Quantification and Epistemic Modality

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1. A lottery puzzle

Imagine that there is a lottery with only two tickets, a blue ticket and a red ticket. The tickets are also numbered 1 through 2, but we don't know which color goes with which number. The winner (there is only one) has been drawn, and we know that the blue ticket won. But since we don't know whether the blue ticket is ticket #1 or ticket #2, we don't know the number of the winning ticket.[1]

We now reason as follows (in what follows, *might* is to be read epistemically):

(1) Ticket #1 is such that it might be the winning ticket. $(\lambda x.\Diamond x = w)(t_1)$

(2) Ticket #2 is such that it might be the winning ticket. $(\lambda x.\Diamond x = w)(t_2)$

(3) Those are all the tickets. $\forall x(x = t_1 \vee x = t_2)$
So:

(4) Any ticket might be the winning ticket. $\forall x \Diamond x = w$
So:

(5) The red ticket is such that it might be the winning ticket. $(\lambda x.\Diamond x = w)(r)$

Two comments:

- On a static, truth-conditional theory, the validity of each inference follows given fairly modest assumptions about the semantics of the *non-modal fragment* of the language.
- Neither inference remains valid (according to standard theories) if these *de re* modal predications are replaced with their *de dicto* counterparts. Sentence (4), for example, does not entail (5′) according to standard theories:

  (5′) It might be that the red ticket is the winning ticket. $\Diamond r = w$

Two possible responses

(i) Accept (5). But what about:

  (6) The red ticket is such that it might be the blue ticket.
  (7) The losing ticket is such that it might be the winning ticket. (Yalcin, 2015)
  (8) The losing ticket is such that we might discover that it is the winning ticket.
  (9) The losing ticket might turn out to be the winning ticket.

(ii) One of (1) and (2) is false, but we don't know which. But what about:

  (10) It might be the case that ticket #1 is the winning ticket, but we don't know whether or not ticket #1 is such that it might be the winning ticket.

[1] Examples with this structure were first discussed in Aloni (1997, 2001) and Gerbrandy (1997, 1998). Moss (2016) discusses similar phenomena.

Also: given relatively modest assumptions, one of the following is predicted to be true:

(11) The blue ticket is such that it might not be the blue ticket.

(12) Ticket #1 is such that it might be ticket #1.

A Quinean insight:

- Suppose that it turns out that ticket #2 is the losing red ticket. Let β be that ticket. Is β such that it might be the winning ticket?
- "being necessarily or possibly thus and so is not a trait of the object concerned, but depends on the manner of referring to the object" (Quine, 1953, 148).

How do we implement the Quinean insight? Two points:

(i) Standard dynamic semantics does not (fully) incorporate the Quinean insight (*contra* Yalcin (2015)).

(ii) The Quinean insight requires us to adopt a non-standard (i.e. non-Kripkean) account of *transworld identification* (e.g. counterpart theory).

2. Dynamic semantics

Dynamic semantics (see appendix for details):

- In dynamic semantics, the meaning of a sentence is not given by the conditions under which it is true, but by its capacity to *update or change a state of information.*
- A state of information is typically construed as a set of possibilities of some kind.
- Given a state of information s and a sentence ϕ, $s[\phi]$ is the result of updating s with ϕ. So "$[\phi]$" denotes a function from states of information to states of information.
- A state of information s *supports* a sentence ϕ just in case updating s with ϕ simply returns ϕ, i.e. just in case $s[\phi] = s$. information.
- An inference is *valid* just in case any state of information that supports the premises supports the conclusion.

Fact 1. *A state s supports (4) just in case, for every object o in the domain, there is a possibility $i \in s$ at which o is the winning ticket.*

$$s[\forall x \Diamond x = w] = \begin{cases} s & \textit{if for all } o \in \mathcal{D}, \textit{ there is an } i \in s \textit{ such that } o = i(w); \\ \emptyset & \textit{otherwise.} \end{cases}$$

Here, $i(w)$ is the extension of w at i.

Fact 2. *The de dicto-de re distinction for individual constants collapses:*

$$s[(\lambda x.\Diamond\phi)(a)] = s[\Diamond\phi(a/x)]$$

From this and the semantics for the modal operator, we have:

Fact 3.

$$s[(\lambda x.\Diamond x = w)(a)] = \begin{cases} s & \textit{there is an } i \in s \textit{ such that } i(a) = i(w); \\ \emptyset & \textit{otherwise.} \end{cases}$$

Some predictions:

- Good news! In the lottery scenario, our state of information supports (1)–(3), but not (5).
- Bad news? Sentences (1)–(3) do not entail (5).

  Both of our inferences – the inference from (1)–(3) to (4) and the inference from (4) to (5) – fail.
- To see why the inference from (4) to (5) fails, consider a domain with two elements, α and β, our two tickets. Consider a state of information s with two possibilities, i and i', which can be depicted as follows:

i	i'
α: #1, blue, winner	α: #1, red, loser
β: #2, red, loser	β: #2, blue, winner

- Terrible news :(The following contradictory-sounding sentences are predicted to be consistent:

  (13) Although any ticket might be the winning ticket, the red ticket is such that it cannot be the winning ticket. $(\forall x \Diamond x = w) \wedge (\lambda x.\neg\Diamond x = w)(r)$

  (14) Although the blue ticket is such that it must the be the winning ticket, no ticket is such that it must be the winning ticket. $(\lambda x.\Box x = w)(b) \wedge (\neg\exists x \Box x = w)$

3. Counterpart theory

Counterpart theory:

- Given a Kripke model $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{D}, \mathcal{I} \rangle$, a *counterpart relation* $\mathcal{K}$ is a reflexive binary relation on $\mathcal{W} \times \mathcal{D}$.
- For any world $v \in \mathcal{W}$, variable assignment g on $\mathcal{M}$, and counterpart relation $\mathcal{K}$ on $\mathcal{M}$:

  $[\![\Diamond\phi]\!]^{v,g,\mathcal{K}} = 1$ iff there is a $v' \in \mathcal{W}$ such that $v\mathcal{R}v'$ and $[\![\phi]\!]^{v',g',\mathcal{K}} = 1$, for some assignment g' such that, for each free variable x in ϕ, $\langle g(x), v\rangle \mathcal{K} \langle g'(x), v'\rangle$.
- The parameter $\mathcal{K}$ here can be thought of as the counterpart relation provided by the context.

Color counterparts and number counterparts:

- A ticket t in world w bears the color counterpart relation to a ticket t' in world w' iff the color of t in w is the color of t' in w'.
- A ticket t in world w bears the number counterpart relation to a ticket t' in world w' iff the number of t in w is the number of t' in w'.

Sentences (1) and (2):

- Ordinarily, when we utter (1) or (2), the utterance context will deliver the *number* counterpart relation.
- Why? Because those sentences pick out the tickets via their numbers, and this will tend to make that particular counterpart relation salient.
- And given that counterpart relation, (1) and (2) will both be true.

Sentence (5):

- Ordinarily, when we utter (5), the utterance context delivers the color counterpart relation. Because that sentence picks out a ticket via its color, and this will tend to make salient the color counterpart relation.
- And given that counterpart relation, (5) will be false.
- So even though the argument from (1)–(3) to (5) is valid, the premises tend to be evaluated in a context in which they are true, whereas the conclusion tends to be evaluated in a context in which it is false.

4. Variation

Imagine that there is a lottery with three tickets. Either there are two blue tickets and a red ticket, or two red tickets and a blue ticket – we don't know which. And either there are two circular tickets and one triangular ticket, or there are two triangular tickets and one circular ticket – we don't know which. And we don't know how the colors correspond to the shapes, i.e. any way of matching colors to shapes consistent with what we've said is compatible with what we know.

We know that a blue ticket won, but because we don't know how the colors correspond to the shapes, we don't know the shape of the winning ticket.

We now reason as follows:

(15) Any circular ticket might be the winning ticket. $\forall x(Cx \supset \Diamond x = w)$

(16) Any triangular ticket might be the winning ticket. $\forall x(Tx \supset \Diamond x = w)$

(17) Every ticket is circular or triangular. $\forall x(Cx \vee Tx)$

So:

(18) Any ticket might be the winning ticket. $\forall x \Diamond x = w$

So:

(19) Any red ticket might be the winning ticket. $\forall x(Rx \supset \Diamond x = w)$

Appendix: dynamic semantics

What follows is a modified version of the semantics presented in Groenendijk *et al.* (1997).

Assume a language $\mathcal{L}$ of quantified modal logic, with $=, \lambda, \neg, \wedge, \exists, \Diamond$ as the primitive logical symbols. The other logical symbols are defined in the usual way, e.g. $\forall x\phi$ is $\neg\exists x\neg\phi$.

Definition 1. A *dynamic model* $\mathcal{M}$ for $\mathcal{L}$ is a triple $\langle \mathcal{W}, \mathcal{D}, \mathcal{I} \rangle$, where $\mathcal{W}$ and $\mathcal{D}$ are non-empty sets, and $\mathcal{I}$ is an interpretation function that assigns to each individual constant a total function from W into D, and to each n-ary predicate P a total function from worlds to subsets of D^n.

Definition 2. A *possibility* on a dynamic model $\mathcal{M}$ is a *pair* of a world and a variable assignment. A *state of information* on a dynamic model $\mathcal{M}$ is a set of possibilities on $\mathcal{M}$.

Definition 3. Given a possibility $i = \langle v, g \rangle$ on a dynamic model $\mathcal{M}$:

(i) if P is an n-ary predicate, $i(P) = \mathcal{I}(P)(v)$,

(ii) if a is an individual constant a, $i(a) = \mathcal{I}(a)(v)$, and

(iii) if x is a variable, $i(x) = g(x)$.

Definition 4.

For any possibility $i = \langle v, g \rangle$ and object $o \in \mathcal{D}$, $i[x/o] = \langle v, g[x/o] \rangle$.

For any state s and object $o \in \mathcal{D}$, $s[x/o] = \{i[x/o] : i \in s\}$.

Definition 5.

For any possibility $i = \langle v, g \rangle$ and individual constant a, $i[x/a] = \langle v, g[x/i(a)] \rangle$.

For any state s and individual constant a, $s[x/a] = \{i[x/a] : i \in s\}$.

Definition 6. Let s be any state in any dynamic model $\mathcal{M}$. *The update of s by ϕ relative to $\mathcal{M}$*, $s[\phi]$, is defined as follows:

$$s[P(t_1, ..., t_n)] = \{i \in s : \langle i(t_1), ..., i(t_n) \rangle \in i(P)\}$$

$$s[\neg\phi] = s - s[\phi]$$

$$s[\phi \wedge \psi] = s[\phi][\psi]$$

$$s[(\lambda x.\phi)(a)] = \{i \in s : i[x/a] \in s[x/a][\phi]\}$$

$$s[\exists x \phi] = \{i \in s : \text{there is an } o \in D \text{ such that } i[x/o] \in s[x/o][\phi]\}$$

$$s[\Diamond\phi] = \begin{cases} s & \text{if } s[\phi] \neq \emptyset; \\ \emptyset & \text{otherwise.} \end{cases}$$

Definition 7. A state of information s on a dynamic model $\mathcal{M}$ *supports* a sentence ϕ just in case $s[\phi] = s$. An inference is *valid* just in case for every model $\mathcal{M}$ and every information state s on $\mathcal{M}$, if s supports the premises of the inference, s supports the conclusion of the inference.

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