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MODELING THE VOLATILITY OF RETURNS ON COMMODITIES: AN APPLICATION AND EMPIRICAL COMPARISON OF GARCH AND SV MODELS

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Abstract

Seven GARCH and stochastic volatility (SV) models are used to model and compare empirically the volatility of returns on four commodities: gold, copper, oil, and natural gas. The results show evidence of fat tails and random jumps created by supply/demand imbalances, international instability episodes, geopolitical tensions, and market speculation, among other factors. We also find evidence of a leverage effect in oil and copper, resulting from their dependence on world economic activity; and of an inverse leverage effect in gold and natural gas, consistent with the former's role as safe asset and with uncertainty about the latter's future supply. Additionally, in most cases there is no evidence of an impact of volatility on the mean. Finally, we find that the best-performing return volatility models are GARCH-t for gold, SV-t for copper and oil, and SV with leverage effects (SV-L) for natural gas.

JEL Codes: C11, C52, G15.

Keywords: Returns, Volatility, GARCH, Stochastic Volatility, Commodities, Bayesian Estimation, Fat Tails, Jumps, Leverage.

Resumen

Se utilizan siete modelos GARCH y de volatilidad estocástica (SV) para modelar y comparar empíricamente la volatilidad de los rendimientos de cuatro productos: oro, cobre, petróleo y gas natural. Los resultados muestran evidencia de colas anchas y saltos aleatorios creados por desequilibrios de oferta/demanda, episodios de inestabilidad internacional, tensiones geopolíticas y especulación de mercado, entre otros factores. También se encuentra evidencia de un efecto de apalancamiento en el petróleo y el cobre, como resultado de su dependencia de la actividad económica mundial; y de un efecto de apalancamiento inverso en oro y gas natural, consistente con el papel del primero como activo seguro y con incertidumbre sobre el suministro futuro de este último. Además, en la mayoría de los casos no hay evidencia de un impacto de la volatilidad en la media. Finalmente, se encuentra que los modelos de volatilidad de retornos de mejor desempeño son GARCH-t para oro, SV-t para cobre y petróleo, y SV con efectos de apalancamiento (SV-L) para gas natural.

Clasificación JEL: C11, C52, G15.

Palabras Claves: Returns, Volatility, GARCH, Stochastic Volatility, Commodities, Bayesian Estimation, Fat Tails, Jumps, Leverage.

Modeling the Volatility of Returns on Commodities: An Application and Empirical Comparison of GARCH and SV Models[1]

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1 Introduction

Like with other financial series, over the last three decades commodity spot markets have experienced varying levels of volatility, with negative real-sector implications in trade-dependent economies. As of 2015, Peru and Chile's copper exports amounted to 19% and 23% of the value of total export value, respectively; and Colombia and Ecuador's oil exports reached 34% of total export value. Additionally, gold amounted to 16% of Peru's total export value. Moreover, refined oil and oil derivatives represent around 10% of Latin America's total import value. In Peru, commodities represented 66% of total exports in 2016. Gold amounted to 43%, 24%, and 20% of international reserves in Venezuela, Colombia, and Bolivia, respectively, in 2000-2017 according to the World Gold Council (2017). Global shocks in the form of fluctuations in the price of metals, energy, and agriculture represent around one-third of variability in the economic cycle; see Fernández et al. (2016) and Jacks et al. (2011).

In the financial sector, commodities play a role as investment instruments; i.e., assets yielding higher returns than stock or bonds. They also serve as hedging instruments against market risks under increased uncertainty.[4] Jafee (1989) argues that inclusion of gold and/or gold shares in investment portfolios reduces uncertainty and increases expected yields. Additionally, commodity funds' higher yields and inverse correlation with returns on shares provide greater coverage during market volatility episodes; see Edwards and Caglayan (2001). This is confirmed by Gorton and Rouwenhorst (2006) and Erb and Harvey (2006), who suggest that asset portfolios including commodities perform better under unexpected inflation surges. For example, the Argentina, Brazil, and Peru stock exchanges respond contemporaneously to changes in the returns on energy, metals, and agriculture; and the Chile and Peru stock exchanges are the most affected by changes in the prices

[1] This paper is drawn from Jean Pierre Fernández Prada Saucedo's thesis to obtain a Master's degree in economics from the Pontificia Universidad Católica del Perú (PUCP). The authors thank the useful comments by Paul Castillo (Central Reserve Bank of Peru-BCRP and PUCP), David Florián (BCRP), and participants in the 36th BCRP Meeting of Economists (Lima, October 30-31, 2018) and the 23rd Latin American Meeting of the Econometric Society (LAMES, Guayaquil, Ecuador, November 8-10, 2018). The views expressed in this paper are those of the authors and do not reflect necessarily the position of the Fiscal Council of Peru. Any remaining errors are our responsibility.

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[4] Information from the Bank for International Settlements (BIS, 2018) suggests that in 1998-2017 the notional value and the gross value of over-the-counter (OTC) market of commodity derivatives reached peaks of USD 221 and USD 1,322 billion, respectively.

of industrial and precious metals; see Jhonson and Soenen (2009). For the impact of oil prices on the U.S. economy, see Federer (1996) and Guo and Kliesen (2005). Moreover, negative commodity price shocks exacerbate financial vulnerability and increase the likelihood of bank crises; see Kinda et al. (2016).

Volatility changes over time have been conventionally modeled using GARCH models, where the conditional variance is a deterministic function of the parameters of the model and of lagged data. For example, Akgiray et al. (1991) estimate a GARCH model with generalized errors applied to spot price series for gold and silver; and find that the estimated coefficients change in the four sub-periods under study due to political tensions. Bracker and Smith (1999) compare the ability of three asymmetric GARCH models and a standard GARCH model to explain the volatility of returns on copper futures; and finds that the standard GARCH model and the EGARCH model proposed by Nelson (1991) provide a better adjustment. Narayan and Narayan's (2007) estimation of asymmetric GARCH models applied to returns on oil in 1991-2006 point to the existence of regime changes. Additionally, Ewing and Malik (2010) include structural changes identified by Inclán and Tiao's (1994) algorithm in a group of GARCH models; and find that persistence decreases significantly. Using a Markov-Switching GARCH model, Fong and See (2002) find that exclusion of regime changes produces "long memory" volatility in returns on oil. Using a similar approach, Choi and Hammoudeh (2010) find that the duration of high- and low-volatility regimes differs between series for gold, copper, oil, and the S&P 500. Moreover, using a semi-parametric statistical approach, Charles and Darné (2014) find that outliers in crude oil markets may create a bias in the parameters of the volatility equation in GARCH models, the identification of structural changes, and the estimation of volatility persistence. Nomikos and Andriasopoulos (2012) study volatility in futures contracts for eight energy markets using an EGARCH model with a modified equation for returns. The results show that shocks on oil and gas returns take 998 and 155 days to revert, respectively, while the component for return jumps takes longer to dissipate for gas than for oil (72 vs. 36 days, respectively). They also highlight that all commodities, except oil, show an inverse leverage effect; i.e., positive returns today announce an increase in volatility tomorrow. The presence of jumps in two natural gas series (UK and U.S.) is examined by Mason and Wilmot (2014), who find that jumps are more significant in the UK than in the U.S., reflecting structural dissimilarities between both markets. Hammoudeh and Yuan (2008) study the volatility of gold, copper, and silver and their response to oil and interest rate shocks; see also Watkins and McAleer (2008). The results indicate that copper shows leverage effects, while the contrary holds for gold and silver, which turns the latter two into safe assets in uncertainty scenarios.[5]

Alternatively, other research works consider stochastic volatility (SV) models, where volatility is a latent variable governed by a stochastic process. For example, Vo (2009) proposes a model that considers regime changes to explain the dynamics of oil prices. The results indicate that this inclusion prevents overestimation of the parameters for volatility persistence. Larsson and Nossman (2011) model returns on WTI oil in 1989-2009 using an SV model with correlated jumps (SV-CJ) introduced by Duffie et al. (2000). The results indicate that extreme changes in oil returns during the 1990s were governed by price jumps, while in the 21st century they were dominated by volatility jumps. Additionally, the authors argue that models that do not include SV or jumps do not provide a good representation of oil volatility during stress periods like the Gulf War and the 2009 crisis. Du et al. (2011) use an SV-J model to find that inventories and speculation are relevant to explain

[5]For further references see Fong and See (2001), Ramírez and Fadiga (2003), and Lucey and Tully (2006).

oil volatility. Brooks and Prokopczuk (2013) study volatility in three commodity market segments (metals, energy, and agriculture) and the S&P 500 index using three SV models (SV, SV-J, and SV-CJ). The authors use the DIC[6] to find that SV-J models provide a better adjustment; and confirm that the correlation between commodity and S&P 500 volatilities is low, suggesting that commodities can be instrumental in diversifying risk. Additionally, Liu et al. (2014) study the volatility of returns on copper and aluminum spot and futures markets using the SV-J and SV-CJ models; and find that models that include jumps provide a better measure of risk than the standard SV model.[7]

GARCH and SV are two kinds of non-nested models that have been compared for their forecasting performance. However, literature comparing the goodness of fit and inference capabilities of both model families is scarce. Taylor (1994), Ghysels et al. (1996), Andersson (2001), Carrasco and Chen (2002), and Bai et al. (2003) use a theoretical approach to compare both families based on the similarity of kurtosis and first-order autocorrelation of the squared returns generated by the models relative to the properties of the original series. Other studies like Garcia and Renault (1998), Lehar et al. (2002), Fleming and Kirby (2003), and Pederzolli (2011) compare both families under a value-at-risk approach, while works like Danielson (1994), Kim et al. (1998), Gerlach and Tuyl (2006), and Nakajima (2009) compare both families using a Bayesian approach. In this line, Chan and Grant (2016a) recently compared seven pairs of GARCH and SV models applied to weekly spot prices for ten energy products using marginal likelihoods. They find that: (i) in general SV models perform better than their GARCH counterparts; (ii) inclusion of jumps or fat tails substantially improve the performance of GARCH models, but are less beneficial to SV models; (iii) the risk-return relationship is not significant for most estimations; (iv) the leverage effect is important for modeling the WTI series; and (v) the SV-MA model provides a better adjustment for modeling volatility in energy series.

This paper uses Chan and Grant's (2016a) approach to compare seven pairs of GARCH and SV models applied to daily return series for four important commodities in South American trade: gold, copper, oil, and natural gas. The authors expect to fill a gap in studies on volatility applied to commodity markets that provide a formal comparison of the goodness of fit of both model families. The main results are that considering fat tails in any model family is substantially beneficial in terms of goodness of fit, although lower for the SV family. The best model for gold is GARCH-t. The model selected for copper and oil is SV-t, while SV-L (SV with lagged effects) is preferred for natural gas. The use of fat tails or random jumps to model the volatility of returns on commodities is due to market sensitivity to macroeconomic instability, geopolitical tensions, market speculation, and supply/demand mismatches. Additionally, the paper finds evidence of leverage effects in copper and oil, as both depend on the global economic cycle. In the case of gold and natural gas, the paper finds inverse leverage effects, where future volatility is expected to be higher in response to current positive returns. In the case of gold, this effect is due to its role as safe asset in stress episodes, while in the case of natural gas demand pressures increase uncertainty about future supply. Finally, there is no evidence of an impact on the mean volatility in most goods.

The rest of the paper is organized as follows. Section 2 describes the seven pairs of GARCH

[6] The Deviance Information Criterion (DIC) is a Bayesian criterion proposed by Spiegelhalter et al. (2002) and used to compare SV models using the conditional data likelihoods. This criterion is used and discussed by Chan and Grant (2016b).

[7] Schmitz et al. (2014) arrive at the same conclusion for the soy and wheat markets (and for agricultural markets in general).

and SV models, as well as the Bayesian estimation and comparison methods. Section 3 presents the data and estimation results for the four commodities; and a justification is provided for the selection of the models based on historical events and the nature of each good. Section 4 presents the conclusions.

2 Methodology

This section briefly presents the two classes of models with time-changing volatilities used in the empirical section. The first class are GARCH models developed by Bollerslev (1986) as an extension of the seminal work by Engle (1982). The second group are SV models originally developed by Taylor (1986, 1994).

2.1 GARCH Models

Following the notation used by Chan and Grant (2016a), Bollerslev's GARCH (1,1) model is defined as:

$$\begin{aligned} y_t &= \mu + \epsilon_t, \\ \sigma_t^2 &= \alpha_o + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \\ \epsilon_t &\sim N(0, \sigma_t^2), \end{aligned} \tag{1}$$

where $\epsilon_0 = 0$, $\sigma_0^2 = var(y_t)$, $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$. This model is named GARCH-1. A GARCH (2,1) model is considered next:

$$\sigma_t^2 = \alpha_o + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2, \tag{2}$$

where $\sigma_{-1}^2 = \epsilon_0 = 0$, $\sigma_0^2 = var(y_t)$ is a constant and the same restrictions are applied to the coefficients of σ_t^2 to ensure non-negativity and stationarity. This model is named GARCH-2. The third model preserves the same form as (1), but with t-student innovations; i.e., $\epsilon_t \sim t_\nu(0, \sigma_t^2)$. The selection of this distribution can capture extreme events that may be omitted by the GARCH-1 and GARCH-2 models. This model is named GARCH-t. The fourth model, named GARCH-J, modifies (1) by including a random jump component that allows adjustment to infrequent data changes:

$$y_t = \mu + k_t q_t + \epsilon_t, \tag{3}$$

where q_t is a jump that follows a Bernoulli distribution with a success probability of $prob(q_t = 1) = \kappa$. When $q_t = 1$ a jump takes place in period t with a magnitude determined by $k_t \sim N(\mu_k, \sigma_k^2)$. The fifth model includes σ_t^2 in the equation for the conditional mean; i.e., the returns are dependent on volatility. This model is named GARCH-M:

$$y_t = \mu + \lambda \sigma_t^2 + \epsilon_t, \tag{4}$$

where the parameter λ may be understood as the risk premium. The sixth is a GARCH model that includes a dynamic element in the errors of process y_t via a first-order moving average component (MA(1)); i.e., (1) is:

$$\begin{aligned} y_t &= \mu + \epsilon_t, \\ \epsilon_t &= u_t + \psi u_{t-1}, \end{aligned} \tag{5}$$

where the invertibility of the MA(1) component is ensured by assuming $|\psi| < 1$ and $u_t \sim N(0, \sigma_t^2)$. This model is named GARCH-MA. The last specification uses the asymmetric GARCH structure proposed by Glosten et al. (1993); i.e., introduces an additional impact from the excess negative returns on the variance in (1):

$$\sigma_t^2 = \alpha_o + [\alpha_1 + \delta \mathbf{1}(\epsilon_{t-1} < 0)]\, \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{6}$$

where $\mathbf{1}(.)$ is an indicative function that is triggered when $\epsilon_{t-1} < 0$. In this scenario, the asymmetric effect is represented by $\delta > 0$[8], which measures the additional impact from a lagged negative shock on current volatility. This model is named GARCH-L.

2.2 SV Models

Following Chan and Grant's (2016a) notation, the model named SV-1 is the canonical SV model:

$$\begin{aligned} y_t &= \mu + \epsilon_t^y, \\ h_t &= \mu_h + \phi_{h_1}(h_{t-1} - \mu_h) + \epsilon_t^h, \\ \epsilon_t^y &\sim N(0, e^{h_t}), \\ \epsilon_t^h &\sim N(0, \omega_h^2), \end{aligned} \tag{7}$$

where h_t is the log-volatility following an $AR(1)$ stationary process, as $|\phi_{h_1}| < 1$, disturbances ϵ_t^h and $\epsilon_t^y$are uncorrelated, and the process is initialized with $h_1 \sim N(\mu_h, \frac{\omega_h^2}{(1-\phi_{h_1}^2)})$. The second model, named SV-2, has the same equation for the mean as (7), but includes an additional lag in the log-volatility equation:

$$h_t = \mu_h + \phi_{h_1}(h_{t-1} - \mu_h) + \phi_{h_2}(h_{t-2} - \mu_h) + \epsilon_t^h,$$

where ϕ_{h_1}, ϕ_{h_2} are assumed to lie inside the unit circle and h_1 and h_2 are initialized with: $h_1, h_2 \sim N(\mu_h, \frac{(1-\phi_{h_2})\omega_h^2}{(1+\phi_{h_2})((1-\phi_{h_2})^2-\phi_{h_1}^2)})$. The model named SV-t is similar to SV-1 but with:

$$\epsilon_t^y \sim t_\nu(0, e^{h_t}). \tag{8}$$

The fourth SV model admits the existence of random jumps and is named SV-J. The equation for the mean in (7) is:

$$y_t = \mu + k_t q_t + \epsilon_t^y, \tag{9}$$

where the jump indicator q_t and the jump size k_t have the same characteristics of a GARCH-J model. The fifth model, Koopman and Uspensky's (2002) SV-M model, where the stochastic volatility is included in the equation for the mean, is:

$$y_t = \mu + \lambda e^{h_t} + \epsilon_t^y. \tag{10}$$

[8] A value of $\delta < 0$ indicates that in response to a positive return scenario today, volatility tomorrow is higher. This is evidence of an inverse leverage effect.

An SV model is also specified to include the dynamics of innovations in y_t, represented by an MA(1) process named SV-MA:

$$\begin{aligned} y_t &= \mu + \epsilon_t^y, \\ \epsilon_t^y &= u_t + \psi u_{t-1}, \\ u_t &\sim N(0, e^{h_t}), \end{aligned} \tag{11}$$

where $u_0 = 0$, $|\psi| < 1$. Finally, this paper uses an SV model with leverage effect, named SV-L. In this specification, the mean and log-volatility equations are the same as in (7), but disturbances ϵ_t^h and ϵ_t^y are allowed to be correlated:

$$\begin{pmatrix} \epsilon_t^y \\ \epsilon_t^h \end{pmatrix} \sim N\left[0, \begin{pmatrix} e^{h_t} & \rho e^{\frac{1}{2}h_t}\omega_h \\ \rho e^{\frac{1}{2}h_t}\omega_h & \omega_h^2 \end{pmatrix}\right], \tag{12}$$

where ρ is the parameter indicating a correlation between the shocks. When $\rho < 0$, there is a negative correlation in the returns and their volatility.[9]

2.3 Bayesian Estimation and Comparison Models

Briefly, the Bayesian approach follows Bayes' theorem: $\pi(\theta|y) \propto f(y|\theta)\pi(\theta)$, where $\pi(\theta|y)$ is the posterior distribution of a set of parameters represented by θ, conditional on the data; $f(y|\theta)$ is the likelihood function; and $\pi(\theta)$ is the prior or ex-ante criterion for the behavior of θ. Along these lines, Markov Chain Monte Carlo (MCMC) methods are used to sample the posterior distributions of interest, $\pi(\theta|y)$. Some details of the model estimation and comparison process are discussed next.

2.3.1 The Priors

The same priors as Chan and Grant (2016a) are used to perform inferences. The selected priors share three characteristics: (i) they are the same for the GARCH and SV models; (ii) they are functions whose densities are independent and can be integrated into the unit; and (iii) they are relatively non-informative, and therefore play an unimportant role in using the data to derive a posterior distribution function for the parameters.

First, the assumptions for the GARCH-1 model are $\mu \sim N(\mu_0, V_u)$ and $\log \boldsymbol{\gamma} \sim N(\boldsymbol{\gamma}_0, V_\gamma)\mathbf{1}(\alpha_1 + \beta_1 < 1)$, where $\boldsymbol{\gamma} = (\alpha_0, \alpha_1, \beta_1)'$ follows a truncated log-normal distribution that admits certain parameter space to ensure stationarity. The hyper-parameters have the following values: $\mu_0 = 0$, $V_u = 10$, $\boldsymbol{\gamma}_0 = (1, \log 0.1, \log 0.8)'$ and $V_\gamma = diag(10, 1, 1)$. In GARCH-2, $\widetilde{\boldsymbol{\gamma}} = (\alpha_0, \alpha_1, \beta_1, \beta_2)$, and therefore $\log \widetilde{\boldsymbol{\gamma}} \sim N(\widetilde{\boldsymbol{\gamma}}_0, V_{\widetilde{\gamma}})\mathbf{1}(\alpha_1 + \beta_1 + \beta_2 < 1)$, $\widetilde{\boldsymbol{\gamma}}_0 = (1, \log 0.1, \log 0.8, \log 0.1)$ and $V_{\widetilde{\boldsymbol{\gamma}}} = diag(10, 1, 1, 1)$. In the remaining GARCH models, the priors for μ and $\boldsymbol{\gamma}$ are the same as in GARCH-1. In GARCH-J, the intensity of jumps is assumed to follow a uniform distribution, whereas the average jump and the jump variance, $\boldsymbol{\delta} = (\mu_k, \log \sigma_k^2)'$, behave like a normal bivariate distribution: $\kappa \sim U(0, 0.1)$ and $\boldsymbol{\delta} \sim N(\boldsymbol{\delta}_0, V_{\boldsymbol{\delta}})$. The hyper-parameters are $\boldsymbol{\delta}_0 = (0, \log 10)'$ and $V_{\boldsymbol{\delta}} = diag(10, 1)$. In the case of GARCH-M, $\lambda \sim N(\lambda_0, V_\lambda)$, where $\lambda_0 = 0$ and $V_\lambda = 100$. In the

[9] A value of $\rho > 0$ indicates a positive correlation between the returns and their volatility and, therefore, provides evidence of an inverse leverage effect.

case of GARCH-MA, $\psi \sim N(\psi_0, V_\psi)\mathbf{1}(|\psi| < 1)$, where $\psi_0 = 0$ and $V_\psi = 1$. In GARCH-t, $\nu > 2$ and $\nu \sim U(2, 100)$. Finally, the distribution of the δ parameter in GARCH-L is assumed to be $(\delta|\boldsymbol{\gamma}) \sim U(-\alpha_1, 1 - \alpha_1 - \beta_1)$.

Regarding the SV models, the priors for parameters μ, μ_h, ϕ_{h_1} and ω_h^2 in SV-1 are $\mu \sim N(\mu_0, V_\mu)$, $\mu_h \sim N(\mu_{h_0}, V_{\mu_h})$, $\phi_{h_1} \sim N(\phi_{h_0}, V_{\phi_{h_1}})\mathbf{1}(|\phi_{h_1}| < 1)$ and $\omega_h^2 \sim IG(\nu_h, S_h)$, respectively. The hyper-parameters are $\mu_0 = 0, \mu_{h_0} = 1, V_\mu = V_{\mu_h} = 10, \phi_{h_0} = 0.97, V_{\phi_{h_1}} = 0.1^2, \nu_h = 5$ and $S_h = 0.16$. The same priors are maintained for SV-2, and $\boldsymbol{\theta}_h = (\phi_{h_1}, \phi_{h_2})'$, such that: $\boldsymbol{\theta}_h \sim N(\boldsymbol{\theta}_{h_0}, V_{\boldsymbol{\theta}_h})\mathbf{1}(\boldsymbol{\theta}_h \in A)$, where A is a space where stationarity is ensured. The hyper-parameters are $\boldsymbol{\theta}_{h_0} = (0.97, 0)'$, $V_{\boldsymbol{\theta}_h} = diag(0.1^2, 1)$. Concerning the remaining models: (i) the same priors are assumed as in the SV-1 model for parameters μ, μ_h, $\phi_{h_1}$and ω_h^2; (ii) the priors for the additional parameters are the same as the prior for their GARCH counterparts, and (iii) the ρ parameter is distributed as $U(-1, 1)$.

2.3.2 The Algorithm

Both kinds of models are estimated using the MCMC methods proposed by Chan and Grant (2016a, b). The purpose is taking samples from the posterior distributions of the models by performing Markov-type sampling and using posterior draws to calculate several magnitudes of interest such as the posterior means and marginal likelihoods.[10]

Metropolis-Hastings algorithms are used in the GARCH models to sample from the posterior distributions. The parameters of the models are grouped to perform the estimations in different blocks. In this way, it is possible to begin sampling from the complete conditional densities of the parameters. It is important to mention that, in contrast with Chan and Grant (2016a), who use a normal distribution to sample the parameters of the volatility equation, this paper uses a *Beta* distribution. The reason for this change is that the MCMC chains of these parameters in most GARCH models present a "sticky chain problem." Following Rosenthal (2011) and Junker et al. (2016), this paper sets out to solve it by raising the level of acceptance of the parameters, thereby avoiding stagnation of the chain by diminishing the variance of the generating density. If the problem persists, it is possible to change the generating density of the algorithm. In this case, while diminishing the variance level increases the level of the acceptance ratio, the chains continue to show protracted stagnation during several simulation periods. In response, a *Beta* distribution is substituted for the density of the Metropolis-Hastings algorithm, thereby obtaining well-behaved chains with higher acceptance ratios.

In the SV models it is necessary to simulate the conditional density of the vector of non-observable log-volatilities, $p(\mathbf{h}|\mathbf{y}, \Phi_i)$, where Φ_i represents the parameter vector for each model $i = 1, \ldots, n$. For this purpose, this paper uses the adaptive Metropolis-Hastings algorithm proposed by Chan (2015) and based on Chan and Jeliazkov (2009).[11] Carrying out the simulation of marginal likelihoods requires a generating (or importance) density, from which the target density draws, $p(y|M_i)$, are done, where M_i represents the i model; see Koop (2003). Once all draws are done, a weighted average is calculated, where the weights are assigned according to the contribution of each draw to the decrease in the bias of the proposed density relative to the target density, so as to obtain an unbiased and consistent estimator for the latter. The importance sampling estimator

[10] Complete details about the algorithm may be found in the appendix of Chan and Grant (2016a).

[11] For details about the sampling for SV-MA and SV-t, see Chan (2013) and Chan y Hsiao (2014), respectively.

for $p(y|M_i)$ is defined as $\hat{p}_{IS} = \frac{1}{N}\sum_{n=1}^{N}\frac{p(\mathbf{y}|\boldsymbol{\theta}_n)p(\boldsymbol{\theta}_n)}{g(\boldsymbol{\theta}_n)}$, where $\boldsymbol{\theta}_1 \ldots \boldsymbol{\theta}_N$ are independent draws obtained from the importance density. Along these lines, the selection of this density is based on Chan and Eisenstat (2015), who propose approximating $g(\boldsymbol{\theta})$ through the cross-entropy method, which is instrumental in deriving an optimal density that minimizes the Kullback-Leibler divergence relative to the ideal density.[12] Additionally, the ideal density should be in the same parametric class as the densities for the priors.

2.3.3 The Bayes Factor

Once the marginal likelihoods are obtained, a formal method for comparing the models is the Bayes factor (BF). The goal is comparing a group of models $\{M_1, \ldots, M_i\}$, where $i = 1, \ldots, n$ and each M_k model is made up of two separate components: a likelihood function $p(y|\boldsymbol{\theta}_k, M_k)$ and a prior density $p(\boldsymbol{\theta}_k|M_k)$. The BF provides a criterion for comparing models M_i and M_j, defined as $BF_{ij} = \frac{p(\mathbf{y}|M_i)}{p(\mathbf{y}|M_j)}$, where $p(\mathbf{y}|M_k)$ is the marginal likelihood of model M_k $(k = i, j)$. A possible interpretation is that the marginal likelihood is a predictive density under M_k across the observed data $\mathbf{y}$. Therefore, $BF_{ij} > 1$ will indicate that the observed data are more likely to be obtained using M_i rather than M_j, which can be considered as evidence in favor of M_i. The strength of this evidence is proportional to the value obtained by the BF. The latter is associated with the posterior odds ratio between two models, defined as $\frac{p(M_i|\mathbf{y})}{p(M_j|\mathbf{y})} = \frac{p(M_i)}{p(M_j)} \times BF_{ij}$, where the prior odds ratio is assumed to be equal to one, given that both models are *a priori* equally likely. This implies that the posterior odds ratio between two models is equal to BF_{ij}[13]. Therefore, the BF is essentially a ratio of two marginal likelihoods, which is why it often yields only the values of such marginal values of the models under comparison. This is the method followed by the authors.

3 Empirical Analysis

3.1 The Data

This paper uses four S&P GSCI spot price index time series. The returns are expressed as $y_t = 100 \times (\log P_t - \log P_{t-1})$, where P_t is the closing price in period t. The commodities analyzed are gold and copper (metal items), as well as oil and natural gas (energy items). The daily data are drawn from Bloomberg: January-1983-April 2017 for gold and copper; January 1987-April 2017 for oil; and January 1994-April 2017 for natural gas.

A disaggregated analysis is warranted by the fact that the dynamics of the volatility of returns is governed by peculiar characteristics in each market, and therefore shows a different behavior over time; see Erb and Harvey (2006) and Batten et al. (2010). Figure 1 shows the four return series, which show both turbulence and stability episodes. This behavior is similar to that of financial return series, which show clusters originated by episodes of macroeconomic, financial, or geopolitical stress.

Table 1 shows a summary of statistics for the returns and square returns. Panel A shows that: (i) the average return of the four series is close to zero; (ii) all series except natural gas show

[12] For further details, see appendix B in Chan and Grant (2016a) and appendices A and B in Chan and Grant (2016b).

[13] For a further discussion about the BF, see Koop (2003).

negative returns on average; (iii) the series show high kurtosis (above six on average); and (iv) gold and gas show the narrowest and widest distance between the maximum and minimum returns, respectively (which is consistent with the standard deviation of each series). Panel B shows that the empirical distribution of the square returns on oil have greater symmetry and kurtosis than the other goods, mainly due to the extreme event created by the Gulf War.[14]

3.2 Estimation Results

One hundred thousand simulations were performed for each parameter with a burn-in of 50 thousand to ensure convergence.[15] It is important to note that there is a group of common parameters (i.e., $\alpha_0, \alpha_1, \beta_1, \mu, \mu_h, \phi_{h_1}$, ϕ_{h_2}, and ω_h^2) belonging to the seven GARCH and SV models for each good; and that none of them show a zero value in their credibility intervals. The following sections describe the estimation results for the remaining models taking GARCH-1 and SV-1 as reference.[16]

The results are presented for each commodity, where the even- (2-8) and odd-numbered (3-9) Tables show the results for the GARCH and SV families, respectively.[17]

In Figure 2 the estimated implicit volatilities for the best model in each family (for each commodity) are presented and compared with the square returns. The models capture accurately the dates of the events that caused volatility surges; and the volatilities estimated for both families are highly correlated during their periods of analysis.

3.2.1 Gold

Table 2 shows the results for the GARCH family. The estimation of GARCH-t yields $\nu = 3.95$, suggesting the occurrence of extreme events affecting gold returns (Figure 1). Additionally, in GARCH-J, κ indicates that the jump probability is 0.10; i.e., 25 jumps per year on average,[18] with an average magnitude of $\mu_k = -0.144$, implying that the series experienced more negative jumps over time. Regarding GARCH-M, no evidence is found that market participants demand a risk premium to invest in gold. Similarly, GARCH-MA shows no evidence of first-order serial correlation in the returns; i.e., the persistence of unexpected shocks in the process governing the returns is no greater than one day. The results are consistent with Tully and Lucey (2007), who also find no evidence of ARCH-M and/or first-order serial correlation effects in gold returns. The GARCH-L model shows evidence of an inverse leverage effect ($\delta < 0$) implying that positive returns today generate higher volatility tomorrow. Lucey and Tully (2006) and Hammoudeh and Yuan (2008) find the same findings and conclude that gold can play a role as safe asset in the face of adverse events. Finally, based on the marginal log-likelihoods, GARCH-t provides the best fit, followed by GARCH-J and GARCH-2.

[14] If the Gulf War is excluded from the sample, the descriptive statistics (mainly kurtosis) diminish by two-thirds of their current value.

[15] Trace plots, histograms and autocorrelation figures were generated for each parameters in all GARCH and SV models to confirm the existence of convergence. All figures are available upon request.

[16] Information from the U.S. Geological Survey annual reports was used to identify the historic events that may have affected gold and copper volatility directly. See Earle and Amey (2018) and Edelstein (2018).

[17] The Tables include the Ljung-Box Q and McLeod-Li Q_2 statistics applied to the standardized residuals and their squares, respectively. In both cases the null hypothesis is the absence of autocorrelation.

[18] The average number of jumps is denoted as $j = \frac{\kappa \times n}{N}$, where κ is the jump probability, n is the number of observations, and N is the number of years in the sample.

Table 3 shows the results for the SV models. The SV-t model shows that $\nu = 12.60$, suggesting evidence of extreme events, although to a lesser extent than the GARCH-t model. Regarding SV-J, $\kappa = 0.01$, i.e., there are 2.5 jumps per year with an average magnitude of $\mu_k = -0.35$, implying that returns experienced a greater number of falls over time. In this regard, Brook and Prokopzuk (2013) find a jump intensity of 0.0532; i.e., 13 jumps per year, while for an SV-CJ model they find a jump intensity of 0.0172; i.e., 2.52 jumps per year, similar to the finding in this paper. Additionally, SV-M and SV-MA do not show evidence of a risk premium or serial correlation in the returns. SV-L yields $\rho > 0$, implying a positive relationship between shocks on the returns process and on volatility; i.e., there is evidence of an inverse leverage effect as in GARCH-L. In this respect, Brooks and Prokopzuk (2013) also find that $\rho > 0$ and suggest that this kind of asymmetry, which is the opposite of what is normally found in stock markets, is created by changes in the share of market participants that perform hedging and speculative operations. They argue that when the returns are positive, the number of speculators increases more than the number of hedgers, and therefore future volatility will be higher when today's returns are positive. Thus, positive returns on gold may indicate the beginning of a new stress episode. Finally, SV-t provides the best goodness of fit, followed by SV-L and SV-1.

From an economic perspective, it is arguable that GARCH-t, SV-t, GARCH-J, and SV-L provide a better fit for gold. Unlike other commodities, gold is considered highly liquid in financial markets and plays a role as store of value, portfolio diversifier, and safe asset in adverse macroeconomic scenarios. Therefore, events that trigger increases in gold volatility are more associated with international instability episodes than with developments in the gold industry. Events that caused abrupt falls in gold returns[19] are more relevant in explaining volatility that those that triggered its role as safe asset.[20] The latter is in line with a negative asymmetry in returns (Table 1). Moreover, the role of safe asset is not always triggered as, for example, in the 1990s the strength of the dollar and central bank gold sales deteriorated the role of gold as an investment asset, although events like the Asian and Russian crises took place in that period.[21]

Figure 2 shows relevant episodes in the history of gold volatility fluctuations. The beginning of the sample in 1983 shows a growing trend in volatility, created by the launching of gold futures trading in the London and Tokyo stock exchanges and by the run to safe gold investments in the wake of the 1980s crisis. A jump in volatility took place in 1985, probably triggered by banks' adoption of a net seller position and the weakening of the dollar against European currencies and the yen. The launching of computer trading in gold futures at the Commodity Exchange (COMEX) and the Sydney Exchange prompted yet another jump in 1986. Later the collapse of stock exchanges around the world on October 19th, 1987 (Black Monday) caused a surge in the demand for gold.[22] However, volatility diminished in 1988, as calm returned to the markets and investor sentiment about political stability improved after the U.S. army pulled out of Afghanistan. Volatility increased again in 1989 in response to central banks' gold sales, which resulted in negative

[19] Dollar appreciation episodes, unexpected gold sales by central banks, and appearance of substitute investment instruments, among others.

[20] The 1980s crisis, the Savings & Loan crisis, the 2001 and 2008 financial crises, geopolitical tensions between the U.S. and other countries, among other factors.

[21] Developments like the 1994 Mexican Crisis, the 1997 Asian Crisis, and the 1998 Russian Crisis created financial panic but did not drive investors to use gold massively as safe asset. On the contrary, in this period (1992-1999) gold returns experienced negative jumps.

[22] In that year, futures trading operations increased 20% relative to the months in the run-up to Black Monday (1986).

jumps in returns.

In the mid-1990s, the Savings & Loan crisis abruptly exacerbated the volatility of daily returns and again investors sought safety in gold. Between August 1990 and February 1991, the Gulf War provoked an increasing trend in volatility. According to Dey (2016), the return to calm in the remainder of 1991 was associated with the dissolution of the Soviet Union at the end of the year, which established the dominance of the U.S. and reduced the likelihood of a monetary transition at the global level. In 1992, central banks, producers, and investors carried out massive gold sales in response to the deterioration caused by the recent crisis, resulting in a new increase in volatility. In 1997, uncertainty increased regarding the management of gold reserves by the countries that would later form the European Monetary Union.[23] At end-1999, volatility declined in response to a 400% increase in the demand for hedging instruments caused by the continuous fall in the price of gold. However, the bursting of the Dotcom bubble in 2000 and the 9/11 attacks[24] brought volatility to new peaks and triggered gold's safe asset role in response to a weakening dollar.

Further jumps in volatility occurred in 2003-2004 due to the escalation of U.S.-Iraq geopolitical tensions and the surge in investment flows towards Exchange Trade Funds' (ETFs) gold securities. Other jumps were associated with bank sales decreasing by 23% and 51% in 2004 and 2006 and increasing by 40% and 80% in 2005 and 2007, and with investment flows towards ETFs.[25] Later, the Global Financial Crisis (GFC) (September 2008) seriously affected economic activity in most developed economies and provoked a massive run towards gold, in turn causing a surge in volatility until end-2009. An important development was the greater frequency of clusters such as the ones occurred in the 1980s, associated with the weakening of the dollar during the U.S. recessions from 2001 to a few years after the 2008 crisis. This is consistent with Tully and Lucey (2007), who argue that the weakening of the dollar against other currencies makes gold cheaper, as it is traded in dollars; and with Batten et al. (2010), who indicate that gold responds to monetary variables like inflation and money supply growth.

Another abrupt increase in volatility in 2011 was caused by growing speculative demand for gold in the run-up to the European Union debt crisis. A new peak in volatility occurred in 2013, associated with U.S. economic recovery and the upcoming change in the Fed's monetary stance, which would result in future interest rate increases and the phasing out of Quantitative Easing. This prompted a 50% fall in global gold investment. Moreover, in 2013, 2014, and 2015, central banks' historically high gold purchases (409, 466, and 483 TM, respectively) abruptly diminished returns and increased volatility in response to uncertainty about the imminent hike in Fed rates. Finally, gold volatility jumped in 2016 and 2017 in the wake of the referendum on the UK's exit from the European Union and U.S.-North Korea geopolitical tensions.

The $Q(20)$ and $Q_2(20)$ statistics suggest that only GARCH-t, GARCH-J, SV-2, and SV-t do not reject the null hypothesis of no autocorrelation in the standardized residuals and the square standardized residuals.

[23] During the same year, the Asian Crisis resulted in a 37% fall in Asian demand for gold (20% of global demand). In contrast with past crises, gold's safe asset role was not triggered because the dollar experienced an appreciation period and alternative investment instruments were already in place.

[24] It should be noted that the consolidation of the mining industry reduced the number of firms, limited the gold supply, and made the supply/demand balance more sensitive during the first decade of the new millennium.

[25] Additionally, the 2006 U.N. Security Council sanctions to curb Iran's nuclear program created considerable geopolitical tensions.

3.2.2 Copper

Table 4 shows the results for the GARCH family. The GARCH-t model yields $\nu = 6.38$, suggesting the occurrence of extreme events. The average jump in GARCH-J is $\mu_k = -0.33$ (i.e., the fall in returns is almost twice as large as for gold), indicating that the series for returns on copper experienced more falls than jumps across the sample. Additionally, $\kappa = 0.10$, which translates into 25 jumps per year. GARCH-M shows evidence that market participants demand a risk premium. At the same time, there is no evidence of first-order serial correlation in the disturbances of the returns on copper. GARCH-L yields $\delta > 0$; i.e., there is a leverage effect where, given current negative returns, volatility will be greater in the future. Along these lines, Hammoudeh and Yuan (2008) find that $\delta > 0$ and argue that the returns on copper are asymmetrical because they are linked to global economic activity.[26] Finally, based on the values of the marginal log-likelihoods, GARCH-t provides the best fit, followed by GARCH-J and GARCH-2.

Table 5 shows the results for the SV family. SV-t yields $\nu = 10.52$; . At the same time, SV-J yields $\kappa = 0.04$ and $\mu_k = -0.07$, indicating 10 jumps per year on average and a lower negative jump than for gold. For example, Liu et al. (2014) find a jump probability of 0.05 and $\mu_k < 0$, similar to the results found in this paper. Additionally, there is no evidence of a risk premium and serial correlation in SV-M and SV-MA, respectively. SV-L shows a negative correlation between shocks on copper returns and their volatility, $\rho = -0.12$, similar to the findings by Liu et al. (2014). Finally, SV-t provides the best fit, followed by SV-2 and SV-L.

The history of copper returns can be instrumental in explaining the findings in this paper. As copper is used mainly in manufacturing,[27] its volatility is governed by supply and demand movements. Gerwe (2016) points out that inventories, economic activity and the dollar exchange rate are copper price fundamentals.[28] Figure 2 shows a volatility peak at the beginning of the 1980s, caused by an excess supply of low-grade ore and the aftershock from the 1980s crisis, which brought returns down. Volatility surged in April 1984, driven by speculations about Fed rate hikes. In response, COMEX and the London Metal Exchange (LME) sold copper positions to move to other assets. In October 24-31st, 1985, brass trading operations were suspended, in turn negatively affecting copper returns and exacerbating their volatility. In 1986, volatility surged again because COMEX obtained authorization from the Commodity Futures Trading Commission (CFTC) to trade copper options, while the dollar depreciated against the yen and the Deutsche Mark. In 1987, COMEX and LME inventories experienced a fall caused by developments like the highest copper consumption in eight years, supply disruptions created by miners' strikes in Canada, operative problems in African ports, and concerns about COMEX's ability to fulfill contracts for physical delivery of copper.[29] High volatility persisted in 1988 because of strike announcements in Peru and Papua New Guinea and the closure of foundries in Chile and the U.S. In 1989, the supply/demand balance remained sensitive to inventory movements and industry announcements.

In 1990, copper returns became negative because of an increase in LME inventories and the presence of a differential between LME and COMEX security prices. The downward trend in 1991

[26] In contrast, Bracker and Smith (1999) find an inverse leverage effect, like in the case of gold, for three asymmetric GARCH models.

[27] Gerwe (2016) indicates that, in 2014, 39% of final copper consumption was used to produce electric and electronic goods, while 30% was used in construction.

[28] Gerwe (2016) also indicates that wars cause an increase in the demand for copper; however, the war periods mentioned in the section about gold were not accompanied by considerable volatility surges.

[29] In October 1987, the CFTC unsuccessfully looked for evidence of copper price manipulation.

was caused by U.S. recession pressures, the dissolution of the Soviet Union, and a deterioration in the copper industry. In 1992, as inventories reached its highest peak since 1984, returns dropped and volatility surged. In mid-1993, artificial price rises took place despite excess inventories;[30] and volatility increased after the LME established (September 8) that the price of copper had fallen to a six-year minimum as a result of improper practices. Later (June 1996) the scandal around the Sumimoto Corporation (which reported USD 1.8 billion losses resulting from illicit copper trading operations) brought volatility to levels similar to the 1987 peak. Moreover, volatility remained high in response to concerns about LME regulatory agency practices.

The 1997 Asian Crisis resulted in a considerable contraction in economic activity, in turn bringing inventories to the 1984 historic peak and depressing returns in 1998. However, copper prices recovered somewhat in 1999 despite high inventories, probably in connection with speculation resulting from the announcement of BHP Billington production cuts.[31] Volatility remained stable in 2000-2003 despite the 2001 U.S. recession. However, in 2004 the market became sensitive to industry announcements, especially concerns about growth in China and other Emerging Market Economies. In 2005 volatility reached 2004 levels, reflecting developments like miners' strikes in Chile and the U.S. and market speculation (third and fourth quarters) prompted by short sales of copper futures contracts by a trader seeking to drive the price down to favor China. Volatility jumped again in 2006 in response to falling COMEX and LME inventories and a sustained increase in Asian demand for copper.

Volatility in 2008 was associated with a price rally triggered by a drop in world copper inventories. The abrupt drop in returns at the beginning of the GFC was partially offset by a 38% increase in Chinese consumption, but volatility remained high until end-2009. Volatility diminished somewhat over the next two years, but with mixed jumps caused by Chinese imports and an inventory buildup in metal exchanges. Negative jumps occurred in 2011 despite substantial Chinese imports because of concerns about the European debt crisis. In 2012-2013 volatility was governed by changes in China's consumption pattern, industry announcements, and speculation about Fed cuts in bond purchases, which materialized in January 2014. Finally, in later years, Chinese deceleration and the strengthening of the dollar created new return clusters and greater volatility.

In sum, evidence suggests that volatility in copper returns was governed by changes in world inventories, global consumption, market speculation, and industry announcements. The presence of fat tails and jumps in copper returns is founded on elements such as temporary adverse sentiment in the market for copper securities in response to price manipulation or increased uncertainty about potential production cuts. The results also suggest the presence of leverage effects in copper returns, as during global recession episodes world copper consumption falls relatively more than in a normal macroeconomic environment. Preference for SV-t over SV-L seems to be explained by the fact that the structure of SV-L does not generate sufficient variability to capture the impact of more transitory events compared with crisis events, which have a longer effect on financial market sentiment.

The $Q(20)$ and $Q_2(20)$ statistics suggest that none of the models in the GARCH family reject the no autocorrelation null hypothesis. In the SV family, only SV-2 and SV-J reject the null hypotheses.

[30] The LME futures market entered backwardation, while COMEX was in contango.
[31] At the time, BHP Billington was the world's second largest copper producer.

3.2.3 Oil

Table 6 shows the results for the GARCH family. GARCH-t estimation yields $\nu = 7.63$, suggesting the existence of extreme values. The average jump in GARCH-J is -0.55, indicating that, on average, jumps in returns were negative. Additionally, the jump probability is 0.09, which translates into 23 jumps per year. Compared with copper and gold, the fall in oil returns is more abrupt. GARCH-M shows evidence of a risk premium demanded by market participants to invest in the oil market. Again, there is no evidence of first-order serial correlation in GARCH-MA. Regarding GARCH-L, $\delta > 0$; i.e., there is evidence of asymmetry in oil returns. This is consistent with the literature; see for example Nomikos and Andriasopoulos (2012) and Chan and Grant (2016a). Finally, regarding goodness of fit, GARCH-t is preferred, followed by GARCH-J and GARCH-L. The results for oil in Chan and Grant (2016a) are qualitatively similar to the ones obtained in this research. For example, they calculate $\nu = 10.86$ in GARCH-t; i.e., greater than for gold and copper, like in this paper. The GARCH-J model yields $\mu_k < 0$ and a jump intensity of 0.05, while GARCH-L yields $\delta > 0$. The only exception is GARCH-MA, which indicates the presence of first-order serial correlation and provides the best fit in the GARCH family. The authors' interpretation is that Chant and Grant (2016a) use weekly series where first-order autocorreliation may exist, in contrast with the daily data used in this paper.

Table 7 shows the results for the SV family. The SV-t model yields $\nu = 12.58$, which is much higher that for gold and copper; and indicates that oil returns show innovations that further depart from the t-Student distribution. This is also present in the results obtained by Chant and Grant (2016a), where the value of ν is higher than 40. In SV-J, the jump probability is 0.05 and $\mu_k < 0$. In the same line, Larsson and Nossman (2011) and Brooks and Prokopczuk (2013) find that the jump probability in SV-J is 0.01 and 0.025, respectively; i.e., six jumps per year on average. Additionally, both calculate a negative jump value for the returns. In contrast, neither SV-M nor SV-MA provide an additional contribution in modeling oil volatility. The SV-L model shows a negative correlation between shocks on oil returns and their volatility because $\rho = -0.25$, which is similar to the results found by Brooks and Prokopczuk (2013). Based on the values for the marginal log-likelihood, the selected model is SV-t, followed by SV-L and SV-2. Comparing the results in this paper for the SV family with those obtained by Chan and Grant (2016a), the jump value for the returns in SV-J is positive, $\mu_k > 0$. Additionally, SV-t tails are similar to those in a normal distribution, as $\nu = 56.13$. At the same time, they present SV-MA as the best model, while this paper finds a low goodness of fit like in SV-1. Again, these differences reflect the fact that the authors use weekly data beginning in 1994; i.e., they do not include the impact of the Gulf War and the 1990s crises on oil returns. Moreover, using weekly data implies smoothing out the extreme data that may appear in daily series like the ones used in this paper.

The results suggest that introducing fat tails and a leverage effect is beneficial in the case of oil. While oil depends on supply/demand fundamentals like short-run supply inelasticity in the face of price changes, like in the case of copper, history shows that the geographical coincidence of conflicts between Middle East and Western countries and the presence of important oil sources, as well as announcements by the Organization of Petroleum Exporting Countries (OPEC) cartel, have a substantial impact on expectations about the global availability of oil inventories, and therefore about the volatility of the process governing oil returns. Figure 2 shows that the first volatility peak (also the largest in the whole sample) occurred during the Gulf War in 1990-1991. Another peak occurred, for example, on May 23, 1998, when OPEC agreed to cut production to mitigate the decline in returns created by weak global consumption in the wake of the Asian crisis. Moreover, Operation Desert Fox (a massive bombing campaign against Iraq) was launched in December. The bursting of the Dotcom bubble at end-2000 triggered a recession, which in turn caused a fall in U.S. oil consumption and negative returns in the oil market. The 9/11 attacks and the subsequent U.S. invasion of Iraq further exacerbated volatility. The 2002 oil strike in Venezuela caused a new price jump. Damages in the U.S. Gulf of Mexico oil facilities caused by the 2005 Rita and Katrina hurricanes and sustained Asian demand drove global oil inventories down and triggered widespread uncertainty. In the run-up to the GFC, oil volatility escalated as a result of a speculative commodity boom. When the GFC put an end to it, oil volatility climbed to its highest levels since the Gulf War. In February 2011, the Libyan civil war affected oil exports and created considerable uncertainty in the crude oil market. Chinese deceleration contributed to increasing volatility since 2014; and economic and political instability took the market to considerable peaks in 2016.

In sum, the presence of fat tails originates in unexpected changes in the output of strategic crude producers and U.S.-OPEC political tensions. The results also suggest the presence of leverage effects, implying that negative oil returns today indicate higher volatility tomorrow. This usually takes place in the wake of economic crises, where a downward trend in oil returns prompts massive crude security and inventory selloffs and an increasing volatility trend. Preference of the fat tail model over leverage-effect models may be explained by the inclusion of the Gulf War in the sample.

The $Q(20)$ and $Q_2(20)$ statistics show that none of the models in both families reject the no autocorrelation null hypothesis.

3.2.4 Natural Gas

Table 8 shows the results for the GARCH family. The GARCH-t model yields $\nu = 10.55$, similar to oil. At the same time, GARCH-J yields $\mu_k = 0.45$ and a jump probability of 0.09; i.e., 23 jumps per year. In contrast with previous series, only gas yields $\mu_k > 0$. The results for GARCH-J are consistent with the literature; e.g., Mason and Wilmot (2014) use a GARCH-J model and find that the jump intensity is 0.015, with a positive jump value. Additionally, Nomikos and Andriosopoulos (2012) find a positive average jump with an intensity of occurrence of 0.058. At the same time, GARCH-M does not require modeling a risk premium as requirement to invest in that market; and GARCH-MA does not make a significant contribution. Therefore, in terms of parsimony it is possible to be indifferent between this model and GARCH-1. In parallel, GARCH-L results suggest $\delta < 0$, suggesting an inverse leverage effect like in the case of gold. Nomikos and Andriosopoulos (2012) argue that this is explained by the fact that positive demand shocks dominated supply shocks, and therefore natural gas price increases become an indicator of volatility-enhancing scarcity. Based on the marginal log-likelihoods, the preferred model is GARCH-t, followed by GARCH-J and GARCH-L.

Table 9 shows the results for the SV family. While there is evidence of extreme events in the return series, innovations tend to a normal distribution, as SV-t yields $\nu = 48.95$. SV-J yields a jump probability of 0.05; i.e., 13 jumps per year on average, with a magnitude of 0.44. Again, the risk premium coefficient in SV-M and the serial correlation coefficient in SV-MA include zero in their respective credibility intervals. However, SV-L shows a positive asymmetric relationship between returns and volatility, implying an inverse leverage effect, as explained above. Finally, the marginal likelihoods indicate that the preferred model is SV-L, followed by SV-2 and SV-t.

The results suggest that the importance of the inverse leverage effect dominates the fat tail component. An inspection of high-volatility episodes in this market (Figure 2) shows that the first peak (February 2, 1996) was caused by scarce gas inventories and triggered a substantial price increase. Later, according to Roesser (2009), the Federal Energy Regulatory Commission (FERC) detected price manipulation to drive up the price of Western U.S. gas, which explains the volatility peak between end-2000 and the beginning of 2001. The February 2003 volatility surge was once more the result of supply/demand mismatches caused by severe winters, which affected U.S. gas distribution facilities and led to a peak in demand. The Katrina and Rita hurricanes (August and September 2005, respectively), caused serious damage to the Gulf of Mexico facilities and created considerable volatility in natural gas returns. The 2008 volatility peaks were caused by a mix of supply/demand fundamentals and financial market speculation. In April 2008, a gas leak in the Gulf of Mexico facilities caused concerns about a possible supply reduction in the context of the GFC. Figure 2 shows a growing trend in volatility until mid-2010 caused by global recovery.

The presence of an inverse leverage effect is consistent with the nature of the natural gas market, where production is inelastic to price. Therefore, the volatility of gas returns is highly sensitive to available supply, which may vary due to climatic developments and infrastructure restrictions. Positive returns indicate excess demand and therefore higher future uncertainty about natural gas supply conditions. This is consistent with Mu (2007) and Roesser (2009), who argue that an important fundamental in gas price is the supply/demand balance. More specifically, Mu (2007) finds that unexpected weekly inventory changes and temperature levels may cause uncertainty about future supply conditions. Moreover, Geman (2005) underscores that storage is costlier for gas than for oil, and therefore gas trade takes place in local markets, thereby reducing the industry's capacity to meet demand pressures.

The $Q(20)$ and $Q_2(20)$ statistics indicate that all models in both families reject the no autocorrelation null hypothesis.[32]

3.2.5 Comparative Analysis

Tables 2-9 above show the marginal (log) likelihoods for each commodity and each of the seven GARCH and SV models. This allows calculation of the BFs to identify the best-fitting models.

First, GARCH-t (M_j following the notation in Section 2) is selected within the GARCH family. In this case, $BF_{ij} < 1$ indicates preference for M_j over M_i. The results indicate that GARCH-t is better than all other models for all commodities. For example, in the case of copper, GARCH-t has a BF of 1.07×10^{-103} compared with GARCH-1. Similar values are obtained for the other models. In the case of oil, GARCH-t has a BF of 3.24×10^{-68}, 3.22×10^{-66} and 8.81×10^{-68} when M_i is GARCH-M, GARCH-MA, and GARCH-L, respectively. Similar results are obtained for natural gas. At the same time, GARCH-J is better than all other models except GARCH-t.

Second, SV-t (M_j) is selected from the SV family. This model is preferred for all commodities with the following two exceptions: (i) SV-L is dominant for natural gas; (ii) the BF is equal to one in SV-t and SV-2. SV-L is also better than SV-J for all commodities. The adjustment levels are similar in SV-J and SV-MA for oil and natural gas.

Third, the two model families are compared. The following models are selected: GARCH-t for

[32] An option for ensuring no autocorrelation in the residuals is including more lags in the return or volatility equations. However, this warrants careful assessment in future research, as it would involve models different from the ones used in this paper.

gold; SV-t for copper and oil; and SV-L for natural gas.[33]

It is important to underscore that SV-t is better than GARCH-J for all commodities. Models exhibiting jumps seek to explain at least two moments in the return series (asymmetry and kurtosis). On one hand, these models do not produce the necessary excess kurtosis to explain gold, copper, and oil volatility; i.e., they do not surpass fat-tail models. On the other hand, neither do they produce the necessary asymmetry to explain natural gas volatility. Parameters σ_k^2 and μ_k are higher on average for energy than metal products; i.e., they overestimate σ_k^2 and μ_k to improve goodness of fit. All these results are consistent with Liu et al. (2014). Regarding the fat tail component, excess kurtosis in the distribution of gold, copper, and oil returns relative to a normal distribution translates into a higher probability of occurrence of extreme events. Additionally, the value of ν is usually lower for these products, which explains the preference for the extension of fat tails over other extensions in modeling volatility. However, kurtosis is lower for natural gas than metals returns, thereby explaining the lower relevance of the fat tail component.

In sum, three SV models and one GARCH model were selected. This preference is explained by the structure of SV models, which includes a stochastic disturbance in the volatility process, while its GARCH counterparts assume that volatility is governed by a deterministic process. This makes the SV volatility process more flexible, as it incorporates information on events affecting the volatility process in real time.

3.3 Robustness Analysis

A sensitivity analysis is performed on the change in priors. For the sake of simplicity and brevity, only the best models in each family are re-estimated for each commodity. Robustness of the value and sign of the estimated coefficient is tested, as well as the marginal log-likelihood in a scenario of non-informative priors.[34] Therefore, the following hyper-parameters are assumed: $V_{\mu_0} = 100$, $V_{\gamma_0} = diag(100, 100, 100)$, $V_{\delta_0} = diag(100, 100)$ for GARCH-t and GARCH-J; and $V_{\mu_0} = 100$, $V_{\phi_{h_1}} = 100$, $V_{\mu} = 100$, $V_{\mu_h} = 100$, $\nu_h = 2.5$, $V_{\rho} = 100$ for SV-t and SV-L.

Table 10 shows the results for four commodities and for GARCH-t and GARCH-J. They suggest a slight goodness-of-fit decrease in GARCH-J for oil, equal to 1.4% relative to the initial value. Table 11 shows the same results for SV-t and SV-L. Evidence suggests similar conclusions. For example, marginal log-likelihood in SV-t for gold decreases by 0.21%. Parameters ν and ρ decrease for copper and natural gas in SV-t and SV-L, respectively.

It is possible to re-calculate the BFs using the new marginal log-likelihoods, but the order of preference of models for each commodity remains unchanged. GARCH-t continues to dominate the SV family in modeling gold volatility. SV-t continues to be preferred over GARCH-t for copper and oil, while again SV-L dominates GARCH-t in modeling natural gas.

In sum, use of non-informative priors in both families does not introduce relevant changes in the results. The previously selected models (GARCH-t, SV-t, and SV-L) continue to provide the best fit for their respective commodities. No significant changes were identified in the value of the parameters estimated for both families relative to the baseline scenario. Additionally, there are no changes in the sign of the estimated coefficients.

[33] Tables showing the BFs are available upon request.
[34] A scenario using informative priors was also run. The results are available upon request.

4 Conclusions

Seven GARCH and SV models are compared based on goodness of fit for modeling volatility in four commodity series. Overall, GARCH-t is the best model in both families for modeling gold volatility. However, SV-t provides the best fit for copper and oil, while SV-L is better for natural gas.

The results also suggest that gold and gas show inverse leverage effects; i.e., positive returns today increase volatility tomorrow. In contrast, oil and copper show a standard leverage effect, where negative returns today indicate greater future uncertainty. Additionally, most products do not show evidence of a risk premium or MA-type first-order serial correlation.

Use of daily series over a wide horizon allows better knowledge of commodity volatility associated with abrupt changes from extreme events and negative jumps from global and domestic crises, and from economic and political instability. The historic evolution of commodity volatility allows suggesting that a development strategy based on commodity trade may involve greater volatility in macroeconomic aggregates, in turn translating into higher welfare costs.

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Table 1. Summary Statistics for Returns and Squared Returns

	Mean	Std. Deviation	Skewness	Kurtosis	Min.	Max.	Obs.
Panel A: Returns							
Gold	0.011	1.055	-0.214	9.910	-9.811	8.830	8653
Copper	0.015	1.619	-0.234	7.408	-12.516	11.902	8653
Oil	0.013	2.211	-0.825	18.385	-38.404	13.572	7643
Natural Gas	0.007	3.131	0.035	5.185	-16.698	18.765	5872
Panel B: Squared Returns							
Gold	1.100	3.300	11.900	221.500	0.000	96.300	8651
Copper	2.600	6.600	8.500	121.400	0.000	156.600	8651
Oil	4.900	20.400	51.300	3609.100	0.000	1474.900	7641
Natural Gas	9.800	20.000	6.000	59.300	0.000	352.100	5871

Table 2. Gold: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.007	0.007	0.007	0.012	-0.011	0.007	0.020
	(0.010)	(0.010)	(0.008)	(0.008)	(0.014)	(0.009)	(0.010)
	[-0.012,0.026]	[-0.012,0.026]	[-0.008,0.022]	[-0.005,0.028]	[-0.038,0.017]	[-0.012,0.026]	[0.001,0.040]
α_o	0.034	0.035	0.002	0.003	0.034	0.034	0.042
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[0.034,0.034]	[0.035,0.036]	[0.002,0.002]	[0.002,0.003]	[0.034,0.035]	[0.034,0.034]	[0.042,0.042]
α_1	0.077	0.080	0.024	0.031	0.077	0.077	0.109
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
	[0.076,0.078]	[0.079,0.081]	[0.023,0.025]	[0.031,0.032]	[0.076,0.078]	[0.076,0.078]	[0.106,0.111]
β_1	0.896	0.838	0.954	0.949	0.896	0.896	0.883
	(0.000)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
	[0.895,0.896]	[0.836,0.839]	[0.952,0.955]	[0.948,0.949]	[0.895,0.896]]	[0.895,0.896]	[0.882,0.884]
β_2	-	0.054	-	-	-	-	-
		(0.001)					
		[0.052,0.056]					
κ	-	-	-	0.100	-	-	-
				(0.000)			
				[0.100,0.100]			
μ_k	-	-	-	-0.144	-	-	-
				(0.086)			
				[-0.314,0.024]			
σ_k^2	-	-	-	3.663	-	-	-
				(0.237)			
				[3.208,4.149]			
λ	-	-	-	-	0.021	-	-
					(0.013)		
					[-0.004,0.046]		
ψ	-	-	-	-	-	-0.001	-
						(0.012)	
						[-0.025,0.024]	
ν	-	-	3.947	-	-	-	-
			(0.020)				
			[3.908,3.983]				
δ	-	-	-	-	-	-	-0.050
							(0.002)
							[-0.054,-0.046]
$LogL$	-11930.3	-11928.4	-11154.7	-11245.5	11936.5	-11933.5	-11935.1
	(0.09)	(0.18)	(0.20)	(0.10)	(0.19)	(0.12)	(0.15)
$Q(20)$	0.4339	0.4319	0.3502	0.3719	0.4294	0.3726	0.4070
$Q_2(20)$	0.9999	0.9998	0.9902	0.9992	0.9999	0.9999	1.0000

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively.

Table 3. Gold: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.00 (0.01) [−0.011,0.019]	0.00 (0.01) [−0.012,0.018]	0.01 (0.01) [−0.008,0.021]	0.01 (0.01) [−0.008,0.021]	−0.00 (0.01) [−0.022,0.019]	0.00 (0.01) [−0.011,0.020]	0.01 (0.00) [−0.004,0.032]
μ_h	−0.33 (0.08) [−0.483,0.168]	−0.24 (0.17) [−0.502,0.069]	−0.35 (0.09) [−0.542,−0.168]	−0.36 (0.08) [−0.517,−0.191]	−0.33 (0.08) [−0.480,−0.170]	−0.32 (0.08) [−0.471,−0.169]	−0.310 (0.08) [−0.478,−0.141]
ϕ_{h_1}	0.97 (0.00) [0.956,0.975]	0.94 (0.00) [0.938,0.948]	0.98 (0.00) [0.971,0.981]	0.97 (0.00) [0.968,0.978]	0.97 (0.01) [0.952,0.975]	0.96 (0.01) [0.952,0.974]	0.97 (0.00) [0.963,0982]
ω_h^2	0.05 (0.01) [0.042,0.070]	0.04 (0.00) [0.006,0.054]	0.04 (0.00) [0.040,0.042]	0.04 (0.00) [0.039,0.041]	0.06 (0.01) [0.042,0.077]	0.06 (0.01) [0.043,0.072]	0.04 (0.00) [0.033,0.059]
ϕ_{h_2}	-	0.03 (0.01) [0.038,0.041]	-	-	-	-	-
κ	-	-	-	0.01 (0.00) [0.006,0.021]	-	-	-
μ_k	-	-	-	−0.35 (0.07) [−0.503,−0.253]	-	-	-
σ_k^2	-	-	-	7.78 (1.93) [5.057,10.577]	-	-	-
λ	-	-	-	-	0.01 (0.01) [−0.016,0.040]	-	-
ψ	-	-	-	-	-	−0.02 (0.01) [−0.045,−0.001]	-
ν	-	-	12.60 (1.07) [10.684,14.901]	-	-	-	-
ρ	-	-	-	-	-	-	0.20 (0.032) [0.149,0.265]
$LogL$	−11327.8 (0.12)	−11332.4 (0.18)	−11199.1 (0.69)	−11327.9 (0.57)	−11334.2 (0.13)	−11329.6 (0.19)	−11320.3 (0.23)
$Q(20)$	0.281	0.056	0.141	0.283	0.328	0.362	0.325
$Q_2(20)$	0.000	0.176	0.191	0.001	0.000	0.000	0.000

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 4. Copper: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.013 (0.014) [−0.016,0.040]	0.013 (0.014) [−0.015,0.040]	0.016 (0.013) [−0.010,0.042]	0.032 (0.014) [0.004,0.060]	−0.058 (0.019) [−0.096,−0.020]	0.013 (0.015) [−0.016,0.041]	0.007 (0.014) [−0.020,0.035]
α_o	0.033 (0.000) [0.032,0.033]	0.034 (0.000) [0.034,0.035]	0.008 (0.000) [0.007,0.008]	0.003 (0.000) [0.003,0.004]	0.033 (0.000) [0.033,0.034]	0.033 (0.000) [0.032,0.033]	0.047 (0.000) [0.046,0.047]
α_1	0.058 (0.000) [0.057,0.058]	0.060 (0.000) [0.059,0.061]	0.032 (0.001) [0.031,0.033]	0.040 (0.001) [0.039,0.041]	0.058 (0.000) [0.057,0.058]	0.058 (0.000) [0.057,0.059]	0.057 (0.001) [0.056,0.059]
β_1	0.930 (0.000) [0.929,0.930]	0.874 (0.001) [0.872,0.875]	0.951 (0.001) [0.950,0.953]	0.948 (0.000) [0.948,0.949]	0.929 (0.000) [0.928,0.930]	0.930 (0.000) [0.929,0.931]	0.916 (0.000) [0.915,0.917]
β_2	-	0.053 (0.001) [0.051,0.054]	-	-	-	-	-
κ	-	-	-	0.099 (0.001) [0.095,0.100]	-	-	-
μ_k	-	-	-	−0.326 (0.119) [−0.561,−0.095]	-	-	-
σ_k^2	-	-	-	4.518 (0.362) [3.849,5.272]	-	-	-
λ	-	-	-	-	0.040 (0.007) [0.026,0.055]	-	-
ψ	-	-	-	-	-	−0.005 (0.012) [−0.028,0.018]	-
ν	-	-	6.385 (0.043) [6.302,6.467]	-	-	-	-
δ	-	-	-	-	-	-	0.017 (0.002) [0.013,0.020]
$LogL$	−15081.1 (0.23)	−15081.8 (0.11)	−14844.0 (0.09)	−14864.8 (0.19)	−15088.8 (0.34)	−15085.2 (0.09)	−15097.4 (0.10)
$Q(20)$	0.6003	0.6002	0.6784	0.6988	0.5167	0.5537	0.5498
$Q_2(20)$	0.7914	0.8148	0.1337	0.4436	0.8198	0.7902	0.9013

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively.

Table 5. Copper: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.02 (0.01) [−0.009,0.042]	0.02 (0.01) [−0.008,0.044]	0.02 (0.01) [−0.007,0.043]	0.02 (0.01) [−0.011,0.043]	0.02 (0.02) [−0.022,0.058]	0.02 (0.01) [−0.009,0.041]	0.01 (0.01) [−0.017,0.036]
μ_h	0.60 (0.11) [0.390,0.808]	0.59 (0.12) [0.387,0.788]	0.41 (0.13) [0.139,0.662]	0.55 (0.08) [0.397,0.713]	0.60 (0.11) [0.388,0.815]	0.60 (0.11) [0.387,0.809]	0.60 (0.11) [0.390,0.815]
ϕ_{h_1}	0.99 (0.00) [0.979,0.990]	0.84 (0.08) [0.689,0.98]	0.99 (0.01) [0.969,0.994]	0.97 (0.00) [0.968,0.978]	0.99 (0.00) [0.980,0.991]	0.99 (0.00) [0.980,0.990]	0.99 (0.00) [0.981,0.990]
ω_h^2	0.02 (0.00) [0.013,0.023]	0.04 (0.01) [−0.019,0.285]	0.02 (0.01) [0.007,0.040]	0.04 (0.00) [0.037,0.039]	0.02 (0.00) [0.013,0.022]	0.02 (0.00) [0.013,0.023]	0.02 (0.00) [0.013,0.022]
ϕ_{h_2}	-	0.13 (0.08) [0.024,0.049]	-	-	-	-	-
κ	-	-	-	0.04 (0.02) [0.0130,0.0830]	-	-	-
μ_k	-	-	-	−0.07 (0.20) [−0.4029,0.4312]	-	-	-
σ_k^2	-	-	-	2.04 (0.85) [0.893,4.160]	-	-	-
λ	-	-	-	-	−0.00 (0.01) [−0.021,0.019]	-	-
ψ	-	-	-	-	-	−0.02 (0.01) [−0.046,−0.002]	-
ν	-	-	10.52 (2.38) [7.775,16.597]	-	-	-	-
ρ	-	-	-	-	-	-	−0.12 (0.05) [−0.220,−0.022]
$LogL$	−14893.6 (0.03)	−14890.9 (0.15)	−14839.0 (0.02)	−14920.4 (0.40)	−14900.4 (0.06)	−14895.2 (0.07)	−14893.9 (0.04)
$Q(20)$	0.282	0.227	0.345	0.156	0.301	0.492	0.313
$Q_2(20)$	0.063	0.022	0.519	0.012	0.083	0.047	0.066

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 6. Oil: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.033 (0.001) [−0.005,0.071]	0.034 (0.019) [−0.004,0.072]	0.035 (0.019) [−0.001,0.071]	0.056 (0.021) [0.015,0.097]	−0.023 (0.025) [−0.071,0.025]	0.034 (0.020) [−0.005,0.073]	0.022 (0.020) [−0.015,0.06]
α_o	0.052 (0.001) [0.051,0.053]	0.054 (0.001) [0.053,0.056]	0.024 (0.001) [0.022,0.026]	0.017 (0.001) [0.015,0.020]	0.052 (0.001) [0.051,0.054]	0.052 (0.001) [0.051,0.054]	0.066 (0.001) [0.065,0.067]
α_1	0.078 (0.001) [0.077,0.079]	0.081 (0.001) [0.080,0.082]	0.045 (0.001) [0.044,0.047]	0.055 (0.001) [0.053,0.058]	0.077 (0.001) [0.076,0.079]	0.078 (0.001) [0.077,0.079]	0.070 (0.001) [0.068,0.072]
β_1	0.914 (0.001) [0.913,0.916]	0.859 (0.001) [0.857,0.861]	0.934 (0.001) [0.931,0.936]	0.930 (0.001) [0.928,0.932]	0.915 (0.001) [0.913,0.916]	0.914 (0.001) [0.913,0.916]	0.906 (0.001) [0.905,0.907]
β_2	-	0.052 (0.001) [0.051,0.054]	-	-	-	-	-
κ	-	-	-	0.093 (0.007) [0.075,0.100]	-	-	-
μ_k	-	-	-	−0.547 (0.188) [−0.920,−0.186]	-	-	-
σ_k^2	-	-	-	7.902 (0.951) [6.270,10.034]	-	-	-
λ	-	-	-	-	0.018 (0.005) [0.009,0.027]	-	-
ψ	-	-	-	-	-	0.006 (0.013) [−0.019,0.032]	-
ν	-	-	7.632 (0.061) [7.514,7.747]	-	-	-	-
δ	-	-	-	-	-	-	0.027 (0.002) [0.023,0.031]
$LogL$	−15767.3 (0.10)	−15768.6 (0.19)	−15620.1 (0.08)	−15655.3 (0.05)	−15775.5 (0.15)	−15770.9 (0.09)	−15774.5 (0.15)
$Q(20)$	0.7394	0.7373	0.7653	0.7923	0.7273	0.6871	0.7149
$Q_2(20)$	0.3779	0.3626	0.2612	0.2889	0.4668	0.3761	0.4777

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 7. Oil: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.04 (0.02) [−0.000,0.074]	0.04 (0.02) [0.003,0.081]	0.04 (0.02) [0.000,0.074]	0.05 (0.02) [0.011,0.096]	0.07 (0.03) [0.009,0.122]	0.04 (0.02) [0.000,0.072]	0.02 (0.02) [−0.021,0.054]
μ_h	1.20 (0.11) [0.985,1.415]	1.18 (0.18) [0.852,1.467]	1.05 (0.13) [0.784,1.310]	1.11 (0.14) [0.838,1.370]	1.21 (0.11) [0.984,1.422]	1.20 (0.11) [0.987,1.420]	1.20 (0.11) [0.980,1.421]
ϕ_{h_1}	0.98 (0.00) [0.977,0.989]	0.84 (0.08) [0.675,0.972]	0.99 (0.00) [0.985,0.993]	0.99 (0.00) [0.983,0.993]	0.98 (0.00) [0.979,0.989]	0.98 (0.00) [0.978,0.989]	0.99 (0.00) [0.979,0.990]
ω_h^2	0.02 (0.00) [0.015,0.027]	0.04 (0.01) [0.000,0.302]	0.01 (0.00) [0.009,0.016]	0.02 (0.00) [0.011,0.020]	0.02 (0.00) [0.014,0.026]	0.02 (0.00) [0.015,0.025]	0.02 (0.00) [0.014,0.025]
ϕ_{h_2}	-	0.14 (0.08) [0.029,0.050]	-	-	-	-	-
κ	-	-	-	0.05 (0.02) [0.0132,0.0959]	-	-	-
μ_k	-	-	-	−0.73 (0.36) [−1.3286,−0.1462]	-	-	-
σ_k^2	-	-	-	8.65 (4.44) [3.402,18.930]	-	-	-
λ	-	-	-	-	−0.01 (0.01) [−0.026,0.004]	-	-
ψ	-	-	-	-	-	−0.01 (0.01) [−0.036,0.011]	-
ν	-	-	12.58 (1.80) [9.685,16.597]	-	-	-	-
ρ	-	-	-	-	-	-	−0.25 (0.05) [−0.355,−0.143]
$LogL$	−15625.0 (0.02)	−15623.8 (0.15)	−15598.3 (0.01)	−15628.7 (0.42)	−15631.0 (0.09)	−15628.4 (0.05)	−15616.5 (0.03)
$Q(20)$	0.569	0.531	0.626	0.373	0.568	0.586	0.582
$Q_2(20)$	0.129	0.107	0.214	0.201	0.102	0.134	0.159

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 8. Natural gas: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.017 (0.035) [−0.051,0.089]	0.017 (0.035) [−0.051,0.085]	0.018 (0.035) [−0.049,0.086]	−0.009 (0.044) [−0.096,0.076]	0.024 (0.055) [−0.085,0.131]	0.018 (0.036) [−0.051,0.088]	0.044 (0.035) [−0.025,0.111]
α_o	0.216 (0.004) [0.208,0.225]	0.226 (0.004) [0.218,0.234]	0.164 (0.006) [0.152,0.175]	0.148 (0.005) [0.140,0.159]	0.215 (0.004) [0.207,0.224]	0.216 (0.004) [0.208,0.224]	0.225 (0.004) [0.217,0.233]
α_1	0.078 (0.001) [0.077,0.080]	0.081 (0.001) [0.080,0.083]	0.059 (0.001) [0.057,0.061]	0.069 (0.001) [0.067,0.071]	0.078 (0.001) [0.076,0.080]	0.078 (0.001) [0.077,0.080]	0.097 (0.001) [0.095,0.099]
β_1	0.902 (0.001) [0.900,0.904]	0.846 (0.001) [0.843,0.848]	0.908 (0.002) [0.905,0.912]	0.904 (0.001) [0.902,0.905]	0.902 (0.001) [0.900,0.904]	0.902 (0.001) [0.900,0.904]	0.902 (0.001) [0.900,0.903]
β_2	-	0.052 (0.001) [0.051,0.054]	-	-	-	-	-
κ	-	-	-	0.095 (0.006) [0.079,0.100]	-	-	-
μ_k	-	-	-	0.454 (0.354) [−0.249,1.142]	-	-	-
σ_k^2	-	-	-	13.009 (1.655) [10.012,16.669]	-	-	-
λ	-	-	-	-	−0.001 (0.006) [−0.012,0.010]	-	-
ψ	-	-	-	-	-	0.000 (0.014) [−0.028,0.027]	-
ν	-	-	10.554 (0.130) [10.305,10.800]	-	-	-	-
δ	-	-	-	-	-	-	−0.040 (0.001) [−0.042,−0.037]
$LogL$	−14525.0 (0.05)	−14526.7 (0.09)	−14477.9 (0.14)	−14493.5 (0.08)	−14530.5 (0.23)	−14528.7 (0.16)	−14520.5 (0.21)
$Q(20)$	0.0217	0.0212	0.0200	0.0234	0.0217	0.0163	0.0222
$Q_2(20)$	0.0009	0.0006	0.0005	0.0015	0.0009	0.0009	0.0006

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 9. Natural gas: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.02 (0.03) [−0.048,0.086]	0.01 (0.04) [−0.056,0.082]	0.02 (0.03) [−0.049,0.086]	−0.00 (0.05) [−0.105,0.100]	0.05 (0.07) [−0.083,0.175]	0.02 (0.03) [−0.049,0.085]	0.04 (0.04) [−0.047,0.113]
μ_h	2.05 (0.08) [1.890,2.206]	2.04 (0.11) [1.857,2.208]	2.00 (0.09) [1.833,2.169]	2.01 (0.09) [1.833,2.178]	2.05 (0.08) [1.892,2.205]	2.05 (0.08) [1.891,2.212]	2.05 (0.08) [1.903,2.214]
ϕ_{h_1}	0.98 (0.00) [0.966,0.983]	0.92 (0.08) [0.750,1.053]	0.98 (0.00) [0.969,0.984]	0.98 (0.00) [0.967,0.984]	0.97 (0.00) [0.965,0.982]	0.98 (0.00) [0.967,0.983]	0.96 (0.03) [0.892,0.983]
ω_h^2	0.02 (0.00) [0.015,0.027]	0.03 (0.01) [0.022,0.045]	0.02 (0.00) [0.013,0.023]	0.02 (0.00) [0.015,0.027]	0.02 (0.00) [0.016,0.027]	0.02 (0.00) [0.014,0.025]	0.04 (0.03) [0.015,0.103]
ϕ_{h_2}	-	0.04 (0.08) [−0.106,0.213]	-	-	-	-	-
κ	-	-	-	0.05 (0.03) [0.0047,0.0974]	-	-	-
μ_k	-	-	-	0.44 (0.85) [−1.4073,2.0453]	-	-	-
σ_k^2	-	-	-	6.18 (5.06) [1.271,18.502]	-	-	-
λ	-	-	-	-	−0.00 (0.01) [−0.019,0.011]	-	-
ψ	-	-	-	-	-	−0.01 (0.01) [−0.035,0.015]	-
ν	-	-	48.95 (22.80) [19.830,95.881]	-	-	-	-
ρ	-	-	-	-	-	-	0.18 (0.11) [−0.040,0.322]
$LogL$	−14467.8 (0.03)	−14465.7 (0.14)	−14465.7 (0.02)	−14471.5 (0.13)	−14474.8 (0.04)	−14471.5 (0.01)	−14463.4 (0.07)
$Q(20)$	0.027	0.022	0.024	0.035	0.027	0.022	0.035
$Q_2(20)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 10. Robustness Check: Non-Informative Priors Estimates for the GARCH Models

	Gold		Copper		Oil		Natural Gas	
	GARCH-t	GARCH-J	GARCH-t	GARCH-J	GARCH-t	GARCH-J	GARCH-t	GARCH-J
μ	0.007 (0.008) [−0.008,0.022]	0.012 (0.008) [−0.004,0.028]	0.016 (0.013) [−0.010,0.042]	0.032 (0.015) [0.004,0.061]	0.034 (0.019) [−0.003,0.071]	0.057 (0.021) [0.013,0.098]	0.019 (0.035) [−0.049,0.086]	−0.008 (0.043) [−0.094,0.075]
α_o	0.002 (0.000) [0.002,0.002]	0.003 (0.000) [0.002,0.003]	0.007 (0.000) [0.007,0.008]	0.002 (0.000) [0.002,0.003]	0.024 (0.001) [0.022,0.026]	0.016 (0.001) [0.014,0.019]	0.163 (0.006) [0.152,0.175]	0.146 (0.005) [0.138,0.157]
α_1	0.024 (0.000) [0.023,0.025]	0.031 (0.000) [0.030,0.032]	0.031 (0.001) [0.030,0.033]	0.040 (0.001) [0.039,0.041]	0.045 (0.001) [0.044,0.047]	0.055 (0.001) [0.053,0.058]	0.059 (0.001) [0.056,0.061]	0.069 (0.001) [0.067,0.071]
β_1	0.954 (0.001) [0.953,0.956]	0.950 (0.000) [0.949,0.951]	0.952 (0.001) [0.950,0.954]	0.949 (0.000) [0.949,0.950]	0.934 (0.001) [0.932,0.936]	0.931 (0.001) [0.929,0.933]	0.909 (0.002) [0.905,0.913]	0.904 (0.001) [0.902,0.906]
κ	-	0.100 (0.000) [0.099,0.100]	-	0.100 (0.001) [0.096,0.100]	-	0.093 (0.007) [0.077,0.100]	-	0.094 (0.006) [0.079,0.100]
μ_k	-	−0.146 (0.085) [−0.312,0.021]	-	−0.327 (0.120) [−0.562,−0.092]	-	−0.548 (0.188) [−0.922,−0.171]	-	0.446 (0.351) [−0.253,1.122]
σ_k^2	-	3.646 (0.240) [3.203,4.149]	-	4.480 (0.368) [3.788,5.226]	-	7.861 (0.929) [6.306,9.981]	-	13.135 (1.731) [10.145,16.870]
ν	3.943 (0.019) [3.907,3.980]	-	6.375 (0.043) [6.293,6.457]	-	7.621 (0.061) [7.500,7.737]	-	10.542 (0.128) [10.297,10.784]	-
$LogL$	−11159.0 (0.23)	−11251.1 (0.16)	−14849.0 (0.09)	−14871.9 (0.20)	−15625.0 (0.17)	−15633.6 (0.13)	−14483.3 (0.10)	−14502.6 (0.24)
$Q(20)$	21.873	21.52	16.596	16.198	15.194	14.690	35.049	34.442
$Q_2(20)$	8.479	5.9046	27.8532	21.226	23.675	23.00	47.487	44.056

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 11. Robustness Check: Non-Informative Priors Estimates for the SV Models

	Gold		Copper		Oil		Natural Gas	
	SV-t	SV-L	SV-t	SV-L	SV-t	SV-L	SV-t	SV-L
μ	0.009	0.014	0.018	0.011	0.037	0.016	0.018	0.044
	(0.007)	(0.008)	(0.013)	(0.014)	(0.019)	(0.019)	(0.035)	(0.035)
	[−0.006,0.024]	[−0.002,0.029]	[−0.008,0.044]	[−0.016,0.038]	[−0.001,0.074]	[−0.022,0.053]	[−0.050,0.088]	[−0.025,0.113]
μ_h	−0.383	−0.314	0.392	0.612	1.044	1.208	2.004	2.059
	(0.081)	(0.083)	(0.151)	(0.109)	(0.142)	(0.116)	(0.087)	(0.086)
	[−0.541,−0.223]	[−0.477,−0.152]	[0.093,0.691]	[0.401,0.829]	[0.993,1.424]	[0.980,1.440]	[1.833,2.175]	[1.893,2.230]
ϕ_{h_1}	0.966	0.970	0.993	0.987	0.991	0.986	0.978	0.977
	(0.003)	(0.004)	(0.002)	(0.003)	(0.002)	(0.002)	(0.004)	(0.004)
	[0.961,0.972]	[0.963,0.977]	[0.989,0.996]	[0.980,0.991]	[0.987,0.995]	[0.981,0.991]	[0.970,0.986]	[0.969,0.985]
ω_h^2	0.063	0.049	0.008	0.017	0.011	0.018	0.018	0.019
	(0.001)	(0.005)	(0.001)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)
	[0.061,0.065]	[0.039,0.058]	[0.006,0.011]	[0.012,0.023]	[0.007,0.015]	[0.014,0.023]	[0.013,0.023]	[0.014,0.026]
ν	12.677	-	9.120	-	12.088	-	45.121	-
	(1.107)		(0.906)		(1.701)		(20.571)	
	[10.748,15.084]		[7.537,11.086]		[9.403,16.047]		[18.446,92.466]	
ρ	-	0.196	-	−0.103	-	−0.266	-	0.231
		(0.032)		(0.054)		(0.055)		(0.058)
		[0.123,0.255]		[−0.216,−0.008]		[−0.364,−0.163]		[0.114,0.348]
$LogL$	−11222.7	−11325.5	−14839.5	−14898.0	−15600.4	−15621.0	−14469.9	−14468.2
	(1.07)	(0.17)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)
$Q(20)$	21.280	22.180	20.843	22.698	17.372	17.742	34.304	33.962
$Q_2(20)$	28.485	56.579	16.696	29.954	25.881	25.449	49.150	51.158

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

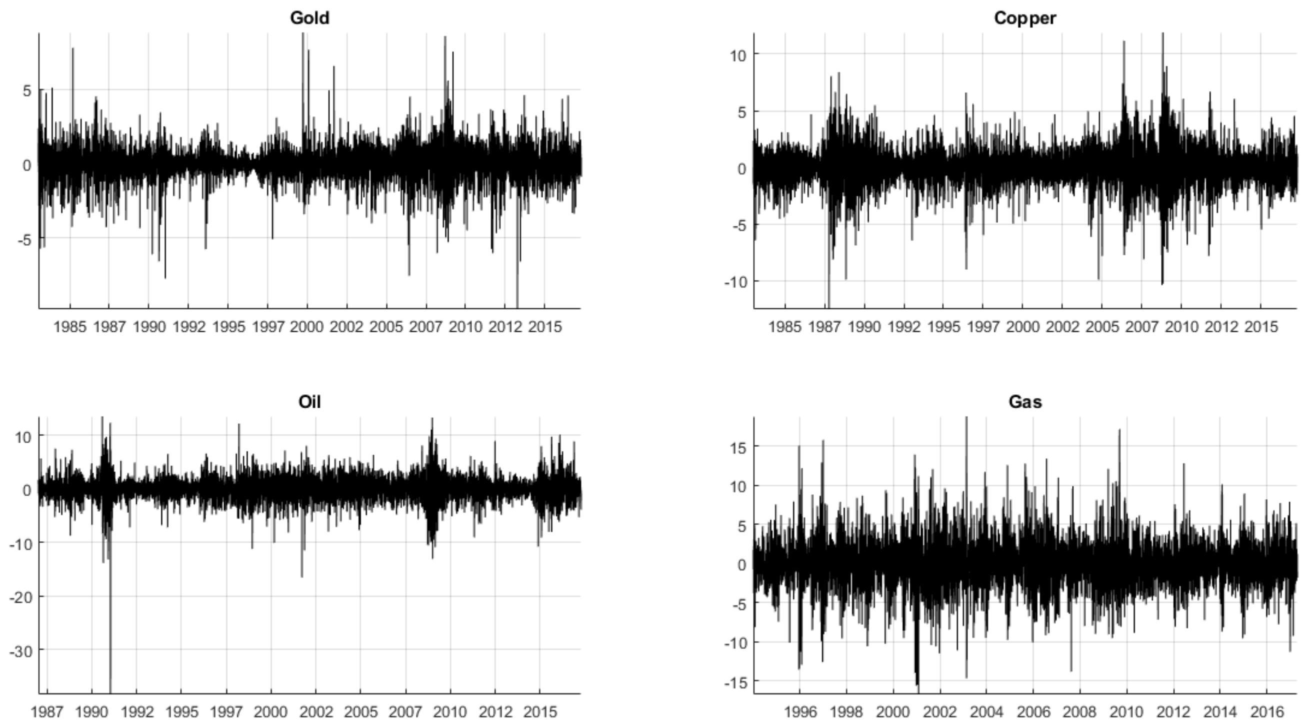

Figure 1. Commodity Daily Returns

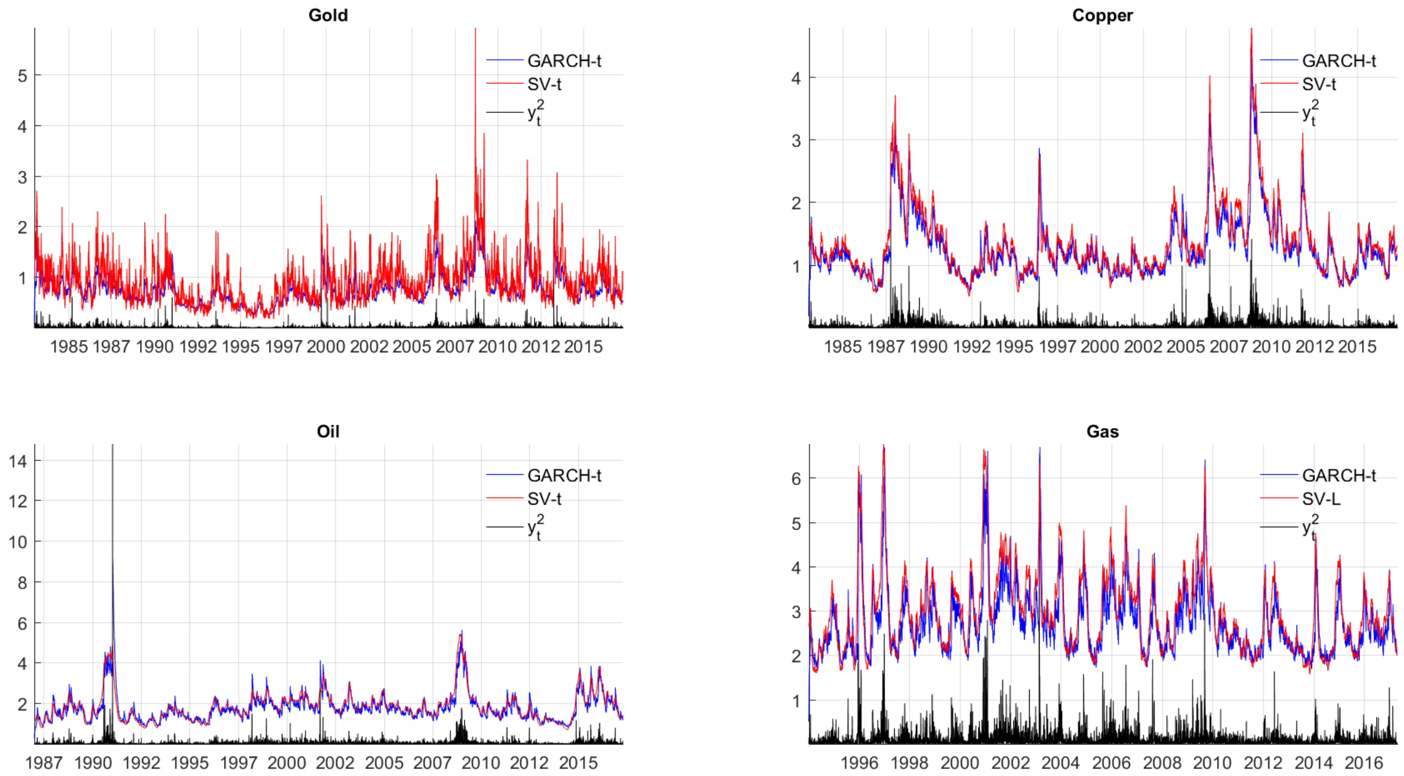

Figure 2. GARCH and SV Estimated Volatilities and Squared Returns

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