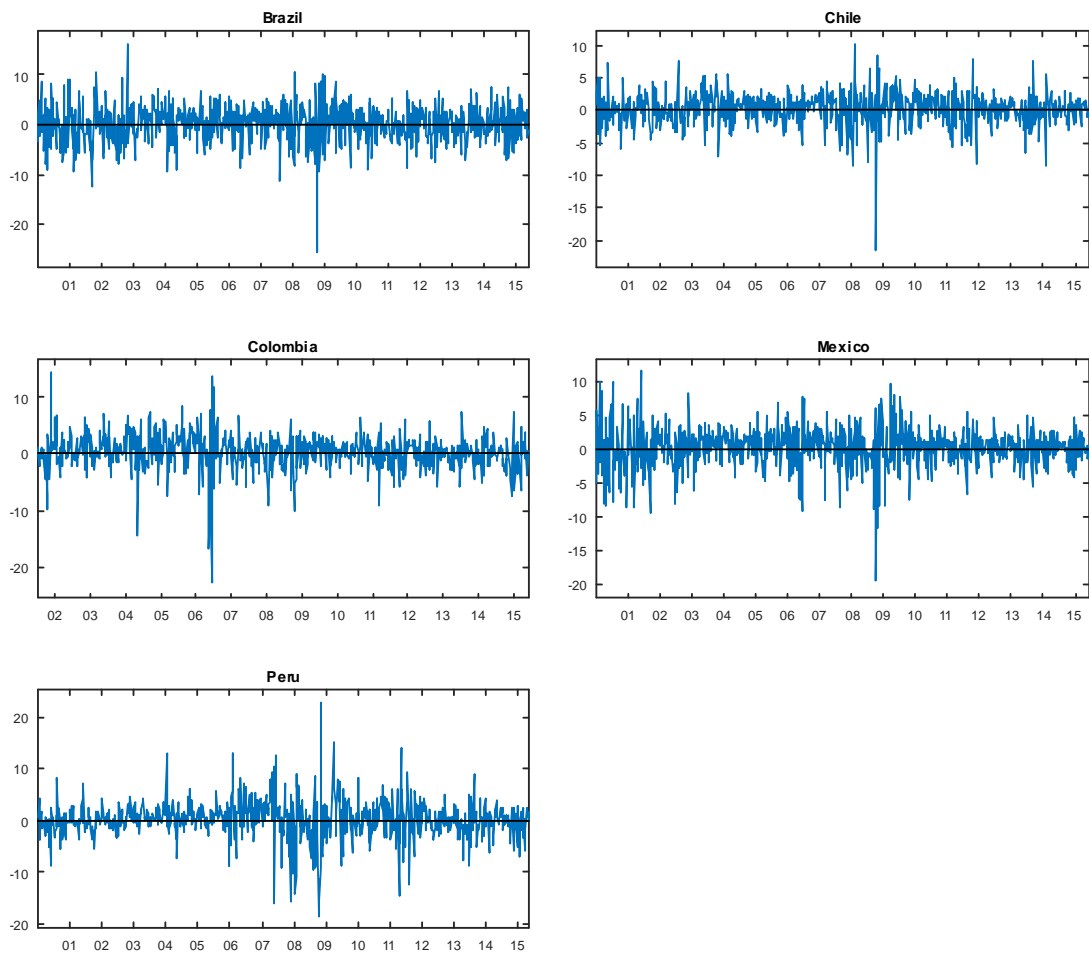
Figure 1. Latin American Stock Market Returns
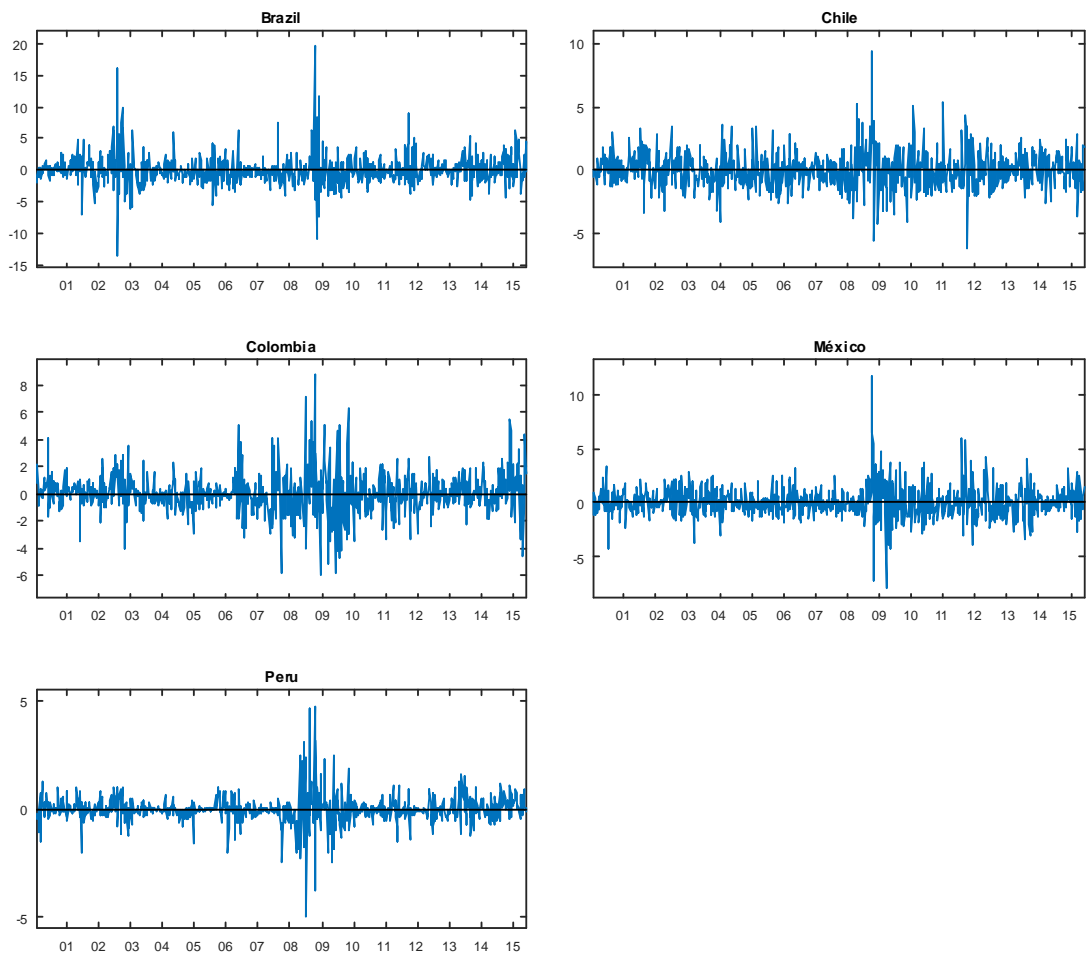
Figure 2. Latin American Forex Rate
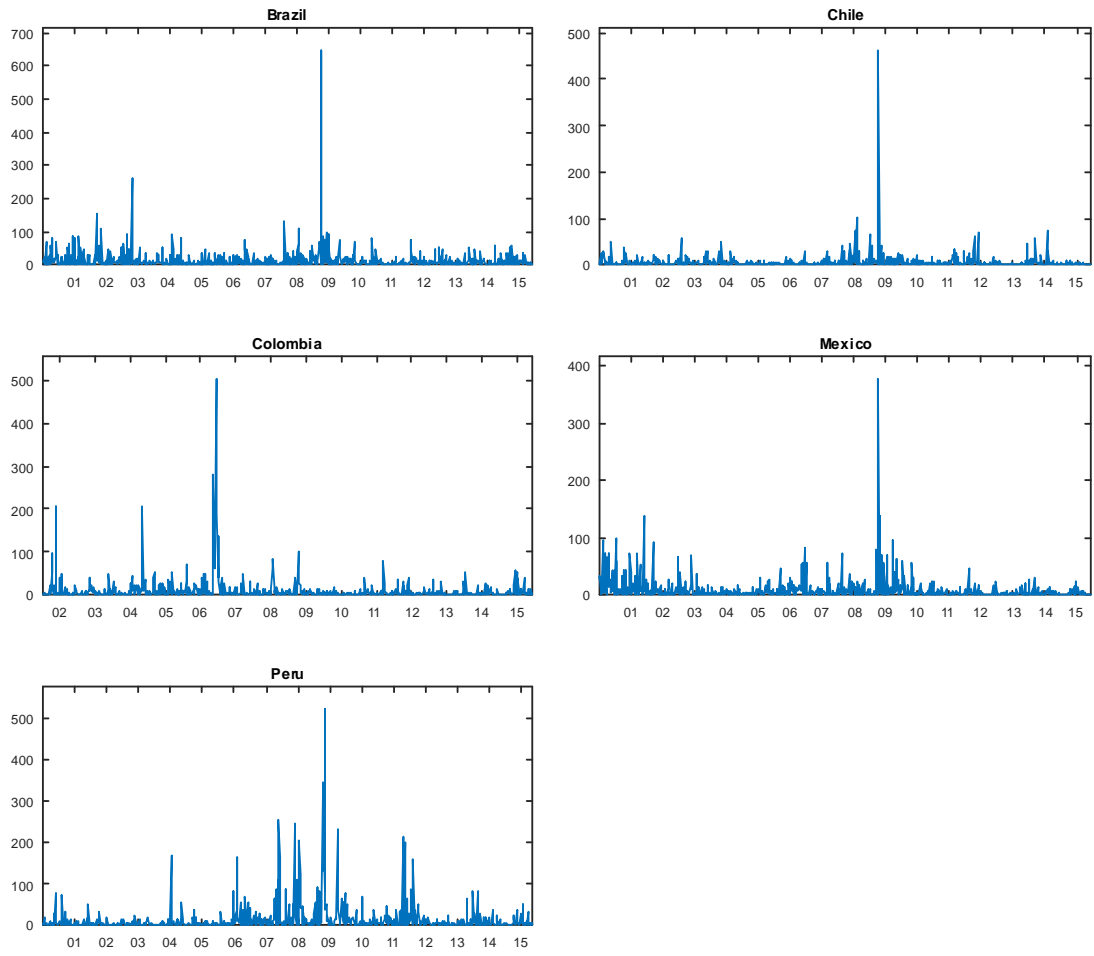
Figure 3. Latin American Stock Market Squared Returns
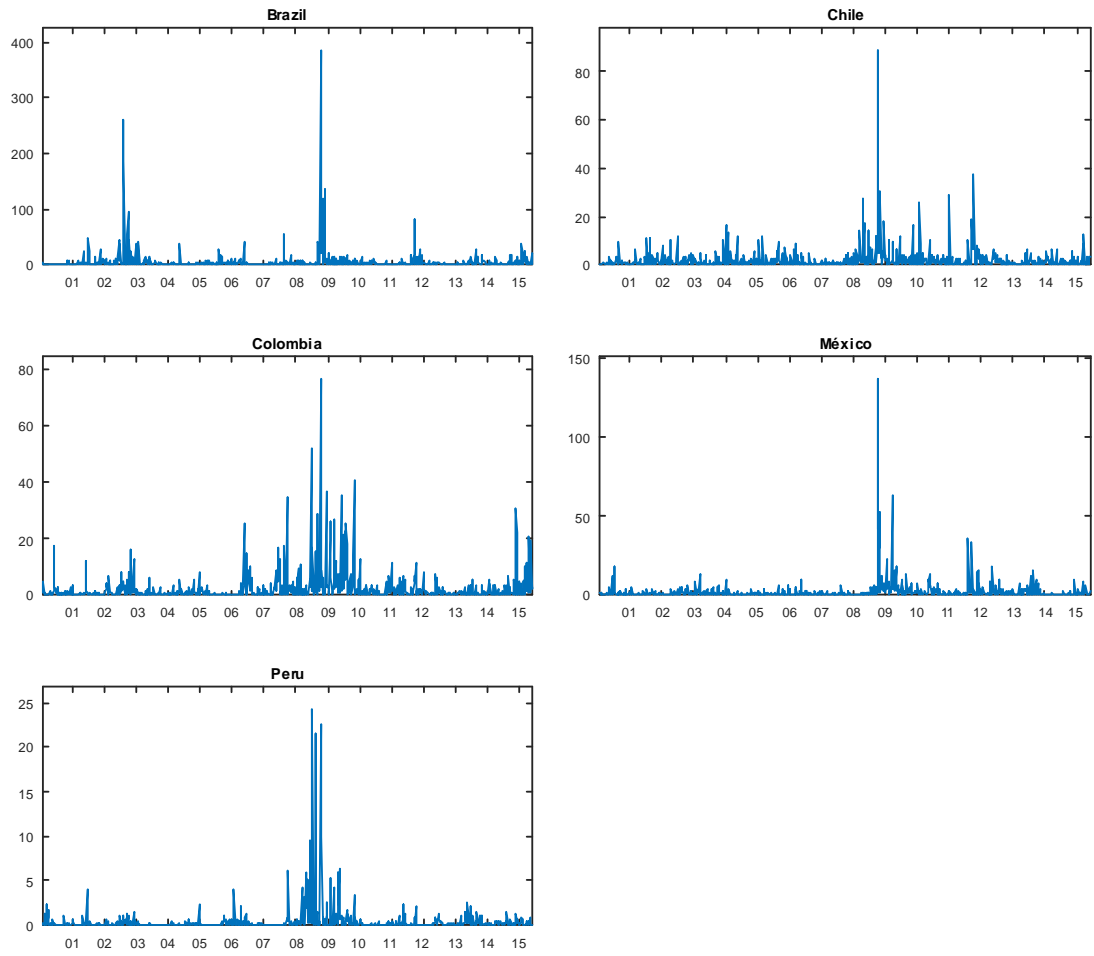
Figure 4. Latin American Squared Forex Rate
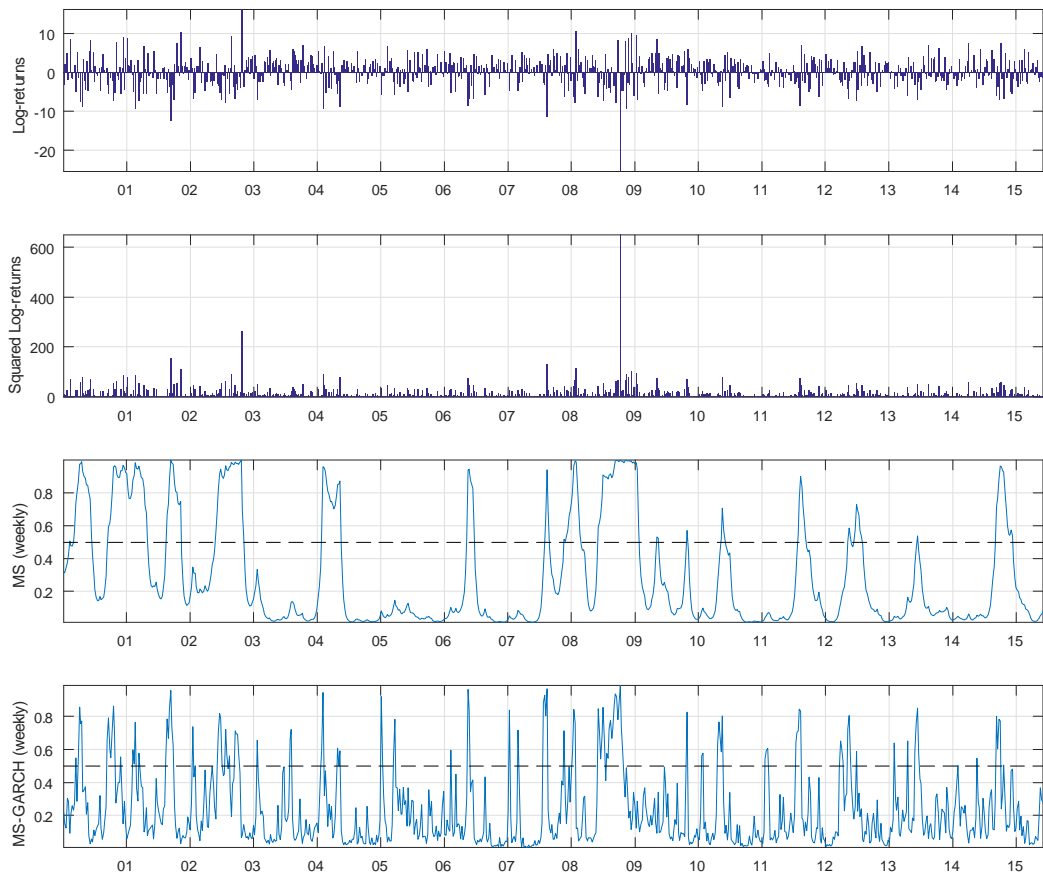
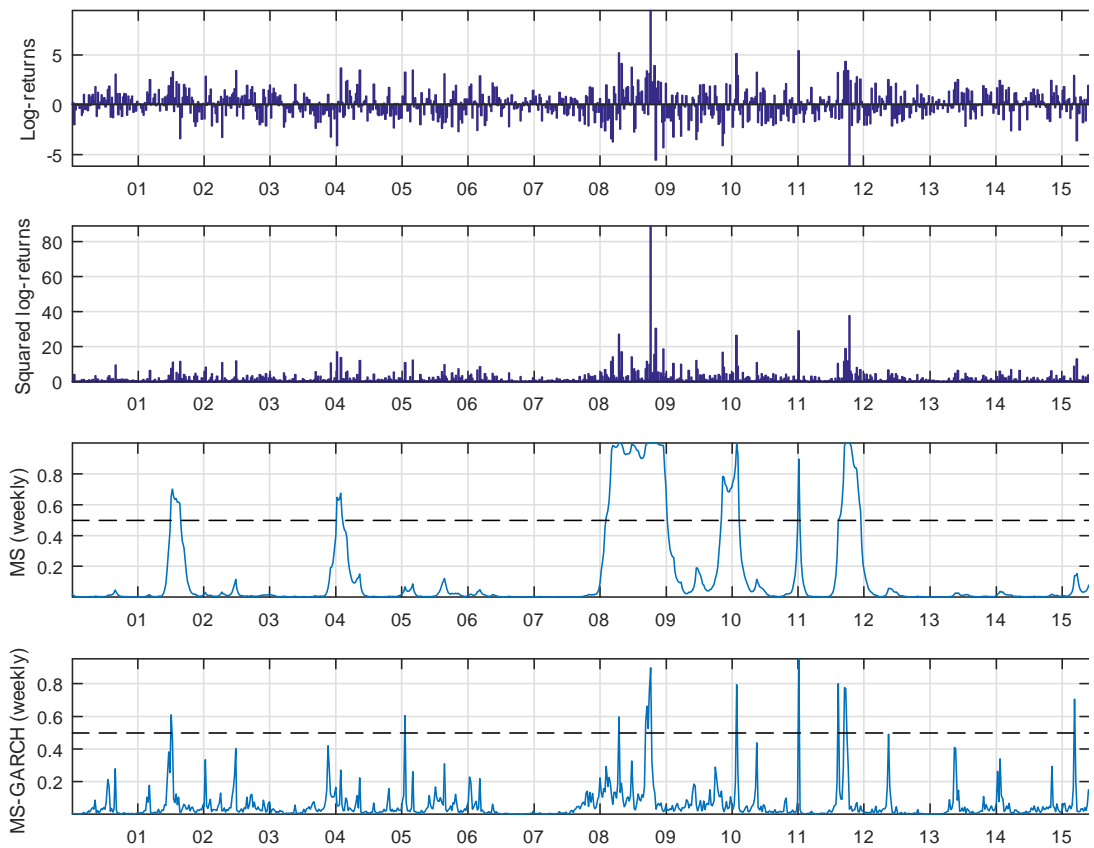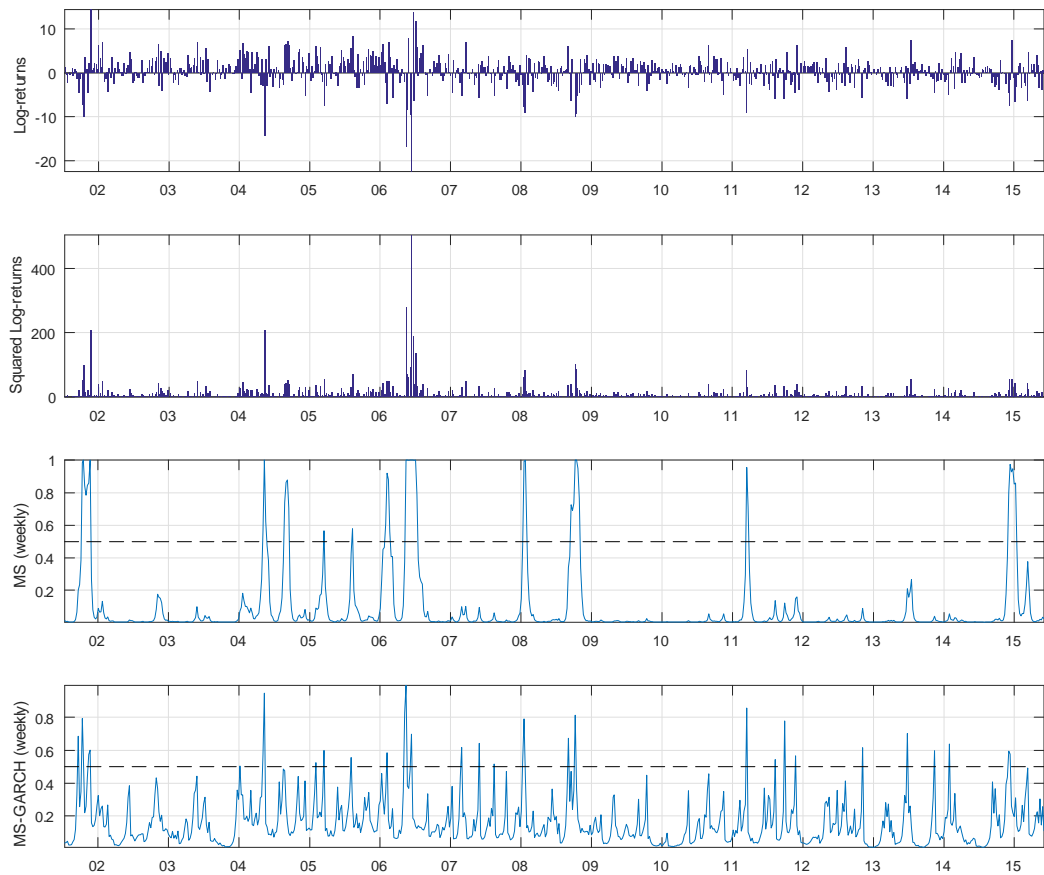**Figure 5.** Brazil: Smoothed Probabilities of beign in regime two (high volatility) in Stock Market
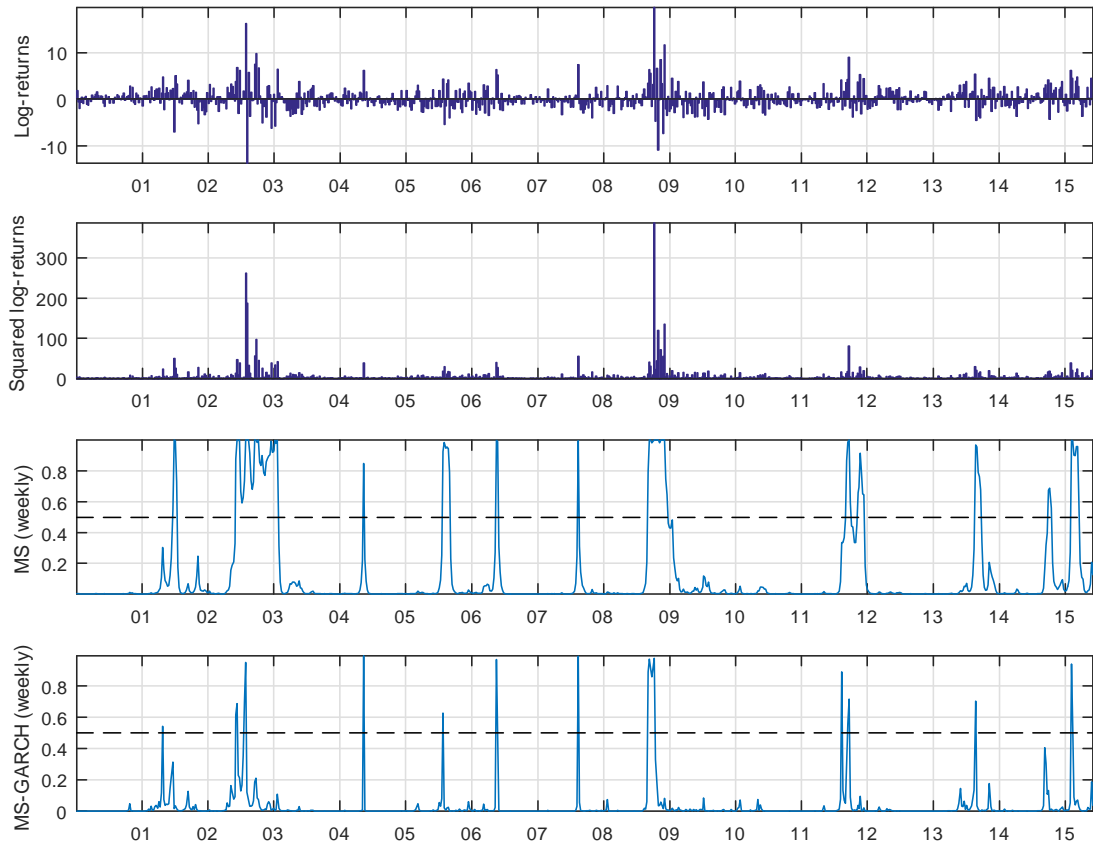
Figure 6. Brazil: Smoothed Probabilities of beign in regime two (high volatility) in Forex Market
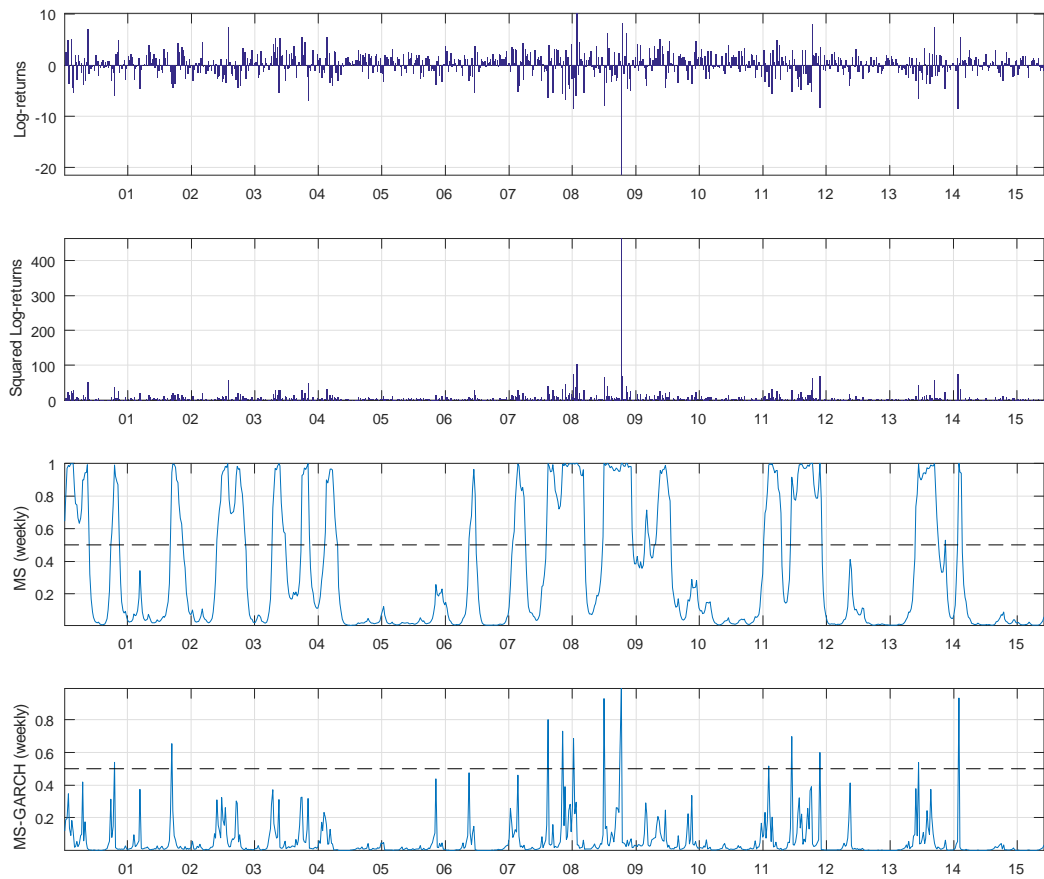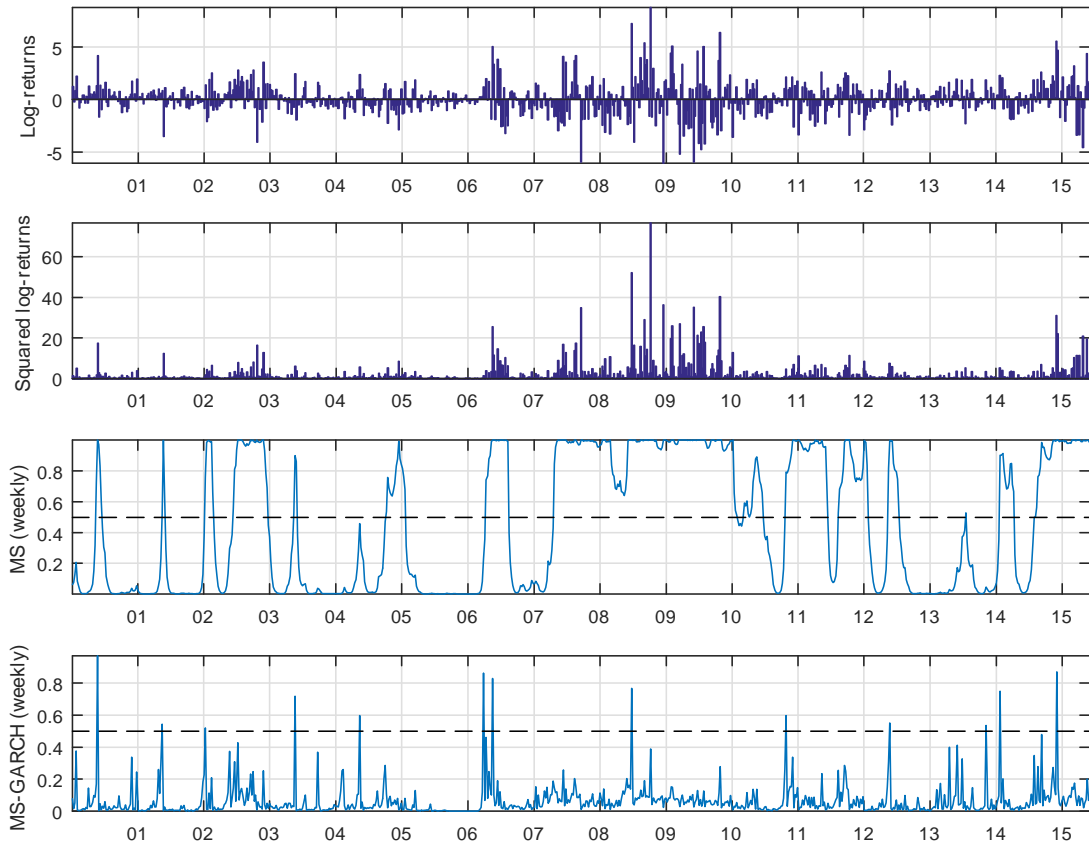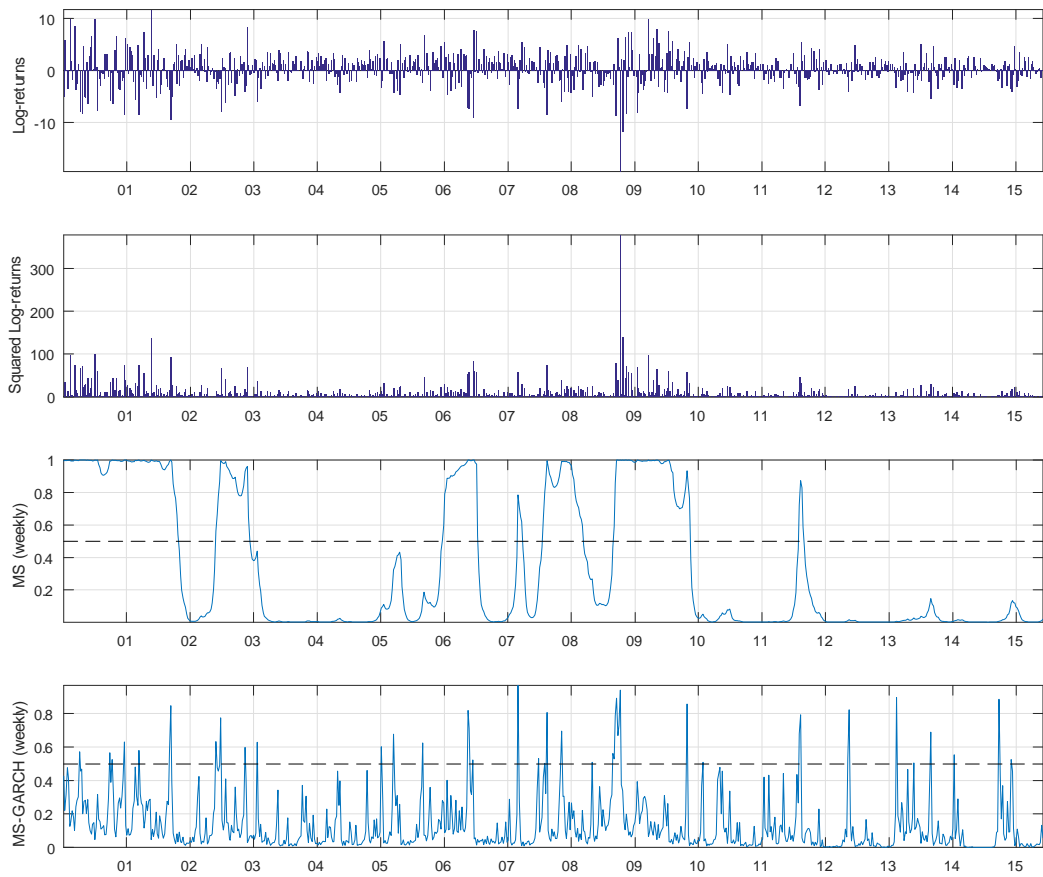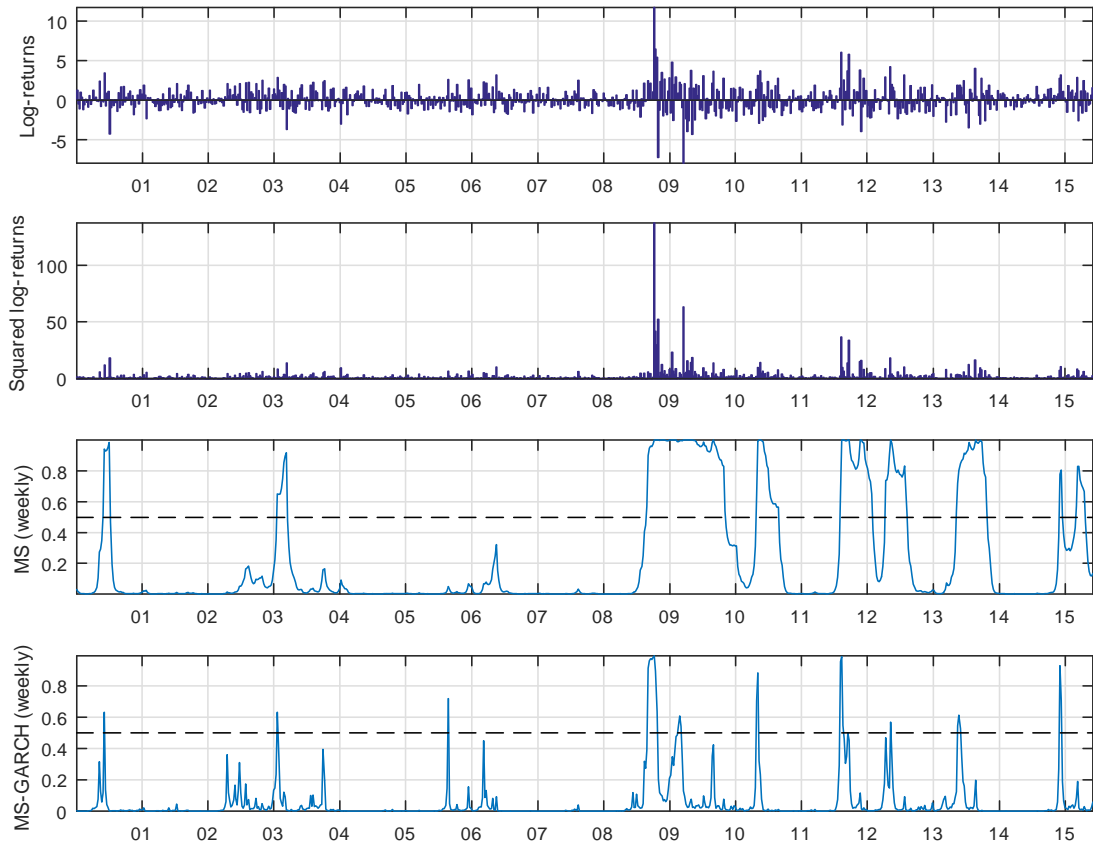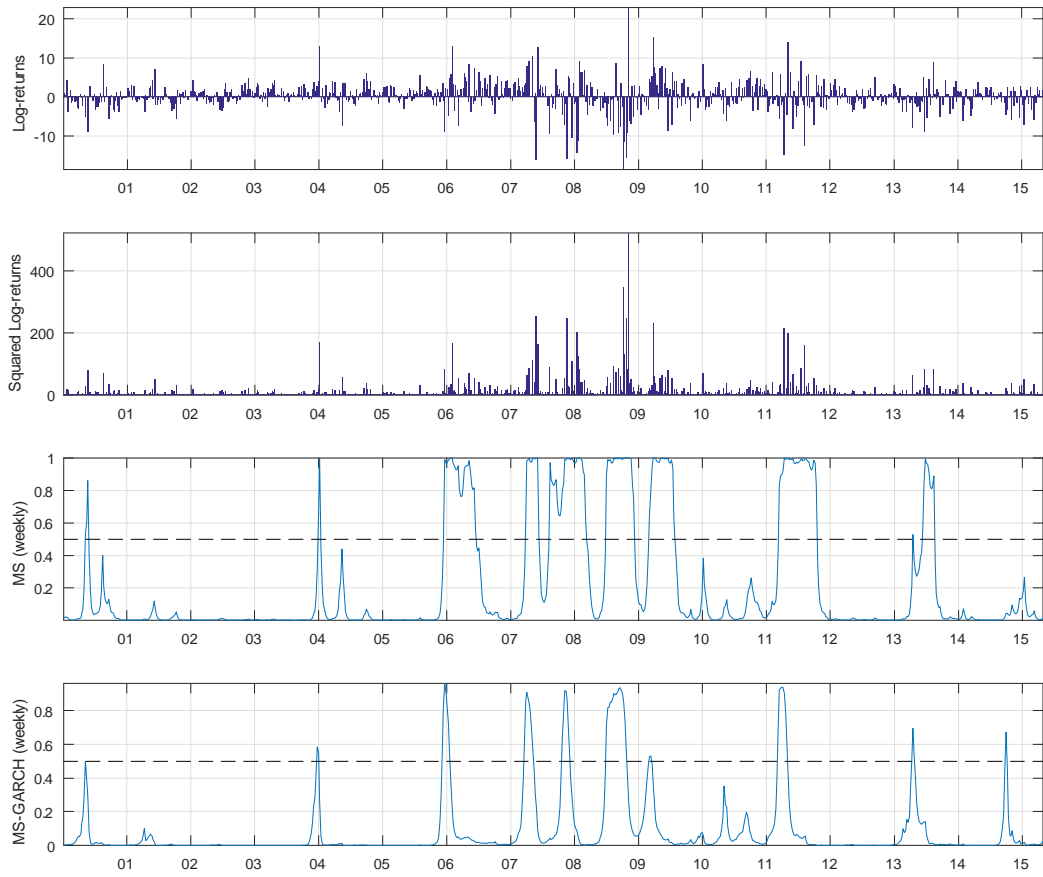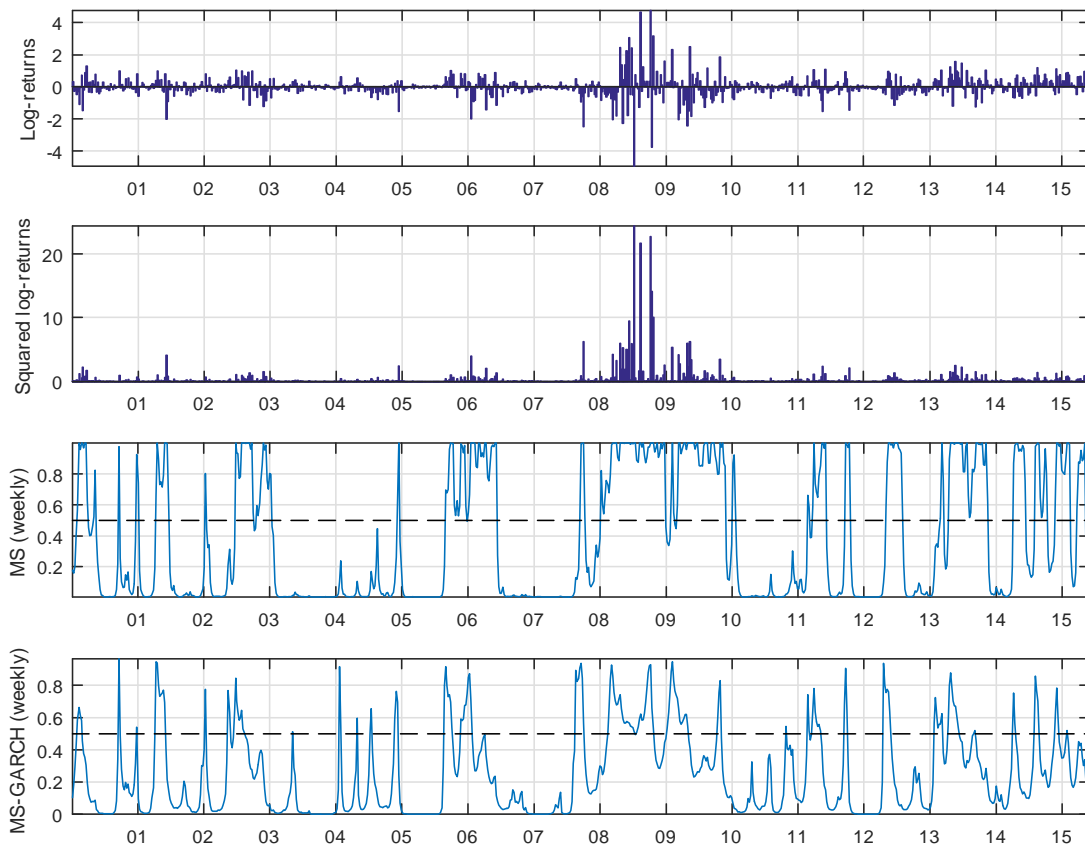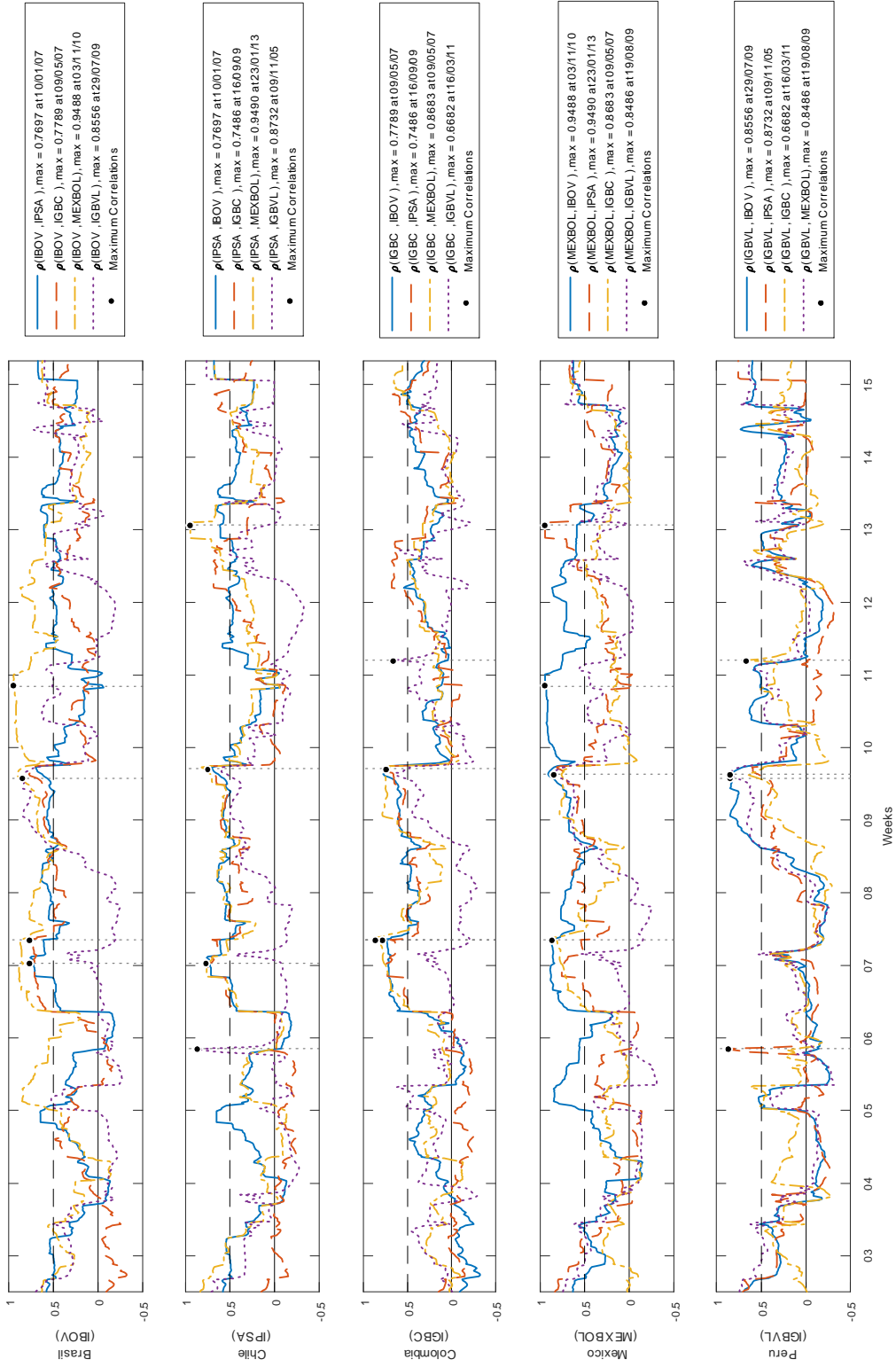
Figure 7. Chile: Smoothed Probabilities of beign in regime two (high volatility) in Stock Market

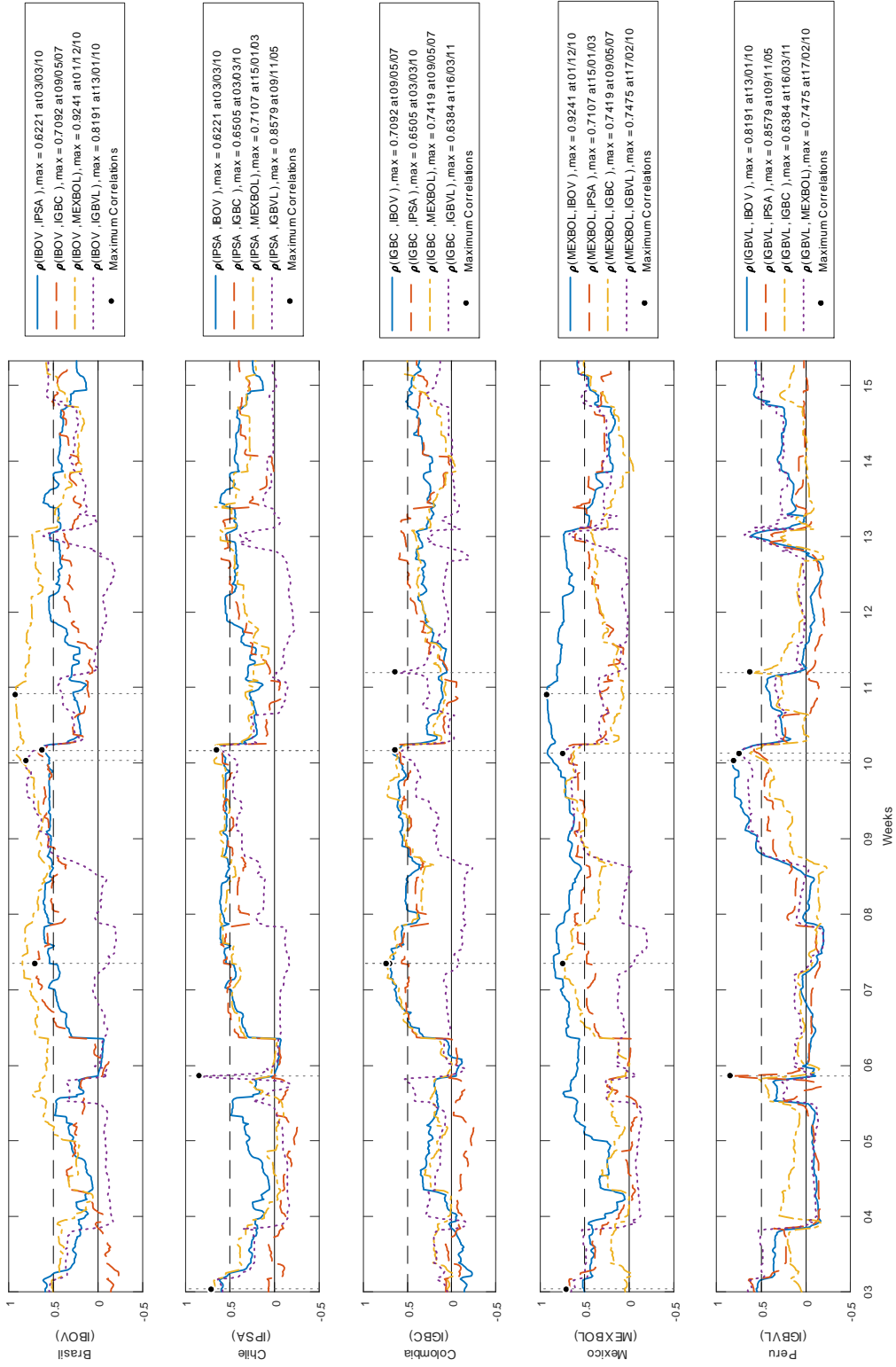**Figure 8.** Chile: Smoothed Probabilities of beign in regime two (high volatility) in Forex Market

Figure 9. Colombia: Smoothed Probabilities of beign in regime two (high volatility) in Stock Market

Figure 10. Colombia: Smoothed Probabilities of beign in regime two (high volatility) in Forex Market

Figure 11. Mexico: Smoothed Probabilities of beign in regime two (high volatility) in Stock Market

Figure 12. Mexico: Smoothed Probabilities of beign in regime two (high volatility) in Forex Market

Figure 13. Peru: Smoothed Probabilities of beign in regime two (high volatility) in Stock Market

Figure 14. Peru: Smoothed Probabilities of beign in regime two (high volatility) in Forex Market
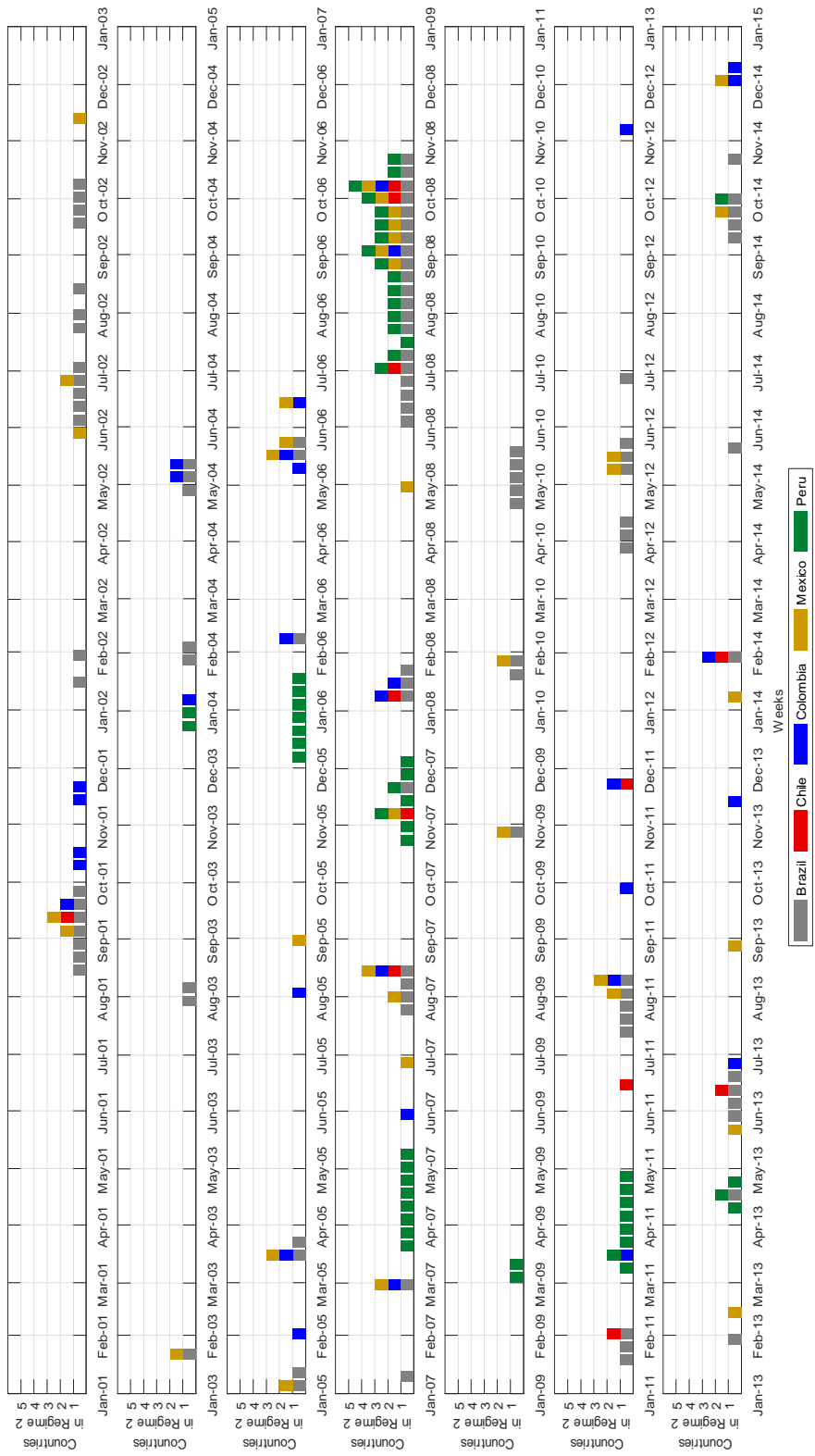
Figure 15. Weekly mapping of Latin American countries facing the regime 2. As shown in the second panel, the MS-GARCH shows that in the second week of October 2008 all the countries studied faced high volatility, this due to crash widespread than was given on October 10 in all world stock markets.

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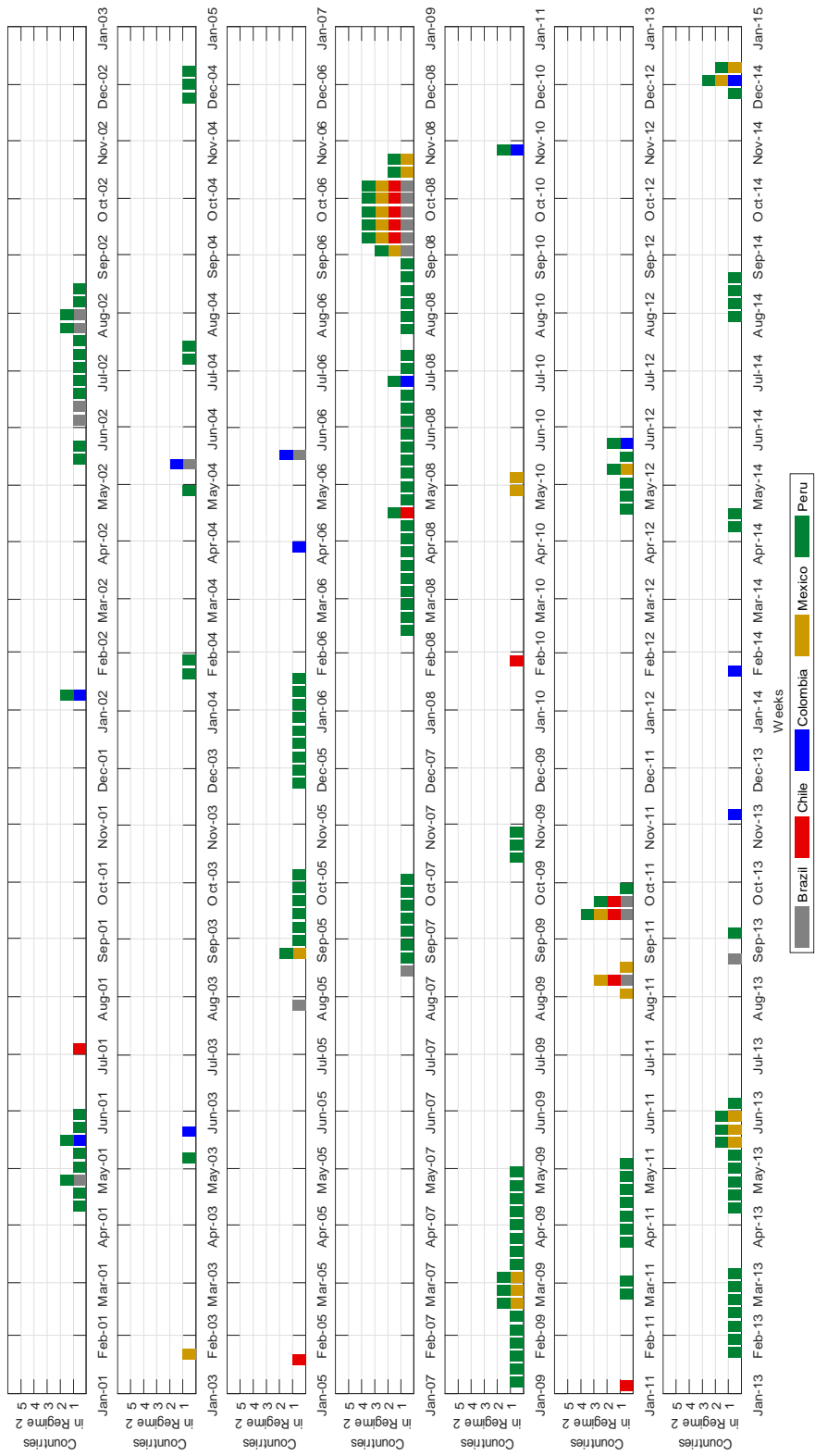
Figure 16. Weekly mapping of Latin American countries facing the regime 2. As shown in the second panel, the MS-GARCH shows that in the second week of October 2008 all the countries studied faced high volatility, this due to crash widespread than was given on October 10 in all world Forex markets.
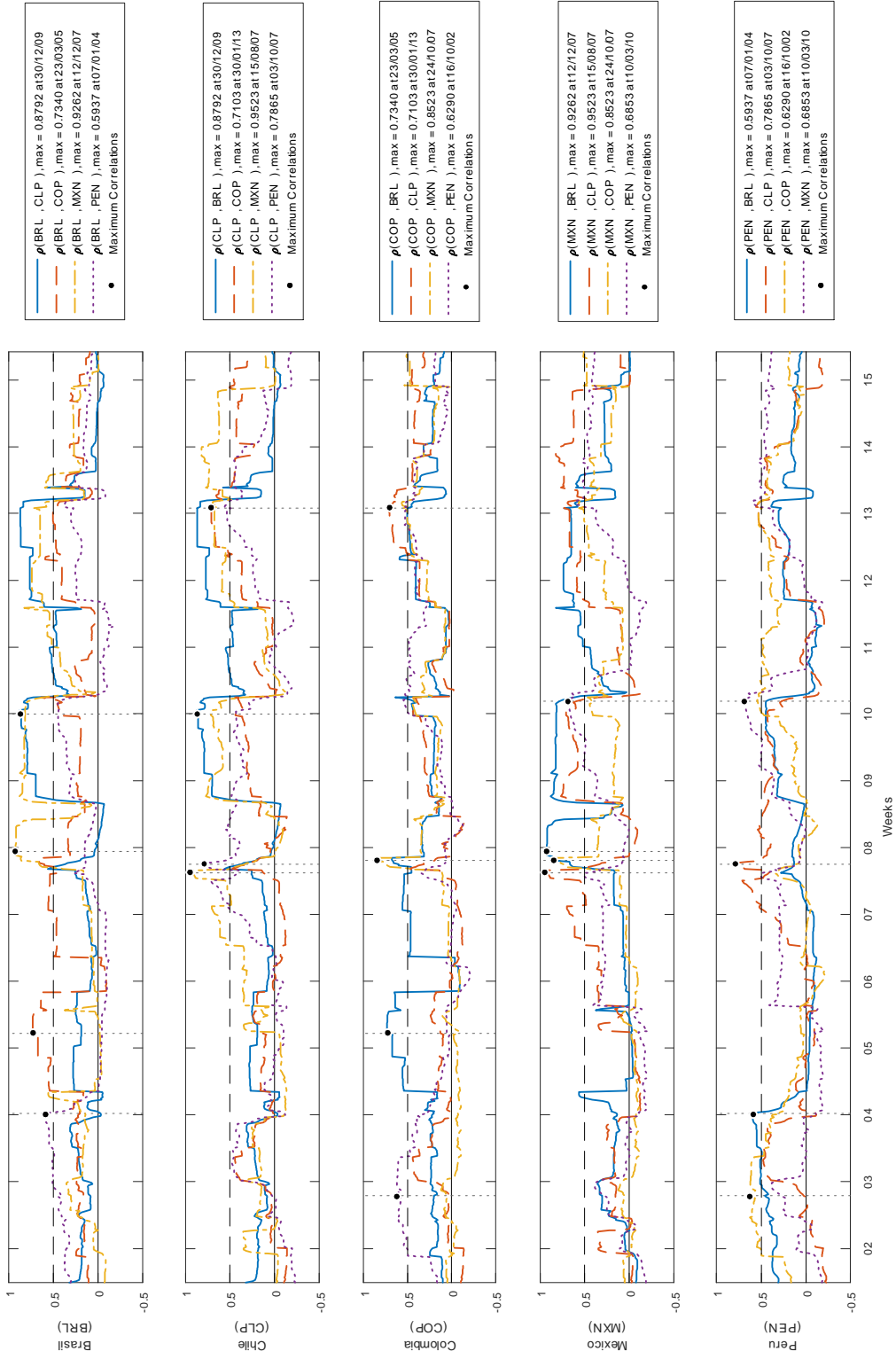
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Figure 17(a) Stock Market Correlations between Smoothed Probabilities of negative returns - high volatility using a window of 52 weeks

Figure 17(b) Stock Market Correlations between Smoothed Probabilities of negative returns - high volatility using a window of 78 weeks
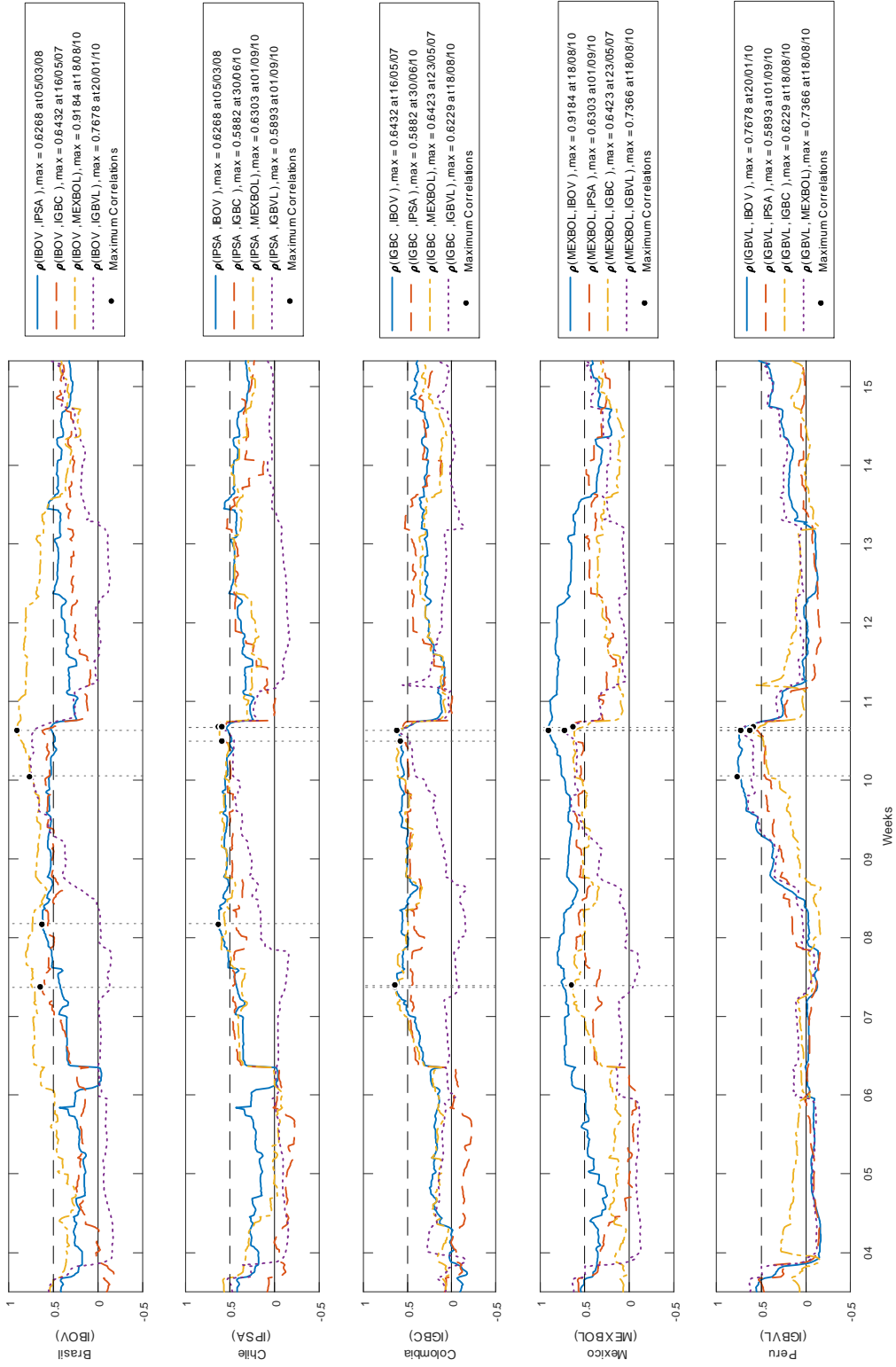
Figure 17(c) Stock Market Correlations between Smoothed Probabilities of negative returns - high volatility using a window of 104 weeks
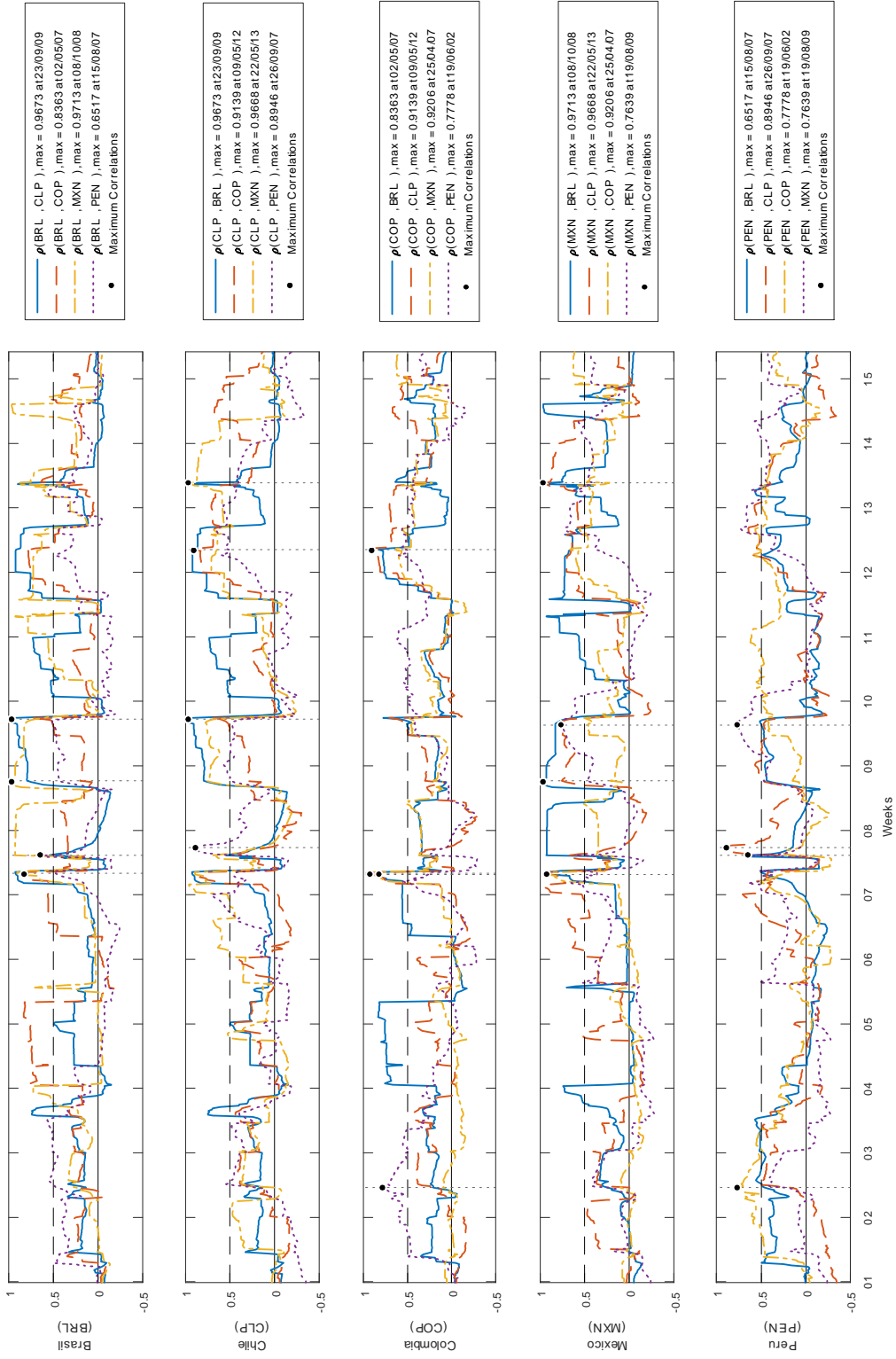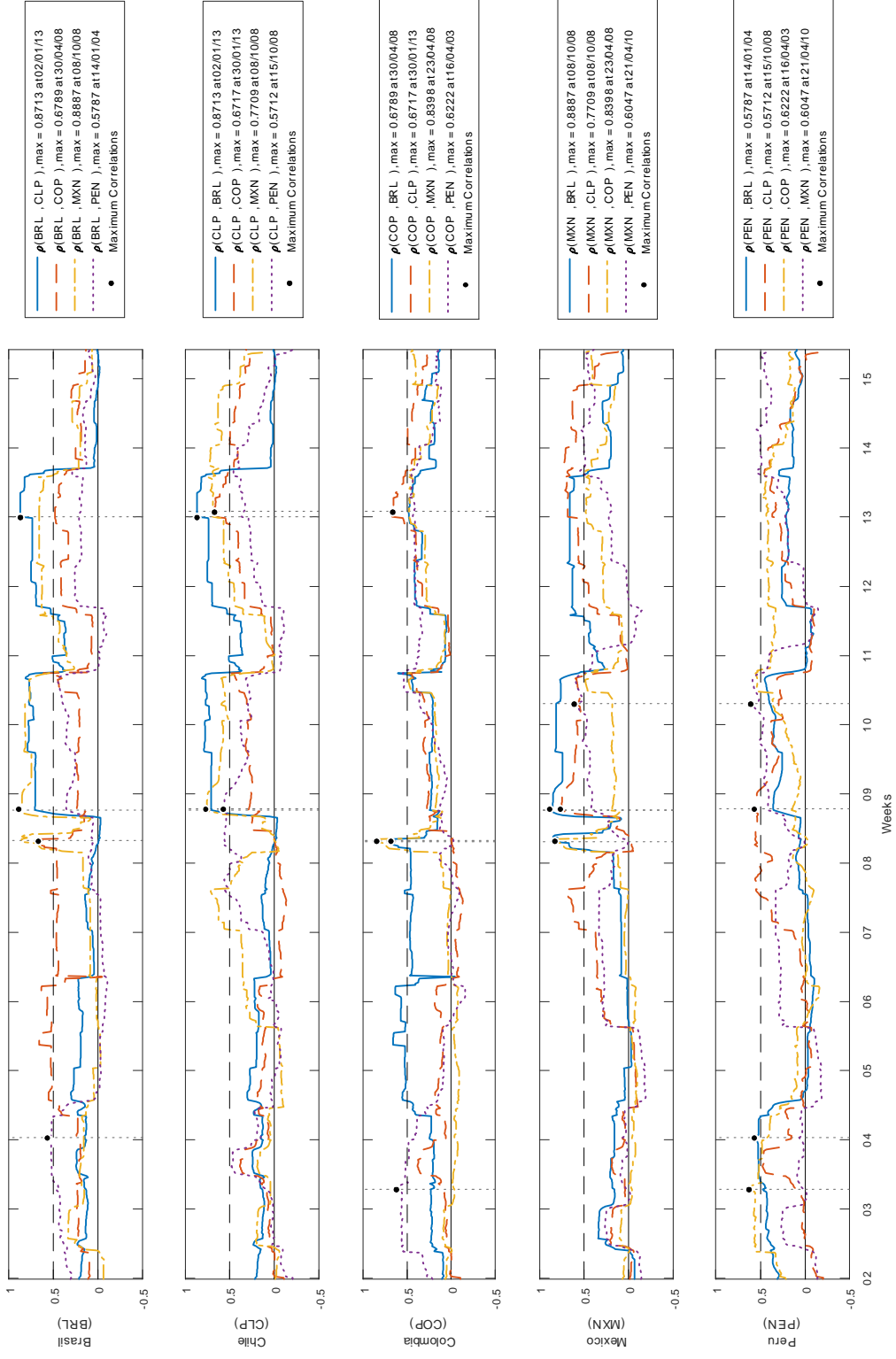
Figure 18(a) Forex Rate Correlations between Smoothed Probabilities of positive returns - high volatility using a window of 52 weeks
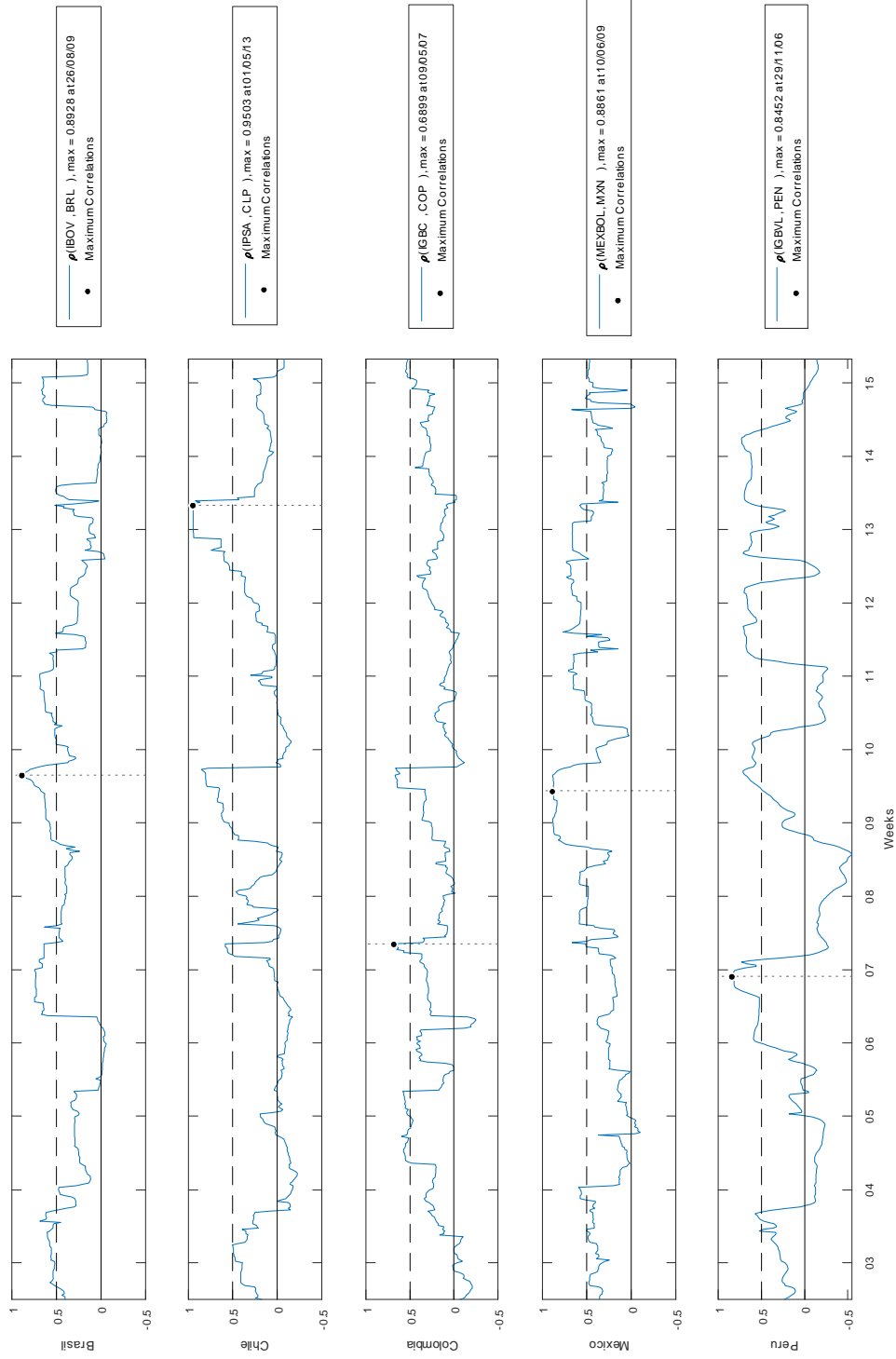
Figure 18(b) Forex Rate Correlations between Smoothed Probabilities of positive returns - high volatility using a window of 78 weeks
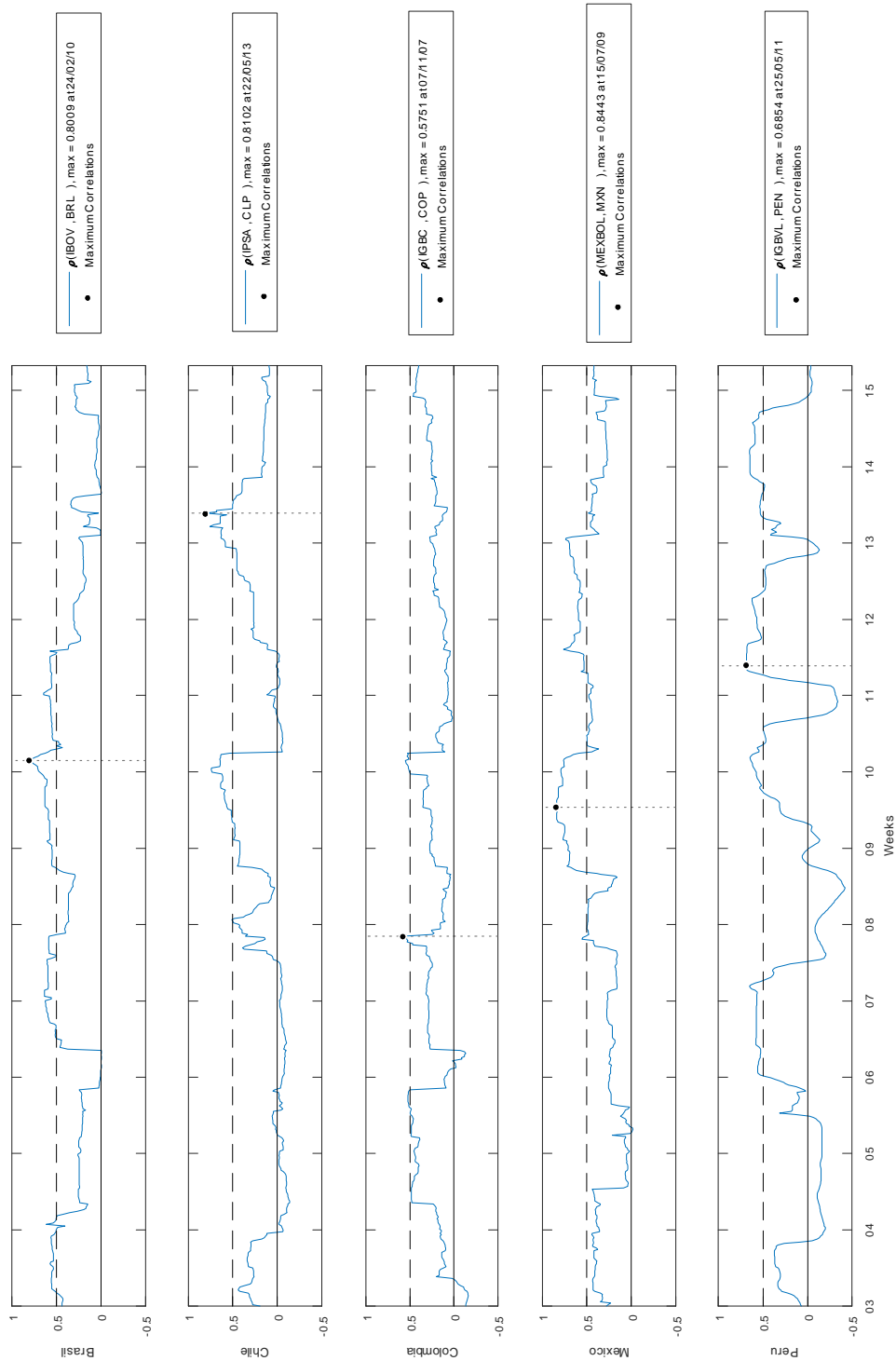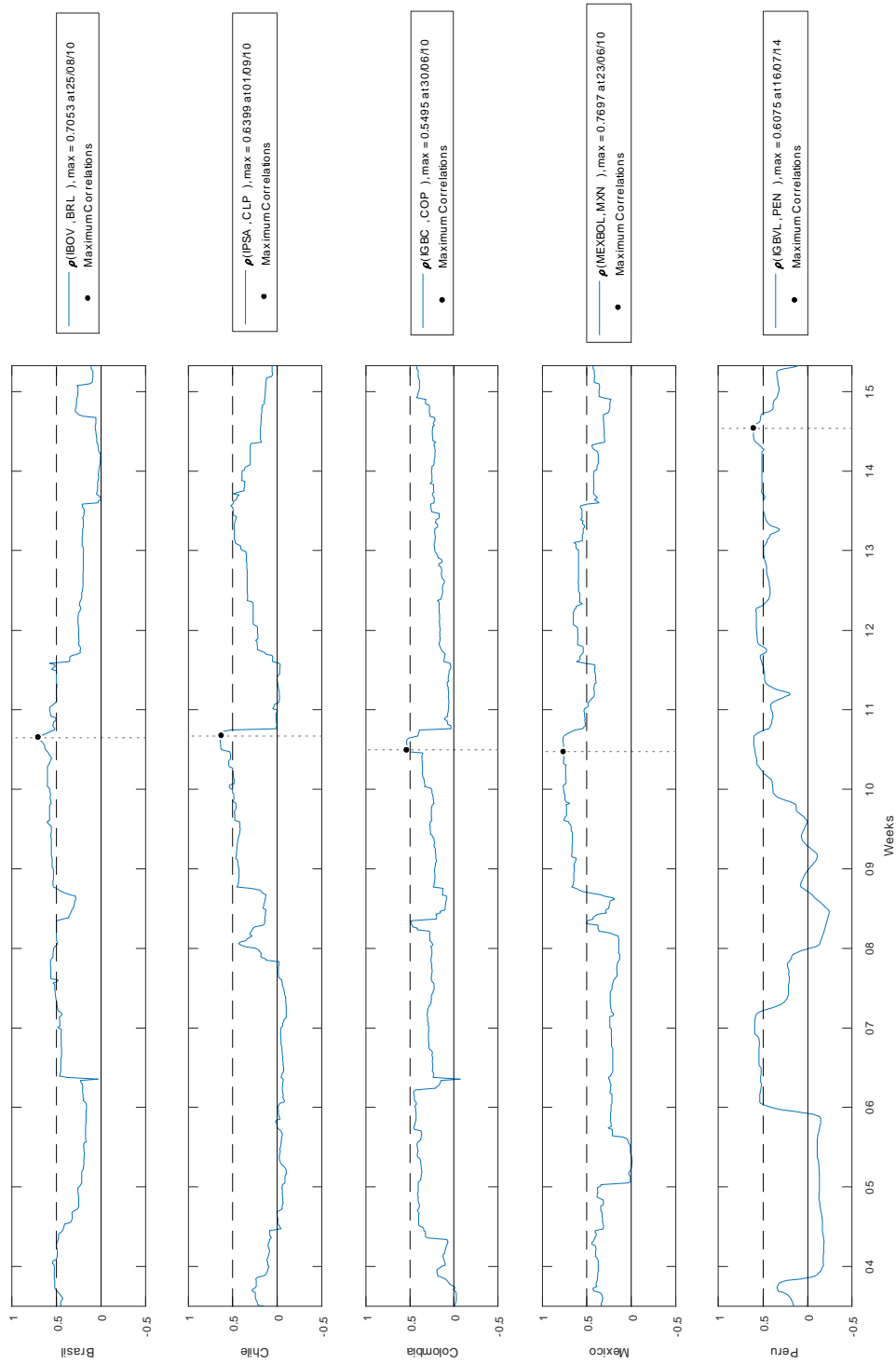
Figure 18(c) Forex Rate Correlations between Smoothed Probabilities of positive returns - high volatility using a window of 104 weeks

Figure 19(a) Correlations between Smoothed Probabilities of Forex Rate positive returns - high volatility and Stock Market Returns negative returns - high volatility for each Latin American country using a window of 52 weeks

Figure 19(b) Correlations between Smoothed Probabilities of Forex Rate positive returns - high volatility and Stock Market Returns negative returns - high volatility for each Latin American country using a window of 78 weeks

Figure 19(c) Correlations between Smoothed Probabilities of Forex Rate positive returns - high volatility and Stock Market Returns negative returns - high volatility for each Latin American country using a window of 104 weeks

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Felix Jiménez
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